CKM Note 57

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Overview of the CKM physics opportunities beyond $K^+ \to \pi^+ \nu\bar{\nu}$[1]

Abstract

A new experiment CKM is accepted by Fermilab for a very sensitive study of rare kaon decay $K^+ \to \pi^+ \nu\bar{\nu}$ on a very intense separated kaon beam. The high quality of the beam and CKM setup make it possible to perform an important set of other kaon decay measurements in parallel with the main research program: 1) the search for new physics effects beyond the Standard Model (search for new P, S, T interactions and lepton flavor violation in kaon decays); 2) further search for direct CP violation in kaon decays; 3) study of low energy hadron physics in pure conditions of decay processes ($K^+ \to \pi^+ l^+ l^-$; $\pi\pi l \nu_l$; $\pi^0\gamma\gamma$, etc). Expected results of these studies are compared to other experimental programs.

[1] The extended version of the author's talk on the Meeting of the CKM Collaboration, Univ. of Michigan, Ann Arbor, July 2001

1 Introduction

In 2001 a new experiment "Charge Kaon at Main injector" [1] (CKM, or E921) was accepted by Fermilab and is now in the stage of technical design and preparation. The main aim of this experiment is to measure the branching ratio of the ultra-rare charge kaon decay $K^+ \to \pi^+ \nu\bar{\nu}$ by observing approximately 10^2 events of this decay with small background.

The CKM experiment will open the possibility to determine the magnitude of the Cabibbo-Kobayashi-Maskava matrix element $|V_{td}|$ with statistical uncertainty of about 5% and total precision around 10%. This decay mode is known to be theoretically clean with only significant correction due to charm contribution (see details in [1] and in the references therein). It will play a crucial role in the testing of the Standard Model predictions for the mechanism of CP violation and for properties of FCNC processes.

Evidence for $K^+ \to \pi^+ \nu\bar{\nu}$ was obtained at BNL E787 experiment with kaon decays at the rest where one clear event of this decay was observed and the branching ratio $BR(K^+ \to \pi^+ \nu\bar{\nu}) = (1.5^{+3.4}_{-1.2}) \cdot 10^{-10}$ was measured [2]. In the new version of this experiment with upgraded setup (BNL E949), which is now beginning at BNL [3], it will be possible to obtain several events of $K^+ \to \pi^+ \nu\bar{\nu}$ in the kaon decays at the rest. But really large increase of statistics is expected in a new type of experiment — the CKM at Fermilab.

The experimental technique which will be used in this experiment is decay in flight in a $\sim$ 22GeV/c RF separated K^+-beam with kaon flux of 30MHz and a purity of higher than 70%. K^+'s are generated by about $5 \cdot 10^{12}$ 120GeV/c protons per spill from Main Injector. The CKM detector will have redundant measurements for both the primary K^+ and secondary π^+ with high rate magnetic spectrometers and RICH velocity spectrometers based on phototube ring imaging Cherenkov counters for each particle. A hermetic photon veto system capable of rejection π^0-decays from background processes with an inefficiency less than $1.6 \cdot 10^{-7}$ and muon range hodoscope veto system with inefficiency less than $1 \cdot 10^{-5}$ are also required. The expected signal acceptance of the CKM setup for $K^+ \to \pi^+ \nu\bar{\nu}$ after all cuts will be $\sim$ 2%. A date collection period of 2 years should yield a signal $\sim 10^2$ events over a background of less than 10 events (assuming theoretical branching ratio $BR(K^+ \to \pi^+ \nu\bar{\nu}) \simeq 1 \cdot 10^{-10}$).

The possible agreement between the results of independent measurements for kaon and B meson decay processes would implemented a very stringent test of the Standard Model CP violation mechanism associated with the complexness of the quark-mixing CKM matrix. But if there are some other CP violation mechanisms beyond the Standard Model these new mechanisms quite probably would affect the properties of different processes and all set of measurements will demonstrate these differences quite unambiguously.

But in any case it is important to stress here that quite recently in the BaBar and Belle experiments the study of charge asymmetry in $B^0_d(\bar{B}^0_d) \to \Psi K^0_S$ decay gave the value of $a_{\Psi K_S} = \sin 2\beta = 0.79 \pm 0.10$ [4] which is in a very good agreement with the Standard Model prediction. As I understand it is the first precision test of the CP violation in the Standard Model and may be of considerable importance in the future understanding of leading mechanisms of the CP violation. It increases also the significance of $K^+ \to \pi^+ \nu\bar{\nu}$ results of the CKM and BNL E949.

The study of $K^+ \to \pi^+ \nu\bar{\nu}$ is the main aim of the CKM experiment. But the large intensity of the charge kaon beam and high quality and capability of the apparatus open also the possibility to perform in parallel or in special short dedicated runs other important measurements in kaon physics. These possibilities are considered in some details below.

2 CKM apparatus

The general layout of the CKM setup is presented in Fig.1. The CKM detector includes:

1. Primary K^+ meson identification system and measuring double spectrometer with Upstream Magnetic Spectrometer (UMS) with proportional chambers and scintillation Kaon Entrance Angle Tracker (KEAT) and Kaon RICH velocity spectrometer (double redundant measurement of primarily kaon).

2. High vacuum 30m long decay region with quite effective photon Vacuum Veto System (VVS) and Beam Interaction Veto System (BIVS).

3. Downstream Magnetic Spectrometer (DMS) with straw tubes in vacuum and with two Exit Time Planes (ETP) of scintillation hodoscopes and Pion RICH velocity spectrometer (double redundant measurement of the secondary π^+).

4. High efficiency photon Forward Veto System (FVS) — γ spectrometer with high resolution CsI scintillation counters.

5. Muon Veto — hodoscopical rang stack system for muon rejection with inefficiency $< 10^{-5}$.

6. Photon Hole Veto System (PHVS) after additional magnet in the end of the setup to increase the hermetism of photon veto system.

The experiment will be performed on a new RF separated charge kaon beam with momentum 22GeV/c and full intensity 50MHz (30MHz kaons, 7.5MHz pions, 7.5MHz muons).

Photon Veto is characterized by high efficiency which is presented in Table 1. These values are important for estimation of background level for many processes.

Table 1: Required inefficiency performance of the CKM photon veto systems.

Energy Low-High (GeV)	(VVS) Inefficiency	(FVS) Inefficiency	(HVS) Inefficiency
0.0 - 0.02	4×10^{-1}	1	1
0.02 - 0.04	3×10^{-2}	5×10^{-1}	1
0.04 - 0.06	3×10^{-3}	5×10^{-2}	1
0.06 - 0.10	7×10^{-4}	5×10^{-3}	1
0.10 - 0.20	4×10^{-4}	2×10^{-3}	2×10^{-2}
0.20 - 0.40	1×10^{-4}	5×10^{-4}	5×10^{-3}
0.40 - 1.00	1×10^{-4}	2×10^{-4}	2×10^{-3}
1.00 - 10.00	3×10^{-5}	1×10^{-4}	2×10^{-3}

Table 2: Background estimates in terms of effective branching ratio.

Background Source	Effective BR ($\times 10^{-12}$)
$K^+ \to \mu^+\nu_\mu$	< 0.04
$K^+ \to \pi^+\pi^0$	3.7
$K^+ \to \mu^+\nu_\mu\gamma$	< 0.09
$K^+A \to K_L X$ followed by $K_L \to \pi^+ e^- \nu$	< 0.14
$K^+A \to \pi^+ X$ in trackers	<4.0
$K^+A \to \pi^+ X$ in residual gas (10^{-6} torr)	< 2.1
Accidentals (2 K^+ decays)	0.51
Total	< 10.6

The possibility of the CKM setup for background rejection for $K^+ \to \pi^+\nu\bar{\nu}$ are characterized by Table 2, in which the number of background events from different processes are presented. We see that this expected background is favorable for measuring of $BR(K^+ \to \pi^+\nu\bar{\nu})$ on the statistics of 10^2 events. The selection of $K^+ \to \pi^+\nu\bar{\nu}$ events and the suppression of $K^+ \to \pi^+\pi^0$ background is illustrated by Fig. 2. The expected result of the CKM experiment for the determination of the Standard Model (SM) unitary triangle parameters is presented in Fig. 3 together with the data of other kaon and B-meson experiments.

3 General description of other kaon decays

The CKM setup is characterized by excellent spatial and time resolution and includes two magnetic spectrometers and two RICH velocity spectrometers that will provide particle identification through precise velocity measurements. The CKM setup includes also quite efficient photon veto system, photon spectrometers, muon veto system and muon identification as well as quite complicated and reliable trigger and DAQ system of pipe line type. The setup will provide the redundant measurement capabilities necessary for contemporary rare kaon decay studies with sensitivity up to 10^{-12}.

While the CKM experiment will accept no compromises in either the sensitivity or background rejection required for the main $K^+ \to \pi^+\nu\bar{\nu}$ measurement, several other interesting physics measurements should be possible with this apparatus.

The main directions of these additional measurements are:

1. The search for new physical effects beyond the Standard Model (lepton flavor violation processes, search for new S, P and T weak interactions in $K^+ \to e^+\nu, e^+\pi^0\nu_e$ decays, etc.).

2. Further search for direct CP violation in K^+ decays (charge asymmetry in $K^\pm \to \pi^\pm\pi\pi; \pi^\pm\pi^0\gamma$; T-odd correlations in $K^+ \to \pi^0 l^+\nu_l\gamma$ decays). The observable effects in these searches are also possible only beyond the SM.

3. Low energy hadron physics studies in the pure conditions of kaon decays ($K^+ \to \pi^+l^+l^-; \pi^+\pi^0\gamma; \pi\pi l^+\nu_l; l^+\nu\gamma$, $\pi^0\gamma\gamma$, etc.). The importance of these investigations is connected with fundamental quark confinement problem and with further development of lattice QCD, CHPT and other hadron theories.

General description of the CKM experimental possibilities for studying of several kaon decays is illustrated by Table 3, which presents the efficiency calculations and expected statistical evaluation of these processes. For the efficiency estimations in Table 3 it was assumed that all decay photons are detected and measured in FVS system and all charge decay products are detected in DMS and can be identified (if it is necessary) in Pion RICH and muon system.

To increase the efficiency for photon detection in multiphoton events it was assumed that in some cases the VVS system can be used as a photon spectrometer with reduced precision for coordinate and energy measurements (see section 5). This question needs further GEANT simulation studies and experimental investigation for final justification.

At this stage we will not consider the trigger signals for the registration of several rare kaon decays in parallel with the main $K^+ \to \pi^+\nu\bar{\nu}$ measurements. As was clear from some preliminary discussions the future development of the CKM DAQ system will allow to use a very intensive data flux and will provide many possibilities to develop the higher trigger levels to select the desired processes.

It is instructive to compare the date of Table 3 with the possibilities of the OKA experiment on the separated charge kaon beam with momentum 12.5GeV/c which is now in preparation at the IHEP accelerator (Protvino, Russia) [5]. The corresponding sensitivity estimations for the OKA measurements are presented in Table 4 (see [5]).

As is seen from this Table 3 and Table 4, the expected decay rate at the Fermilab kaon beam ($3.6 \cdot 10^{11}\, K^+$-decay/week) exceeds the same value for IHEP beam ($1.6 \cdot 10^{10}\, K^+$-decay/week) by factor ~ 20.

Additional factor ~ 2.5 can be expected from longer duration of the CKM runs. But the acceptance of the CKM for most of additional measurements are smaller than in the OKA by factor $\frac{1}{4} \div \frac{1}{10}$ (especially for multiphoton events).

Thus the expected ratio of integral detected decay numbers for these two experiments is N(CKM)/N(OKA) $\sim 10 \div 15$ in favor of the CKM. But wider acceptance of the OKA setup gives some advantages in studying of the characteristics for some decay channels in this experiment.

The comparison of the future possibilities of the CKM and OKA allows one to make several conclusions:

1. CKM experiment has definite advantage in the searches of new processes like lepton flavor nonconservation and new mechanisms of CP violation in charge kaon decays.

2. The advantage in the acceptance of the OKA setup makes it possible to do more careful studies of the Dalitz plot distributions, formfactors and other properties for decays with intermediate branching

Table 3: The efficiency for the detection and sample size of several kaon decay modes in the CKM detector.

Decay	BR	Efficiency	Events/week
$K^+ \to \pi^0\mu^+\nu\gamma$	$\sim 2\times10^{-5}$	0.0080	5.8×10^4
$K^+ \to \pi^0 e^+\nu\gamma$	$(2.62\pm0.20)\times10^{-4}$	0.022	2.1×10^6
$K^+ \to \pi^+\pi^+\pi^-$	$(5.59\pm0.05)\times10^{-2}$	0.20	4.0×10^9
$K^+ \to \pi^+\pi^0\pi^0$	$(1.73\pm0.04)\times10^{-2}$	0.0040	2.5×10^7
$K^+ \to \pi^+\pi^\gamma$	$(2.75\pm0.15)\times10^{-4}$	0.00112	1.1×10^6
$K^+ \to \pi^+e^+e^-$	$(2.88\pm0.13)\times10^{-7}$	0.012	1.2×10^3
$K^+ \to \pi^+\mu^+\mu^-$	$(7.6\pm2.1)\times10^{-8}$	0.086	2.3×10^3
$K^+ \to \mu\nu\gamma$	$(5.50\pm0.28)\times10^{-3}$	0.050	1.0×10^8
$K^+ \to \pi^+\pi^-e^+\nu$	$(3.91\pm0.17)\times10^{-5}$	0.066	9.4×10^5
$K^+ \to \pi^0\pi^0e^+\nu$	$(2.1\pm0.4)\times10^{-5}$	0.0093	7.2×10^4
$K^+ \to \pi^+\pi^-\mu^+\nu$	$(1.4\pm0.9)\times10^{-5}$	0.28	1.4×10^6
$K^+ \to \pi^0\pi^0e^+\nu$	$\sim 0.7\times10^{-5}$	0.0024	6.1×10^3
$K^+ \to \pi^+\pi^0\gamma$	$(2.75\pm0.15)\times10^{-4}$	0.011	1.1×10^6

Notes:

a) For expected statistics evaluation we used MC values for efficiencies of different decays, the K^+ flux $3\cdot10^7\,K^+$/spill, decay region of 30m (17% K^+ decays) and effective number of spill/week $1.4\cdot10^5$ (70% effective time). These numbers correspond to $0.71\cdot10^{12}\,K^+$ decays/week. To take into consideration a dead time effects (30%) and possible additional cuts to suppress background we use in these estimations the reduced effective number $0.36\cdot10^{12}\,K^+$ decays/week.

b) In the efficiency calculations it was assumed that all photons are measured in the FVS. If some of the photons in $\pi^0\pi^0$ decays are measured in VVS the efficiency of corresponding process will increase in $2-3$ times.

ratios (like $K^+\to\pi\pi l^+\nu_l$, $K^+\to\pi^+\pi^0\gamma$, search for scalar and tensor interactions in $K^+\to\pi^0 l^+\nu_l$, etc).

3. Thus both experiments are complimentary to each other and together will give the opportunity for careful studies of many decays. Besides it is well to bear in mind that the OKA experiment will begin for several years earlier that the CKM. A large group of physicists are participating in both experiments and that gives us a suitable opportunity to obtain in the OKA measurements first physical results for several decay channels, important information about the separation criteria, the background situation and systematics for some processes under investigation and to continue these studies on a new level in the CKM experiment.

Now we will discuss several of kaon decay processes in more details (see also [1, 6, 7]).

4 Lepton flavor violating processes in K^+ decays in CKM

The main kaon decay processes with lepton flavor violation (LFV) are summarized in Table 5 where upper limits for these decays obtained mainly in E865 BNL experiment [8, 9] are presented. For completeness we also included in this table the last LFV results from K_L^0 decays [10–12]

From the statistical consideration there is a clear opportunity for the CKM experiment to perform (in parallel with the main $K^+\to\pi^+\nu\bar\nu$ study) much more sensitive searches of all those K^+ LFV decays up to the level $BR\sim10^{-12}$. These limits correspond to effective number of K^+ decays in 2 years of measurement $N(K^+\text{ decays}) = (3.6{\cdot}10^{11}\text{ decays/week}){\cdot}(78\text{ weeks}) = 2.81{\cdot}10^{13}(K^+\text{ decays})$. These numbers include a safety

Table 4: The efficiency for registration of several kaon decay channels for the OKA experiment on the separated $K^{\pm}$ beams of IHEP accelerator [5]

BR		New magnet		Old SPHINX magnet		New magnet
		ϵ_1	(events/week)$_1$	ϵ_2	(events/week)$_2$	events/5.76×10^{11} K^+ decays (36 weeks of measurements)
$K^+ \to \pi^+\pi^+\pi^-$	$(5.59 \pm 0.05) \times 10^{-2}$	0.76	6.8×10^8	0.64	5.7×10^8	2.4×10^{10}
$K^+ \to \pi^+\pi^0\pi^0$	$(1.73 \pm 0.04) \times 10^{-2}$	0.47	1.3×10^8	0.32	8.9×10^7	4.7×10^9
$K^+ \to \mu^+\nu_\mu\gamma$	$(5.50 \pm 0.28) \times 10^{-3}$	0.54	4.8×10^7	0.43	3.8×10^7	1.7×10^9
$K^+ \to \pi^0\mu^+\nu_\mu\gamma$	2×10^{-5}	0.44	1.4×10^5	0.30	9.6×10^4	5.1×10^6
$K^+ \to \pi^0 e^+\nu_e\gamma$	$(2.62 \pm 0.20) \times 10^{-4}$	0.24	1.0×10^6	0.16	6.7×10^5	3.6×10^7
$K^+ \to \pi^+\pi^0\gamma$	$(2.75 \pm 0.15) \times 10^{-4}$	0.47	2.1×10^6	0.32	1.4×10^6	7.4×10^7
$K^+ \to \pi^+\pi^- e^+\nu_e$	$(3.91 \pm 0.17) \times 10^{-5}$	0.27	1.7×10^5	0.21	1.3×10^5	6.1×10^6
$K^+ \to \pi^0\pi^0 e^+\nu_e$	$(2.1 \pm 0.4) \times 10^{-5}$	0.18	6.0×10^4	0.12	4.0×10^4	2.2×10^6
$K^+ \to \pi^+\pi^-\mu^+\nu_\mu$	$(1.4 \pm 0.9) \times 10^{-5}$	0.73	1.6×10^5	0.61	1.4×10^5	5.9×10^6
$K^+ \to \pi^0\pi^0\mu^+\nu_\mu$	$\sim 0.7 \times 10^{-5}$	0.44	4.9×10^4	0.30	3.4×10^4	1.8×10^6
$K^+ \to e^+\nu_e$	$(1.55 \pm 0.07) \times 10^{-5}$	0.53	1.3×10^5	0.44	1.1×10^5	4.7×10^6
$K^+ \to \pi^0 e^+\nu_e$	$(4.82 \pm 0.06) \times 10^{-2}$	0.32	2.5×10^8	0.23	1.8×10^8	8.9×10^9
$K^+ \to \pi^0\mu^+\nu_\mu$	$(3.18 \pm 0.08) \times 10^{-2}$	0.48	2.4×10^8	0.34	1.7×10^8	8.8×10^9
$K^+ \to \pi^+ e^+ e^-$	$(2.88 \pm 0.13) \times 10^{-7}$	0.12	5.5×10^2	0.07	3.2×10^2	2.0×10^4
$K^+ \to \pi^+\mu^+\mu^-$	$(7.6 \pm 2.1) \times 10^{-8}$	0.43	5.2×10^2	0.29	3.5×10^2	1.9×10^4

Notes:

a) For expected statistics evaluation we used MC estimations for the efficiencies of different decay modes, K^+ flux 5×10^6 K^+/spill; decay region 12 m (12 % K^+ decay probability) and effective number of spill/week 5.2×10^4 (70 % effective time). These numbers correspond to 3.2×10^{10} K^+ decays/week. To take into consideration effects of dead time losses and additional cuts to suppress background we will use in the effective number calculations the effective flux 1.6×10^{10} K^+ decays/week, reduced by additional factor two.

b) To estimate the decay rates in the OKA setup we used geometry with old SPHINX magnet (events/week)$_2$ and with possible new magnet (events/week)$_1$.

c) It must be bear in mind that with a new magnet it will be possible to realize more effective setup arrangement and to obtain additional gains in the decay rates which were not considered in this table.

d) In the efficiency calculations it was assume that all photons for secondary particle identification would be measured in two lead glass γ-spectrometers of the OKA setup. The effective photon veto system would not be used for photon spectroscopy aims.

factor 0.5 to take into account dead time effects and additional cuts. Certainly very careful investigations of possible background limitations for these searches must be done. Some of these works have already begun.

Even sensitivity of 10^{-12} for kaon LFV processes looks now modest to compare with ambitious new proposals for the search for $\mu \to e\gamma$ and $\mu^- + (A,Z) \to e^- + (A,Z)$ LFV processes on the level $BR < 10^{-14} \div 10^{-16}$ [13]. The difference in the sensitivities of future muon and kaon LFV experiments would be even larger due to additional factor $\tau_\mu/\tau_K \sim 10^2$ included in the limitations for the corresponding partial decay widths: $\Gamma(\text{kaon LFV})/\Gamma(\text{muon LFV}) = [BR(\text{kaon LFV})/BR(\text{muon LFV})]\cdot(\tau_\mu/\tau_K) \simeq (10^2 \div 10^4)\cdot 10^2 \simeq 10^4 \div 10^6$.

But it is very important to stress that the searches for LFV in kaon decays are complimentary to the $\mu \to e\gamma$ type of the experiments. Kaon decays are the only processes which are sensitive to $s \to d\mu\bar{e}$ interactions which may be exhibit of very exceptional properties and open an unique possibility to study

Table 5: Lepton flavor violation in kaon decays.

Mode	BR limit	Expected CKM BR limits
$BR(K^+ \to \pi^+\mu^+e^-)$	2.8×10^{-11} [8]	2.2×10^{-12}
$BR(K^+ \to \pi^+\mu^-e^+)$	5.2×10^{-10} [9]	2.2×10^{-12}
$BR(K^+ \to \pi^-\mu^+e^+)$	5.0×10^{-10} [9]	2.2×10^{-12}
$BR(K^+ \to \pi^-\mu^+\mu^+)$	3.0×10^{-9} [9]	7.2×10^{-13}
$BR(K^+ \to \pi^-e^+e^+)$	6.4×10^{-10} [9]	5.6×10^{-12}
$BR(K_L^0 \to \mu e)$	4.7×10^{-12} [10]	
$BR(K_L^0 \to \pi^0\mu e)$	4.4×10^{-10} [11]	
$BR(K_L^0 \to e^\pm e^\pm \mu^\mp \mu^\mp)$	1.23×10^{-10} [12]	

lepton flavor violating decays with more favorable conditions for a hypothetical selection rule connected with a fundamental generation number conservation. In other words, for these decays a change of generation in the leptonic sector can be compensated by changing of generation in the quark sector ([14], see also [15]). Let us introduce the generation number $G_1 = 1$ for the first fundamental fermion generation and $G_2 = 2$ for the second one (for antifermions $G_1 = -1; G_2 = -2$). Then, as is seen from diagrams in Fig.4, it is possible to classify different processes according to changing $\triangle G$, as it follows from Table 6. The strength of hypothetical $\triangle G$ selection rule is unclear now. Certainly, the new possibilities for the search for LFV processes in kaon decays will occur only if this strength is important and every next order on $\triangle G$ will lead to significant suppression of corresponding process to compare with lower order. We must bear in mind an unexpected difference between $K^+ \to \pi^+e^-\mu^+$ and $K^+ \to \pi^+e^+\mu^-$ in the model with conserved generation number according to $\triangle G$ selection rule.

Table 6: Classification of LFV processes by change of generation quantum number ΔG in the model [14].

First order	$\lvert\Delta G\rvert = 0$	$K^+ \to \pi^+\mu^+e^-$; $K_L^0 \to \mu^\mp e^\pm$; $K_L^0 \to \pi^0\mu^\mp e^\pm$
Second order	$\lvert\Delta G\rvert = 1$	$\mu \to 3e$; $\mu \to e\gamma$; $\mu^-N \to e^-N$
Third order	$\lvert\Delta G\rvert = 2$	$K^0 \to K^0$ $(\Delta M(K_1 - K_2))$; $\mu^-e^+ \to \mu^+e-$; $K^+ \to \pi^+\mu^-e^+$

In the diagrams in Fig.4 the decay processes are governed by exchange of "horizontal" intermediate boson H^0. But it is also possible to consider kaon decay LFV processes which are governed by exchange of leptoquarks Y^* (see Fig.5).

Amplitudes for $s \to d\mu\bar{e}$ process of pseudoscalar and axial-vector type (for $K_L^0 \to \mu\bar{e}$ decay) or with vector, scalar and tensor structure (for $K \to \pi\mu\bar{e}$ decay) with the "horizontal" H^0 boson exchange can be presented with the help of general generation number conserving operators [16]

$$\left.\begin{array}{r} Q_{V,A} = \frac{g_H^2}{2M_H^2}\bar{d}\gamma_\mu[C_{Lq}P_L + C_{Rq}P_R]s \quad \cdot \quad \bar{\mu}\gamma^\mu[C_{Ll}P_L + C_{Rl}P_R]e + \text{h.c.} \\ Q_{S,P} = \frac{g_H^2}{2M_H^2}\bar{d}[C'_{Lq}P_L + C'_{Rq}P_R]s \quad \cdot \quad \bar{\mu}[C'_{Ll}P_L + C'_{Rl}P_R]e + \text{h.c.} \end{array}\right\} \qquad (1)$$

Here $P_L = \frac{1}{2}(1+\gamma_5); P_R = \frac{1}{2}(1-\gamma_5); C, C'$ — constants.

The results of calculation of branching ratios $K \to \mu e$ and $K \to \pi\mu e$ decays for interactions (1) are presented for illustration on Fig. 6 as function of the scale parameter M_H (for $g_H = g$ — a weak constant corresponding $C, C' = 1$).

There are other interesting models for $s \to d\mu e$ processes in the supersymmetry theories with R-parity nonconservation [17, 18], in left-right models with heavy Majorana neutrino [19] in leptoquark exchange model [20].

LFV processes are known to take place both in the minimal supersymmetric model (MSSM) and in its modifications which include violation of R-parity. As for the context of the MSSM, rare kaon decays allow one to correlate squark and slepton mixing, while in R-violating supersymmetry one can probe non-trivial products of Yukawa couplings.

Within framework of the MSSM the magnitudes of the predicted rates depend on the details of the masses and mixings of sparticles including the sneutrinos. Maximal contributions for kaon LFV decays are expected from the box diagrams involving chargino and neutralino exchange.

However, the universal GUT-scale parameters for sfermions and gauginos, when limited from the similar pure lepton decays $\mu \to e\gamma$ and $\mu \to e^+e^+e^-$, lead to a hopelessly low kaon LFV decay branchings in the order of $10^{-16} \div 10^{-18}$, and only fine tuning of GUT parameters makes it of interest the search for lepton flavor violating K-decays.

But the situation can be much different in R-parity violating supersymmetry models as pointed out in [17, 18]. In this case the gauge symmetry of the SUSY model allows additional Yukawa coupling terms in the superpotential of the form $\lambda L_i L_j \bar{E}_k$, $\lambda' L_i Q_j \bar{D}_k$, $\lambda'' \bar{U}_i \bar{D}_j \bar{D}_k$.

Here the $L(Q)$ are the left-handed lepton (quark) superfields and the $\bar{E}(\bar{D}, \bar{U})$ are corresponding right-handed fields. Each of these terms give three Yukawa couplings. For example the first term is

$$\lambda_{ijk}(L_i L_j \tilde{\bar{E}}_k + L_i \tilde{L}_j \bar{E}_k + \tilde{L}_i L_j \bar{E}_k)$$

It may be seen that this terms involve a single SUSY state and breaking the R-parity of MSSM. For electroweak SU(2) and color SU(3) symmetry it was shown that there are 45 R-violating couplings. Certainly, there are many experimental constrains on this set of constants (nonobservation of proton decay and some exotic processes, lepton universality, nonobservation of possible modification of different SM processes). In spite of these limitations it was stated in [17] that it is still possible that several R-violating operators may be large enough and useful limitations for some of them can be obtained from existing upper limits for LFV kaon decays and from their future searches. In this class of models, whilst $\mu \to e\gamma$ occurs at the one loop level, $\mu \to 3e, \mu \to e$ conversion, $K_L^0 \to \mu^\pm e^\mp$ and $K \to \pi\mu^\pm e^\mp$ decays may occur at the tree level via different combinations of couplings. Thus LFV kaon decays have the possibility to provide an important complimentary information for this R-parity violating processes.

For the illustration it is possible to consider $d\bar{s} \to l_j^- l_k^+$ interaction with effective Lagrangian written in terms of quark and lepton fields [17]

$$\begin{aligned} L_{d\bar{s}\to l_j^- l_k^+} &= \frac{1}{m^2_{\tilde{\nu}_i}}[\lambda^*_{ijk}\lambda'_{i12}(\bar{s}_R d_L)(\bar{l}_{jL} l_{kR}) + \lambda_{ikj}\lambda'^*_{i12}(\bar{s}_L d_R)(\bar{l}_{jR} l_{kR})] - \\ &- \frac{\lambda'^*_{ji1}\lambda'_{ki2}}{2m^2_{\tilde{u}_i}}(\bar{s}_R\gamma_\mu d_R)(\bar{l}_{jL}\gamma^\mu l_{kL}) \end{aligned} \quad (2)$$

The tree diagrams for $K^0 \to l_j^- l_k^+$ and $K \to \pi l_j^- l_k^+$ decays are presented on Fig.7.

The calculations with (2) and upper limits for $K_L^0 \to \mu^\pm e^\mp$ decays (Table 5) give limitations for the values of $\lambda\lambda'/m_{\tilde{\nu}}$ and $\lambda\lambda'/m_{\tilde{u}}$ with sneutrino ($\tilde{\nu}$) and up squark ($\tilde{u}$) contributions

$$\left.\begin{aligned} \lambda_{i21}\lambda'_{i12}(\tfrac{100GeV}{m_{\tilde{\nu}}}) &\le 6.2\cdot 10^{-9} \\ \lambda_{i21}\lambda'_{i12}(\tfrac{100GeV}{m_{\tilde{u}}}) &\le 1.9\cdot 10^{-7} \end{aligned}\right\} \quad (3)$$

From $K \to \pi l_j^- l_k^+$ decays the upper limits for $\lambda\lambda'/m_{\tilde{\nu}}$and $\lambda\lambda'/m_{\tilde{u}}$ are by 1-2 orders of magnitude larger than in (3).

It is also important to mention the unique properties of double muon decay process $K^+ \to \pi^-\mu^+\mu^+$ which were discussed [18, 21–23] in the model of cascade kaon decays with rather heavy Majorana neutrino (with mass $\sim$ 300 MeV) $K^+ \to \nu_h + \mu^+; \nu_h \to \pi^-\mu^+$ and with small admixture of ν_h to the muon neutrino ν_μ (this small admixture based for the well-known sea-saw mechanism). These is also possible to use $K^+ \to \pi^- e^+ e^+$ decay for the search of the same type Majorana neutrino in the processes $K^+ \to \nu_h e^+; \nu_h \to \pi^- e^+$ (due to small admixture of ν_h in ν_e based also on sea-saw mechanism — see [22]).

Other possibilities for double muon decay of kaons (with double charge Higgs boson exchange [24] and with heavy Majorana neutrino mechanism [23]) were also considered.

Thus, further searches for LFV kaon decays are very important since they can offer complementary information for possible theories with lepton flavor nonconservation which is not covered even with very sensitive new projects on the LFV searches in muon processes.

It is my opinion that because of unique character of LFV kaon decay process it is very important to consider a new generation of such studies on a new high current accelerators with energy of $\sim 20-70$ GeV and with very high acceptance and precision setups with maximal redundancy measurements for background subtraction to increase the existing sensitivity in Table 5 by $2-3$ order of magnitude for the future possibilities. These very difficult experiments must be considered as an important part of further efforts to study lepton nonconservation and kaon physics.

But in the end of this paragraph we will try to achieve more modest aims and to study several possibilities of the search for kaon decays with lepton flavor violation in the framework of the CKM experiment. As was shown in Table 5 the expected statistical sensitivity of the CKM for the search for LFV kaon decays is on the level of $BR \sim 10^{-12}$. But the real sensitivity can be reduced due to background for these searches. Here we consider only one example — the decay $K^+ \to \pi^-\mu^+\mu^+$ were the sensitivity of E865 was the worst one because of the background from $K^+ \to \pi^-\pi^+\pi^+$ with two pion decays in flight ($\pi^+ \to \mu^+\nu$). The sensitivity of E865 is limited by this background — see Fig. 8. We studied this background for the CKM setup with CKM GEANT MC simulation on the statistics of 10^7 $K^+ \to \pi^-\pi^+\pi^+$ events with forced $\pi^+ \to \mu^+\nu$ decays for both π^+ and with corresponding weights ($\sim 10^{-2}$) for proper normalization. We also performed MC simulation of the main LFV decay under study $K^+ \to \pi^-\mu^+\mu^+$.

As a first step we used "standard cuts" (some reasonable cuts on χ^2 values for the quality of the reconstructed tracks and decay vertexes, cuts for the secondary particles energies in the magnetic spectrometer and calorimeters, and a cut on total transverse momentum $P_T^2 < 1.8 \cdot 10^{-4} GeV^2$($P_T$ — sum of transverse momenta of secondaries compared to incident K^+ meson). These results are presented on Fig. 9a. They correspond to the expected sensitivity (controlled by background level) $BR(K^+ \to \pi^-\mu^+\mu^+) < 2.5 \cdot 10^{-9}$ (90% c.l.), that is on the same level as in E865 [9].

For further suppression of this background we used 2 additional cuts:

a) In the assumption that all 3 charged secondary particles are pions we calculate the effective mass $M_{3\pi}$ and reject the events with $M_{3\pi} < 0.52$GeV (for the signal reaction assigning a pion mass to muon tracks produces a mass spectrum lying above this threshold). This cut reduces the secondary pion decays after DMS spectrometer. The results with this cut in addition are presented in Fig. 9b. The background is reduced significantly, but in the dangerous region of $M(\pi^-\mu^+\mu^+)$ it practically does not change.

b) We performed a double measurement of the momentum of a fast muon (with $P_\mu \gtrsim 10$GeV what is over the threshold for the Pion RICH counter) in the DMS and in the velocity RICH spectrometer and we selected events with

$$1.27 < R = \frac{P_\mu(DMS)}{P_\mu(RICH)} < 1.35$$

(in the existing program for Pion RICH $R = 1$ corresponds to a pion and $R = 1.33$ — to a muon).

The background simulation results after the cut (b) are presented in Fig. 9c and the results with both cuts (a) and (b) are presented in Fig. 9d. As is seen in these pictures the RICH cut reduced the background level drastically although it also reduced the efficiency of $K^+ \to \pi^-\mu^+\mu^+$ registration because of the implicitly applied momentum threshold $P_\mu \gtrsim 10$GeV for a fast muon (from $\epsilon = 0.086$ to $\epsilon = 0.03$). Thus the possibility of using the RICH velocity spectrometer is very important for the search for LFV processes in the CKM experiment.

We have not enough MC statistics for the simulation of background level for the measurements with the RICH selection cut. From comparison of Fig. 8 (E865) and 9c,d in the region where we see some background (near the mass $M(\pi^-\mu^+\mu^+) = 0.46$ GeV) it is possible to conclude that expected background level of the CKM experiment would be $\gtrsim 2$ orders of magnitude smaller than in E865 (that is around $\sim 10^{-11}$).

Study of background limitations for other LFV kaon decays in the CKM experiment are now in operation.

5 Study of $K^+ \to \pi^0 l^+ \nu_l \gamma$ decays in the CKM experiment and the search for T_{odd} correlations

The T-invariance is one of the fundamental symmetries in physics (see, for example [25–27]). It is still an open problem to be tested experimentally with high precision. If CPT-invariance take place, as is usually assumed, the CP-violation is directly connected with T-violation to conserve CPT. T-invariance has been studied in many experiments. Typical example is the search for the electric dipole moments of elementary particles. Another possibility is to look for the T-odd correlations in particles decays, such as triple correlations including spin in the neutron decay. A well known example of the T-odd correlation in the kaon decays is the polarization of the muon normal to the decay plane in $K^+ \to \mu^+ \nu \pi^0 (K_{\mu 3})$:

$$\sigma_\perp^\mu = \frac{\vec{\sigma}_\mu \cdot (\vec{p}_\pi \times \vec{p}_\mu)}{|\vec{p}_\pi \times \vec{p}_\mu|} \tag{4}$$

It was first suggested by J.J.Sakurai [28] to look for T-violation by $\sigma_\perp^\mu$ measurement. The unique feature of these process is that the "fake" T-odd correlation due to the final state interaction (FSI) is very small: $\sigma_\perp^{FSI} = 5 \cdot 10^{-6}$ [29]. Although the SM predicts zero effect, in some models (like 3HDM of Weinberg type) values higher than (several units)$\cdot 10^{-3}$ or even up to 10^{-2} were predicted [30, 31]. The best measurement of the $\sigma_\perp$, performed about 20 years ago at the BNL-AGS [32] gives: $\sigma_\perp = 0.0031 \pm 0.0053$. Current result from KEK E246 [33] is $\sigma_\perp < 10^{-2}$ (90% C.L.) and they hope to reach the sensitivity $\sigma_\perp < 1.3 \cdot 10^{-3}$ at the end of the experiment. Even more sensitive projects to measure $\sigma_\perp$ were considered [34].

To measure effectively $\sigma_\perp^\mu$ polarization one needs to stop muons from $K^+ \to \pi^0 \mu^+ \nu_\mu$ in thin layers of matter and to measure the electron asymmetry in their $\mu \to e$ decays. We do not plan at present to measure $\sigma_\perp^\mu$ in the CKM experiment: the kaon energy of 22GeV and the energies of secondary muons look too high for that. On the contrary, we are seriously considering the possibility of measuring another triple T-odd correlation:

$$T_{\pi\mu\gamma} = \frac{\vec{p}_\gamma \cdot (\vec{p}_\pi \times \vec{p}_\mu)}{|\vec{p}_\gamma| \cdot |\vec{p}_\pi \times \vec{p}_\mu|} \tag{5}$$

in the decay $K \to \pi^0 \mu \nu_\mu \gamma$. Such a measurement was proposed for the first time by J.Gevais, J.Iliopoulous and J.Kaplan in 1966 [35]. There are no theoretical predictions for this effect at present. The "fake" FSI correlation must not be strong: naively it must be similar to $\sigma_\perp^{FSI}$ in $K \to \mu\nu\gamma$, which is $\sim 10^{-4}$. The $K^\pm$ beam gives a principle possibility of subtraction of the FSI: the FSI-correlation has the same sign in $K^\pm$, while the true effect changes the sign.

The other way to check the existence of direct effect of T violation and to have an estimation for possible "fake" asymmetry is to compare the triple correlations (5) $T_{\pi\mu\gamma}$ and $T_{\pi e\gamma}$ for $K^+ \to \pi^0 \mu^+ \nu_\mu \gamma$ and $K^+ \to \pi^0 e^+ \nu_e \gamma$ decays. In multihiggs doublet models the real measurable T-violation effect is expected only through the interactions of Higgs bosons with muons in $K^+ \to \pi^0 \mu^+ \nu_\mu \gamma$ decays. For $K^+ \to \pi^0 e^+ \nu_e \gamma$ in this model $T_{\pi e\gamma}$ asymmetry is practically caused by FSI effects. Thus the comparison of $T_{\pi\mu\gamma}$ and $T_{\pi e\gamma}$ correlations will provide us with good criterion for observation the real T-violation effect and for evaluation of "fake" asymmetry. As it was calculated in [36], the "fake" asymmetries must be $T_{\pi e\gamma}^{FSI} = -0.59 \cdot 10^{-4}$ and $T_{\pi\mu\gamma}^{FSI} = 1.14 \cdot 10^{-4}$.

It seems also that kinematic cuts for photons in $K \to \pi^0 \mu \nu_\mu \gamma$ to separate direct emission radiative process and to suppress IB decay events will be important: we expect possible real T-violation effects in direct photon emission and can use IB events for the fake asymmetry tests.

The decay $K \to \pi^0 \mu \nu_\mu \gamma$ has not yet been observed, the prediction of the SM is [37]: $BR(K \to \pi^0 \mu \nu_\mu \gamma) \simeq 2 \cdot 10^{-5}$ (with the cuts $E_\gamma > 30$MeV and $\vartheta_{\mu\gamma} > 20^\circ$ in kaon rest frame, which are used to suppress IB events). The only experimental measurement of the T-odd correlation in the decay $K \to \pi^0 e \nu_e \gamma$ is from ISTRA set-up at IHEP 70GeV accelerator [38]: $\frac{\vec{p}_\gamma \cdot (\vec{p}_\pi \times \vec{p}_e)}{|\vec{p}_\gamma| \cdot |\vec{p}_\pi \times \vec{p}_e|} = 0.03 \pm 0.08$; it is based on 192 reconstructed events, which correspond to the $BR = (2.7 \pm 0.2) \cdot 10^{-4}$ (in the kaon rest frame $E_\gamma^{c.m.} > 10$ MeV; $0.6 < \cos\theta_{e,\gamma} < 0.9$). Now it is planed to do the measurement of $T_{\pi\mu\gamma}$ correlation in the next IHEP kaon experiment "OKA" with the expected statistics of $\sim 10^6$ $K \to \pi^0 \mu \nu_\mu \gamma$ decays [5]. The next step with increased statistics can be done in CKM experiment [1, 7].

The IHEP theorists are performing now detailed calculations of the correlation (5) in SM and in the Weinberg's 3HDM. This model is known to be a benchmark for the CP violation searches beyond the SM. There are several parameters in the model relevant for our case: $M_{H^{+-}}$ the mass of the lightest charged Higgs; $v_1; v_2; v_3$ – the VEV's of the 3 Higgs fields: $v_1^2 + v_2^2 + v_3^2 = 2\sqrt{2}G_F$; $\alpha_i, \beta_i, \gamma_i (i = 1, 2)$ – six complex Yukawa coupling constants which are related to each other by the conditions [39]:

$$\left.\begin{array}{rl} \frac{Im(\alpha_2\beta_2^*)}{Im(\alpha_1\beta_1^*)} = & \frac{Im(\beta_2\gamma_2^*)}{Im(\beta_1\gamma_1^*)} = \frac{Im(\alpha_2\gamma_2^*)}{Im(\alpha_1\gamma_1^*)} = -1 \\ \frac{Im(\alpha_1\gamma_1^*)}{v_2^2} = & -\frac{Im(\beta_1\gamma_1^*)}{v_1^2} = -\frac{Im(\alpha_1\beta_1^*)}{v_3^2} \end{array}\right\} \quad (6)$$

Further constraints on the model parameters can be extracted from the experimental data on d_n, the neutron electrical dipole moment; $\epsilon; \epsilon'; \triangle M_D$ etc. (see review [31]). Given all the constraints, one still can expect a large value for $T_{\pi\mu\gamma}$, which soon will be estimated. Let us remind that these models predict a significant value for $\sigma_\perp^\mu$ in $K \to \pi^0\mu\nu_\mu$ decays [30, 31].

To illustrate the possibilities for CKM experiments we estimated the expected efficiency for registration of $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ and $K^+ \to \pi^0 e^+\nu_e\gamma$ and expected statistical precision for measurements of $T_{\pi\mu\gamma}$ and $T_{\pi e\gamma}$ asymmetry parameters. It is more convenient to use another normalization for this correlation $T'_{\pi\mu\gamma} = \frac{\vec{p}_\gamma\cdot(\vec{p}_\pi\times\vec{p}_\mu)}{M_K^3}$, which takes into account the information on momentum distribution of the particles in the final states. We performed the calculations for $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ decays with cuts (in the rest frame of kaon) $E_\gamma > 30\text{MeV}$ and $\theta_{l\gamma} > 20^\circ$ to separate the direct photon emission process and to suppress IB radiation.[2]

As will be show below we expect to obtain $1.1 \cdot 10^7$ $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ events and $> 10^8$ $K^+ \to \pi^0 e^+\nu_e\gamma$ events in the CKM experiment in parallel with the main $K^+ \to \pi^+\nu\bar{\nu}$ studies. For estimation of the statistical sensitivity for average asymmetry of $T'_{\pi\mu\gamma} = \frac{\vec{p}_\gamma\cdot(\vec{p}_\pi\times\vec{p}_\mu)}{M_K^3}$ distribution in $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ decay we did a MC simulation for this distribution in the SM and have obtained the sensitivity $T'_{\pi\mu\gamma} = \pm 2.5 \cdot 10^{-5}$ (for more common normalization $T_{\pi\mu\gamma} = \frac{\vec{p}_\gamma\cdot(\vec{p}_\pi\times\vec{p}_\mu)}{|\vec{p}_\gamma|\cdot|\vec{p}_\pi\times\vec{p}_\mu|} = \pm 3 \cdot 10^{-4}$). For $K^+ \to \pi^0 e^+\nu_e\gamma$ the corresponding sensitivity is $T'_{\pi e\gamma} = \pm 7 \cdot 10^{-6} (T_{\pi e\gamma} = \pm 1 \cdot 10^{-4})$ [7].

The main problems for T-asymmetry measurements in $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ are background processes and systematic uncertainties. It seems that the main background processes are $K^+ \to \pi^0\pi^0\mu^+\nu_\mu$ with one lost photon, $K^+ \to \pi^+\pi^0\gamma$ with $\pi^+ \to \mu^+\nu_\mu$ decay in flight, $K^+ \to \pi^+\pi^0\pi^0$ with one lost photon and with $\pi^+ \to \mu^+\nu_\mu$ decay in flight and $K^+ \to \pi^0\mu^+\nu_\mu$ with accidental "photon". We considered here the first 3 background processes.

In our first approach we consider only $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ decays with all 3 photons detected in the Forward Veto System of the CKM setup (FVS, see Fig.1) for which we will have the best energy and coordinate resolutions for photon detection (see estimations in Table 3). But we met with serious problems in this approach:

1. The efficiency for $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ registration was small enough ($\epsilon = 0.008$) even without additional cuts to suppress the main background processes.

2. The muon spectrum for these $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ events is very soft — practically all muons have momenta $P_\mu < 10\text{GeV}$. They are below the threshold for pion RICH of CKM. Pion RICH can not be used to reduce the background from $\pi \to \mu\nu$ decays in flight by double muon momenta measurement and the background from $K^+ \to \pi^+\pi^0\pi^0$ (with $\pi^+ \to \mu^+\nu$; one $\pi^0 \to \gamma(\gamma)$) and from $K^+ \to \pi^+\pi^0\gamma$ (with $\pi^+ \to \mu^+\nu$) was too big.

Thus we used another approach which opened more opportunities for background suppression with the help of additional cuts. In our GEANT simulation [7] we selected the events with 3 photons from $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ decays detected in FVS and VVS[3] and with a charge particle which was registered in DMS spectrometer and passed through pion RICH and Muon Veto System (MVS) of the CKM setup.

We used the requirements for signal photon registration in FVS $E_\gamma > 150\text{MeV}$ and in VVS $E_\gamma > 100\text{MeV}$ and included the energy and coordinate resolutions in our simulation. For both system the energy resolution

[2] It must be bear in mind that may be we will need even more stringent cuts for this purpose.

[3] As was stated above this possibility must be further studied by GEANT simulation and special experimental investigation to understand the influence of the photon impact angles for VVS.

was assumed to be $\delta E_\gamma/E_\gamma = 0.10/\sqrt{E_\gamma[GeV]} + 0.01$[4]. The coordinate resolution for VVS was $\delta\phi = \pm\pi/16$ and $\delta z = \pm 12.5$ cm (this resolution is due to the mechanical structure of VVS, see [1]) and for FVS — δx; $\delta y = \pm 5\text{mm}/\sqrt{E_\gamma[GeV]} + 1$ mm. For two signal photons the constraint for the table value of π^0 mass was imposed. The photon veto requirements for additional photons were used in the accordance with Table 1. For charge particle we imposed the momentum cut $P > 10$ GeV.

After thorough study of different cuts to optimize the registration of the $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ decay and the background rejection we chose the following additional cuts for the above decay and for background processes $K^+ \to \pi^+\pi^0\pi^0$ ($\pi^+ \to \mu^+\nu$; one $\pi^0 \to \gamma(\gamma)$), $K^+ \to \pi^+\pi^0\gamma$ ($\pi^+ \to \mu^+\nu$), $K^+ \to \pi^0\pi^0\mu^+\nu_\mu$ (one $\pi^0 \to \gamma(\gamma)$):

1. For all the decays we defined 3 systems of particles in final states X_1, X_2, X_3 with different assumption for charge particle (π^+ or μ^+) and we found the missing masses relative to these systems M_1, M_2 and M_3 in $K^+ \to X_i + M_i$ decays:

   (a) $X_1 = \mu^+\pi^0\gamma$ (charge particle was assumed to be μ^+) and M_1 for $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ is the mass of neutrino; we selected events with $-0.01 < M_1^2 < 0.01\text{GeV}^2$.

   (b) $X_2 = \pi^+\pi^0$ (charge particle was assumed to be π^+) for $K^+ \to \pi^+\pi^0\pi^0$ decay $M_2^2 = m_{\pi^0}^2$ and we used the cut $M_2^2 > 0.025\text{GeV}^2$ or $-0.01 < M_2^2 < 0.01\text{GeV}^2$, e.g. we rejected the events with $0.01 < M_2^2 < 0.025\text{GeV}^2$. This cut reduced significantly the background from $K^+ \to \pi^+\pi^0\pi^0$.

   (c) $X_3 = \pi^0\gamma$ and we used the cut $M_3^2 > 0.04\text{GeV}^2$ to reduce the background from $K^+ \to \pi^+\pi^0\gamma$.

2. To reduce the background from $\pi \to \mu\nu$ decays in flight we used double measurement of muon momenta in DMS and in pion RICH velocity spectrometer

$$1.285 < R = \frac{P_{\text{charge part.}}(RICH)}{P_{\text{charge part.}}(DMS)} < 1.355$$

   (in the existing program for pion RICH $R = 1$ corresponds to a pion and $R = 1.33$ — to a muon).

3. To increase the efficiency of photon veto system we used the cut for the coordinate of the kaon decay vertex in the beam $20 < z < 44$ m (this cut is different from the standard CKM cut $14 < z < 44$ m because Beam Interaction Veto System was not included in the simulation procedure. In the future this temporary limitation can be omitted.

In the GEANT simulation procedure to increase statistics for background estimations we forced $\pi^+ \to \mu^+\nu_\mu$ decay and then used corresponding weights (~ 0.1 in average) for proper background evaluation.

The results of the GEANT simulation are presented in the Table 7 from which we conclude that backgrounds are at acceptable level.

It is also instructive to see how general additional cuts reduce the efficiency of $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ detection (Table 8). We see that by reducing the $\epsilon(K^+ \to \pi^0\mu^+\nu_\mu\gamma)$ by factor of 3 we can reduce background by more than an order of magnitude. Certainly it is possible to have some intermediate cuts to optimize the measurements.

In the condition of CKM experiment it is possible to obtain $4.0 \cdot 10^{13}$ K^+ decays for 2 years of measurements ($0.51 \cdot 10^7$ K^+ decays/spill; $1.4 \cdot 10^5$ spill/week; $0.50 \cdot 10^{12}$ decay/week with a dead time losses of 30%; 78 weeks of measurements per 2 years) [1]. This value corresponds to $1.1 \cdot 10^7$ $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ decays which we discussed before. More details of studying T-odd effects in $K^+ \to \pi^0 l^+\nu_l\gamma$ in the CKM experiment see in [7].

6 Searches for New Types of Interactions in K Decays

6.1 General Comments

Searches for new types of weak interactions that are associated with exchange of new intermediate bosons — scalar (S), pseudoscalar (P), tensor (T) and right-handed vector bosons W_R with leptoquark processes or

[4]For FVS it is a very rude approximation because in this CsI spectrometer $\delta E_\gamma/E_\gamma = 0.02/\sqrt{E_\gamma[GeV]} + 0.01$.

Table 7: The result of GEANT simulation of $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ decay and background processes for the CKM setup

Decay	BR	MC Statistics	The number of accepted events	Efficiency, ϵ	Backgr./Signal
$K^+ \to \pi^0\mu^+\nu_\mu\gamma$	$2 \cdot 10^{-5}$	80000	1320	1.4%	
$K^+ \to \pi^+\pi^0\pi^0$ $\hookrightarrow \gamma(\gamma)$ $\hookrightarrow \mu^+\nu$	$1.73 \cdot 10^{-2}$	300000	0	$< 2 \cdot 10^{-6}$ (90% c.l.)	$< 6 \cdot 10^{-2}$ (90% c.l.)
$K^+ \to \pi^+\pi^0\gamma$ $\hookrightarrow \mu^+\nu$	$2.75 \cdot 10^{-4}$	80000	452	$5.5 \cdot 10^{-5}$	$5.4 \cdot 10^{-2}$
$K^+ \to \pi^0\pi^0\mu^+\gamma$ $\hookrightarrow \gamma(\gamma)$	$\sim 0.7 \cdot 10^{-5}$	80000	1	$1.25 \cdot 10^{-5}$	$< 1.1 \cdot 10^{-3}$ (90% c.l.)

Table 8: The influence of additional cuts for the efficiency of $K^+ \to \pi^0\mu^+\nu_\mu\gamma$ detection

Cuts	$\epsilon(K^+ \to \pi^0\mu^+\nu_\mu\gamma),\ \%$
Preliminary	4.7
$-0.01 < M_1^2 < 0.01$ GeV^2	4.4
$1.285 < R < 1.355$	4.0
$M_2^2 > 0.025$ GeV^2 or $-0.01 < M_2^2 < 0.01$ GeV^2	2.8
$M_3^2 > 0.04$ GeV^2	1.4

with other possible mechanisms are of great interest in the quest of new physics beyond the Standard Model. One preliminary comment concerning the sensitivity of experiments devoted to searches for new interactions that could be manifested, for example, in $K^+ \to l^+\nu_l$,$K^+ \to \pi^0 l^+\nu_l$, and$K^+ \to l^+\nu_l\gamma$ decays is in order here. The amplitude of each of these processes can be represented in the form

$$A = A_{SM} + A_{NI} = |A_{SM}| + |A_{NI}|e^{i\varphi}, \tag{7}$$

where A_{SM} is the Standard Model amplitude of the processes, A_{NI} is the amplitude associated with new interactions, and φ is their relative phase. Therefore, we have

$$\begin{aligned} |A|^2 &= |A_{SM}|^2 + |A_{NI}|^2 + 2Re(A_{SM}A^*_{NI}) = \\ &= |A_{SM}|^2 + |A_{NI}|^2 + 2\cos\varphi|A_{SM}||A^*_{NI}| \simeq \\ &\simeq |A_{SM}|^2[1 \pm \underbrace{|A_{NI}/A_{SM}|}_{IT}] \end{aligned} \tag{8}$$

(the estimate was obtained under the assumption that the average value of the interference phase is $2\cos\varphi = \pm 1$). Thus, corrections due to new interactions are determined by the interference term IT, which is in inverse proportion to the square of the mass of the corresponding intermediate boson (IT $\sim M^{-2}_{\text{boson}}$). Experiments of the type being discussed appear to be more sensitive to new interactions than, for example, processes with nonconserving lepton flavor ($\mu^- + (A, Z) \to e^- + (A, Z), K^0 \to \mu e$, etc.), which are characterized by probabilities proportional to $|A|^2 \simeq |A_{NI}|^2 \sim M^{-4}_{\text{boson}}$ (see also [40]).

6.2 $K^+ \to e\nu_e$ decay and the search for pseudoscalar interactions

We will consider here only one possible measurement in the CKM for the quest of new type of interactions — search for pseudoscalar coupling in precise study of $K^+ \to e\nu_e$ decay.

The lepton decays of K mesons are especially sensitive to possible admixtures of this new interaction of the pseudoscalar type. Within the Standard Model involving V-A weak interactions, $K^+ \to l^+\nu_l$ decays are suppressed by helicity conservation (see Fig. 10a). The corresponding matrix element for $K^+(P_K) \to l^+(p_l) + \nu_l(p_\nu)$ decays has the form

$$A(K^+ \to l^+\nu_l)_{SM} = \frac{G_F}{\sqrt{2}} V_{us} f_K P_K^\alpha \bar{u}_\nu \gamma_\alpha (1+\gamma_5) u_l, \tag{9}$$

where $f_K = 159.8 \pm 1.5$ MeV is the K-decay constant. The probability of $K^+ \to l^+\nu_l$ decays has the form

$$\Gamma(K^+ \to l^+\nu_l)_{SM} = \frac{G_F^2}{8\pi} f_K^2 |V_{us}|^2 m_K m_l^2 \Big(1 - \frac{m_l^2}{m_K^2}\Big)^2. \tag{10}$$

Because of helicity conservation, this decay probability is proportional to the lepton mass squared m_l^2. One can find the ratio of the relevant branching fractions

$$\begin{aligned} R(K^+ \to l^+\nu_l)_{SM} &= \frac{BR(K^+ \to e^+\nu_e)}{BR(K^+ \to \mu^+\nu_\mu)}\Big|_{SM} = \frac{m_e^2}{m_\mu^2}\frac{(m_K^2 - m_e^2)^2}{(m_K^2 - m_\mu^2)^2}(1+\delta_r) = \\ &= 2.569 \times 10^{-5}(1 - 0.0378 \pm 0.0004) = \\ &= (2.4719 \pm 0.0010) \times 10^{-5}, \end{aligned} \tag{11}$$

where δ_r is the radiative correction calculated with a high accuracy [41]. By using data on $\tau(K^+)$, $BR(K^+ \to \mu^+\nu_\mu)$, and the value of $R(K^+ \to \mu^+\nu_\mu)_{SM}$ in (11), we derive the Standard Model predictions for the width with respect to the rare decay $K^+ \to e^+\nu_e$ and its relative probability

$$\begin{aligned} \Gamma(K^+ \to e^+\nu_e)_{SM} &= (0.834 \pm 0.003) \times 10^{-18}\text{MeV} \\ BR(K^+ \to e^+\nu_e)_{SM} &= (1.570 \pm 0.005) \times 10^{-5} \end{aligned} \tag{12}$$

For the case of pseudoscalar interaction, the decay $K^+ \to l^+\nu_l$ would not be suppressed by helicity conservation (see Fig. 10b), and its probability would be given by

$$\Gamma(K^+ \to l^+\nu_l)_{PS} = \frac{G_{PS}^2}{8\pi} f_K^2 m_K M^2 \Big(1 - \frac{m_l^2}{m_K^2}\Big)^2, \tag{13}$$

where M is some constant having the dimensions of mass. Here, the pseudoscalar coupling constant G_{PS} is defined by analogy with $G_F = g_2^2/4\sqrt{2}M_W^2$ and has the same dimensionality in (13). The pseudoscalar interactions between the quark and lepton currents were considered in more detail in [42], where the corresponding constant was defined as $(G_{UD}^{l\nu})_{PS} = f_{l\nu} f_{UD}/4\sqrt{2}M_P^2$, where $f_{l\nu}$ and f_{UD} are the constants of lepton and quark coupling to the intermediate pseudoscalar bosons of the mass M_P ($U = u, c, t$ stand for the up quarks, while $D = d, s, b$ stand for the down quarks). In [42], the mass factor M has the form $M = m_K^2/(m_u + m_s)$, where $m_u = 5$ MeV and $m_s = 150$ MeV are the current masses of the quark entering into the composition of the K^+meson.

In the case of pseudoscalar interaction, the probabilities of the decays $K^+ \to e^+\nu_e$ and $K^+ \to \mu^+\nu_\mu$ were close

$$R(K^+ \to l^+\nu_l)_{PS} \simeq (1 - m_e^2/m_K^2)^2/(1 - m_\mu^2/m_K^2)^2 \simeq 1.1 \tag{14}$$

A modest difference between them is explained by the smaller phase space for the decay $K^+ \to \mu^+\nu_\mu$.

Because the decay $K^+ \to e^+\nu_e$ is strongly suppressed within the Standard Model by helicity conservation, it proves to be especially sensitive to even a moderate admixture of pseudoscalar interaction, as was mentioned above. We assume that the amplitude of $K^+ \to l^+\nu_l$ decay is determined by the contributions of $V - A$ (Standard Model) and anomalous PS interaction: $A(K^+ \to l^+\nu_l) = A(K^+ \to l^+\nu_l)_{SM} + A(K^+ \to l^+\nu_l)_{PS}$; as was discussed above, $A(K^+ \to \mu^+\nu_\mu)_{SM} >> A(K^+ \to e^+\nu_e)_{SM}$, while $A(K^+ \to \mu^+\nu_\mu)_{PS} \simeq A(K^+ \to e^+\nu_e)_{PS}$. Thus

$$
\begin{aligned}
R(K^+ \to l^+\nu_l) &= \frac{BR(K^+ \to e^+\nu_e)}{BR(K^+ \to \mu^+\nu_\mu)} \simeq \\
&\simeq R(K^+ \to l^+\nu_l)_{SM}[1 \pm |A_{PS}/A_{SM}|]. \qquad (15)
\end{aligned}
$$

The available experimental data [43] yield

$$
\begin{aligned}
R(K^+ \to l^+\nu_l)_{exp} &= \frac{(1.55 \pm 0.07) \times 10^{-5}}{0.6351 \pm 0.0018} = (2.441 \pm 0.110) \times 10^{-5} = \\
&= R(K^+ \to l^+\nu_l)_{SM} \times (0.987 \pm 0.045), \qquad (16)
\end{aligned}
$$

which corresponds to $\Gamma(K^+ \to e^+\nu_e)_{exp} = (0.824 \pm 0.037) \times 10^{-18}$ MeV. From (15) and (16), it follows that the experimental limits on the pseudoscalar-interaction contribution to the decay $K^+ \to e^+\nu_e$ are

$$
\begin{aligned}
|A(K^+ \to e^+\nu_e)_{PS}| &< 5.8 \times 10^{-2}|A(K^+ \to e^+\nu_e)_{SM}|, \\
\Gamma(K^+ \to e^+\nu_e)_{PS} &< 3.4 \times 10^{-3}\Gamma(K^+ \to e^+\nu_e)_{SM} \simeq 3 \times 10^{-21}\text{MeV}. \qquad (17)
\end{aligned}
$$

It was shown in [42] that

$$
R(K^+ \to l^+\nu_l) = R(K^+ \to l^+\nu_l)_{SM}\Big[1 \pm \frac{|G^{e\nu}_{us}|_{PS}}{G_F V_{us}} \frac{m_K^2}{(m_s + m_u)m_e}\Big], \qquad (18)
$$

where $|G^{e\nu}_{us}|_{PS}$ is the pseudoscalar-interaction constant coupling the us and $e\nu$ states (for the decay $\pi^+ \to e^+\nu_e$, pseudoscalar interaction contribution is determined by another constant, $|G^{e\nu}_{ud}|_{PS}$).

From (17) and(18), one can find an experimental constraint on the pseudoscalar coupling constant: $|G^{e\nu}_{us}|_{PS} < 3.6 \times 10^{-11}\text{GeV}^{-2}$.

In the future kaon experiment OKA in IHEP, the sensitivity of the measurement of $R(K^+ \to l^+\nu_l)$ is expected to be $(1-3) \cdot 10^{-3}$, which corresponds to the search for pseudoscalar interaction on the level

$$
|A(K^+ \to l^+\nu_l)_{PS}|^2 < 10^{-6} - 10^{-5}|A(K^+ \to l^+\nu_l)_{SM}|^2 \simeq 10^{-24} - 10^{-23}\text{MeV} \qquad (19)
$$

It should be point out that to reduce the systematic uncertainties in $R(K^+ \to l^+\nu_l)$ measurements it is very important to have a clear electron identification and precise determination of the efficiencies detection, as well as a very effective photon veto system to reduce the background from $K^+ \to \pi^0 e^+\nu_e$ and $K^+ \to e^+\nu_e\gamma$ decays. The CKM setup would have strong tools for these measurements (very effective photon veto system, good electron detection in pion RICH). Thus it will be reasonable to perform of further studies of $K^+ \to e^+\nu_e$ decay in the CKM experiment to increase the sensitivity in the search for pseudoscalar interactions which is expected in the OKA experiment.

It must be noted that the value $R(K^+ \to l^+\nu_l)_{SM}$ in (11) is really the ratio $BR[K^+ \to e^+\nu_e(\gamma)]/BR[K^+ \to \mu^+\nu_\mu(\gamma)]$ and includes the soft IB photons which are not observed in the experiments [41]. The value $R[K^+ \to l^+\nu_l(\gamma)]_{SM}$ is the ratio, which will be measured in the same conditions for $K^+ \to e^+\nu_e(\gamma)$ and $K^+ \to \mu^+\nu_\mu(\gamma)$. Thus soft IB photons have no influence for the precision of the search for deviation from SM predictions and for possible contribution of a new pseudoscalar interaction which was estimated in (19).

The study of $K^+ \to e^+\nu_e$ is the most effective way for the search for this pseudoscalar coupling. As was shown in [5] this decay is mush more sensitive for pseudoscalar searches to compare for example with $K^0_L \to e^+e^-$ (both these decays are helicity suppressed in the Standard Model and thus are very sensitive to new anomalous interactions).

7 The comparison of $K^+ \to \pi^0\mu^+\nu_\mu$ and $K^+ \to \pi^0 e^+\nu_e$ decays and some new types of weak interactions

Let us also consider the study of $K^+ \to \pi^0\mu^+\nu_\mu$ decay and the possibility to search for some influence of the scalar interactions (Higgs boson effects ?) in the comparison of $K^+_{\mu 3}$ and K^+_{e3} decays.

As is well known (see [43]) in the $V-A$ Standard Model the matrix element of $K^+ \to \pi^0\mu^+\nu_\mu$ decay can be influenced by two formfactors $f_+(q)$ and $f_-(q)$ and can be presented in the form

$$A = \left(G_F/\sqrt{2}\right) \cdot \lambda \cdot f_+(q^2) \left[(P_K + P_\pi)_\rho \cdot \bar{\mu}\gamma^\rho(1+\gamma_5)\nu_\mu + \frac{f_-(q^2)}{f_+(q^2)} m_\mu \bar{\mu}(1+\gamma_5)\nu_\mu\right] \tag{20}$$

As was shown in the paper of R. Peccei [30] the influence of H^- Higgs boson exchange diagram in Fig. 11 will modify the expression (20) as

$$\begin{aligned} A \;&=\; \left(G_F/\sqrt{2}\right) \cdot \lambda \cdot f_+(q^2) \Big[(P_K + P_\pi)_\rho \cdot \bar{\mu}\gamma^\rho(1+\gamma_5)\nu_\mu + \frac{f_-(q^2)}{f_+(q^2)} m_\mu \bar{\mu}(1+\gamma_5)\nu_\mu + \\ &\quad + g_S(q^2) m_\mu \bar{\mu}(1+\gamma_5)\nu_\mu\Big] \end{aligned} \tag{21}$$

As is seen from Fig. 11 $\mathrm{Im}\, g_S = \mathrm{Im}\, \alpha^* \gamma (M_{K^-}/M_{H^-})^2$. There is some relation between normal T-odd muon polarization in $K^+ \to \pi^0\mu^+\nu_\mu$, $\mathrm{Im}\, g_S$ and kinematic factor 0.2

$$\langle p_T^\mu \rangle \simeq 0.2 \cdot \mathrm{Im}\, g_S \tag{22}$$

This condition is a basis for the search for normal muon polarization in kaon decay in multihiggs models of Weinberg type.

But there is another question for which I do not know the precise answer now: may be it is possible to search the influence of scalar interaction $g_S(q^2)$ by studying the anomalous value of the coefficient $\left[\frac{f_-(q^2)}{f_+(q^2)} + g_S(q^2)\right]$ in the second part of equation (21) ? In any case it seems to me that to search very carefully for new types of weak interactions we need precise comparison of different kaon decay modes with muons and electrons.

8 Nonleptonic Charged Kaon decays and the Search for Direct CP Violation in $K^\pm$ decays

Studies of the nonleptonic charge kaons decays, $K^\pm \to \pi^\pm\pi^+\pi^-$, $K^\pm \to \pi^\pm\pi^0\pi^0$, $K^\pm \to \pi^\pm\pi^0\gamma$ and $K^\pm \to \pi^\pm\gamma\gamma$ are important for the understanding of the mechanisms of $\triangle S = 1$ hadronic weak interactions and of the main properties of the chiral perturbation theory (CHPT) for these processes. In addition to precision CHPT tests, new measurements in this field with greatly enlarged statistics can in principle enable the search for direct CP violation effects through measurement of rate asymmetries in $(K^+ \to \bar{f})/K^- \to f)$ or Dalitz plot asymmetries.

In principle it is straightforward to operate the charged kaon beam line at both polarities, allowing K^+ and K^- data sets to be collected at different times. In practice, the precision measurements described below will be vulnerable to the different beam background environments of K^+ and K^- running. In addition, detector efficiencies between different K^+ and K^- running periods would have to be controlled to a very high level. Nevertheless, there is some value in listing the potential measurements with the understanding that a serious attack on these physics topics requires detailed study for which we did only first small steps.

The direct CP violation would manifest itself by the difference between the K^+ and K^- decay matrix elements if (see for example [31, 44]), there are at least two amplitudes with different "weak" phases and different "strong" final state interaction phases.

In more details, let us assume that the amplitudes for charge conjugated decays

$$A_f = \langle f|H|K^+\rangle \,,\; A_{\bar{f}} = \langle \bar{f}|H|K^-\rangle \tag{23}$$

are the sum of the independent components

$$\begin{aligned} A_f \;&=\; a e^{i\delta_a} + b e^{i\delta_b} = |a| \exp(i\delta_a + i\varphi) + |b| \exp(i\delta_b) \\ A_{\bar{f}} \;&=\; a^* e^{i\delta_a} + b^* e^{i\delta_b} = |a| \exp(i\delta_a - i\varphi) + |b| \exp(i\delta_b) \end{aligned} \tag{24}$$

Here δ_a and δ_b are the scattering phases in the final state interactions and $\varphi = \varphi_a - \varphi_b$ — the phase difference for two weak decay amplitudes $a = |a|e^{i\varphi_a}$ and $b = |b|e^{i\varphi_b}$. The squires of the amplitudes are

$$\left.\begin{array}{rcl} |A_f|^2 & = & (|a|\exp[i(\delta_a+\varphi)] + |b|\exp i\delta_b)(|a|\exp[-i(\delta_a+\varphi)] + |b|\exp(-i\delta_b)) = \\ & = & |a|^2 + |b|^2 + |a||b|[\exp[i(\delta_a-\delta_b+\varphi) + \exp[-i(\delta_a-\delta_b+\varphi)] = \\ & = & |a|^2 + |b|^2 + 2|a||b|\cos[(\delta_a-\delta_b)+\varphi] \\ |A_{\bar f}|^2 & = & |a|^2 + |b|^2 + 2|a||b|\cos[(\delta_a-\delta_b)-\varphi] \end{array}\right\} \tag{25}$$

The charge asymmetry of these decays is

$$\begin{aligned} \Delta &= \frac{|A_f|^2 - |A_{\bar f}|^2}{|A_f|^2 + |A_{\bar f}|^2} = \\ &= \frac{2|a||b|\cos[(\delta_a+\delta_b)+\varphi] - \cos[(\delta_a-\delta_b)-\varphi]}{2|a|^2 + 2|b|^2 + 2|a||b|[\cos[(\delta_a-\delta_b)+\varphi] + \cos[(\delta_a-\delta_b)-\varphi]]} = \\ &= \frac{-2|a||b|\cos(\delta_a-\delta_b)\sin\varphi}{|a|^2+|b|^2+2|a||b|\cos(\delta_a-\delta_b)\cos\varphi} \end{aligned} \tag{26}$$

Thus, it is clear from (26), that the charge asymmetry in $K^+ \to f$ and $K^- \to \bar f$ decays, which is determined by direct CP violation, will manifest itself only if, in the final state, there are at least two different dynamical amplitudes a and b with different phase φ_a and φ_b (relative phase $\varphi_a - \varphi_b = \varphi \neq 0$) and with different phase shifts δ_a and δ_b for the final scattering ($\delta_a - \delta_b \neq 0$).

We will consider below the decays $K^\pm(k) \to \pi^\pm(p_1)\pi^\pm(p_2)\pi^\mp(p_3)$ (τ-decays) and $K^\pm(k) \to \pi^0(p_1)\pi^0(p_2)\pi^\pm(p_3)$ (τ'-decays) which satisfy these requirements (see for example [31, 43–45]).

The distribution of $K \to 3\pi$ events in the Dalitz plot is described by the expression

$$|A(K\to 3\pi)|^2 = const[1 + gX + hX^2 + jY + kY^2] \sim 1 + gX \tag{27}$$

Here $X = \frac{s_3 - s_0}{m_\pi^2}$; $Y = \frac{s_2 - s_0}{m_\pi^2}$; $s_i = (k - p_i)^2 = (m_k - m_{\pi i})^2 - 2m_k T_i$; $s_0 = \frac{1}{3}(s_1 + s_2 + s_3) = \frac{1}{3}(m_k^2 + m_\pi^2)$ (i = 1;2;3 where i=3 for nonsymmetric pion); k, p_i and $m_k, m_{\pi i}$ are four momenta and masses of kaon and pion; T_i — kinetic energy of pions. The measured values of the slopes in the variable s_3 of the Dalitz plot distributions are $g_\tau = -0.2154 \pm 0.0035$ and $g_{\tau'} = 0.652 \pm 0.031$ [43].

The amplitudes of $K \to 3\pi$ decays can be presented in accordance with (24) in the form

$$\begin{array}{rcl} A(K^+) & = & |a(s_1;s_2;s_3)|\exp i(\delta_a+\varphi) + |b(s_1;s_2;s_3)|\exp i\delta_a \\ A(K^-) & = & |a(s_1;s_2;s_3)|\exp i(\delta_a-\varphi) + |b(s_1;s_2;s_3)|\exp i\delta_a \end{array} \tag{28}$$

In the main approximation of the momentum expansion amplitudes a and b are determined by $\triangle T = 1/2$ and $\triangle T = 3/2$ transition and the phase φ is related with the phase of CKM matrix.

The direct CP violation in $K^\pm \to 3\pi$ decays can be measured in studies of the asymmetry of the type of (26) in partial decay rates

$$\frac{\Gamma(K^+\to\pi^+\pi^+\pi^-) - \Gamma(K^-\to\pi^-\pi^-\pi^+)}{\Gamma(K^+\to\pi^+\pi^+\pi^-) + \Gamma(K^-\to\pi^-\pi^-\pi^+)} \equiv (\Delta\Gamma)_\tau, \tag{29}$$

or in slopes of the Dalitz plot distributions

$$\frac{g(K^+\to\pi^+\pi^+\pi^-) - g(K^-\to\pi^-\pi^-\pi^+)}{g(K^+\to\pi^+\pi^+\pi^-) + g(K^-\to\pi^-\pi^-\pi^+)} \equiv (\delta g)_\tau. \tag{30}$$

The corresponding values $(\Delta\Gamma)_{\tau'}$ and $(\delta g)_{\tau'}$ are for τ'-decay. The accuracy in measuring the slope g of the distribution is related to the accuracy in determining the total width with respect to $K^\pm \to 3\pi$ decays, $\Gamma(K^\pm \to 3\pi)$, by the equation.

$$(\delta g) = R\cdot(\delta\Gamma) = R\cdot\sqrt{\frac{n(K^+)+n(K^-)}{n(K^+)\cdot n(K^-)}} = R\frac{\sqrt{1+(1/r)}}{2\sqrt{n(K^+)}} \tag{31}$$

Here $n(K^+)[n(K^-)]$ is the number of decays $K^+ \to 3\pi[K^- \to 3\pi]$ in $K^+(K^-)$ beam and $r = n(K^-)/n(K^+)$. Factor R depends on the slope g; $R_\tau = 7.56$ [46] and $R_{\tau'} = 3.0$ [47].

Despite the introduction of the large factor R the sensitivity for the charge asymmetry of the slope (δg) is better than for the charge asymmetry of the rate ($\Delta\Gamma$), where asymmetry effect is reduced by integration over the entire Dalitz plot. Thus, for example, the measurements with τ'-decays are by factor $(R_\tau/R_{\tau'})^2 \simeq 6$ more "effective" for the search for CP violation than with τ-decays.

In Table 9 the sensitivity of the performed and proposed experiments for CP violation asymmetry measurements in $K^\pm \to 3\pi$ decays is presented [5, 47–49].

Table 9: Sensitivity of performed and proposed experiments for CP-violation decay asymmetry in $K^\pm \to 3\pi$ decays

Experiment	Decay	Existing (*) or expected (**) statistics for $K^\pm \to 3\pi$	Existing (*) or expected (**) statistical precision in (δg) asymmetry
	$K^\pm \to \pi^\pm\pi^+\pi^-$	$3.2 \cdot 10^6(K^+ + K^-)$ (*)	$(-0.70 \pm 0.53) \cdot 10^{-2}$ (*)
Hyper CP, Fermilab [48]	$K^\pm \to \pi^\pm\pi^+\pi^-$	$3.2 \cdot 10^8(K^+) + 1.3 \cdot 10^8(K^-)$ (*)	$\sim 6 \cdot 10^{-4}$ (**)
TNF, IHEP [47]	$K^\pm \to \pi^\pm\pi^0\pi^0$	$10^6(K^+) + 10^6(K^-)$ (*)	$\sim 2 \cdot 10^{-3}$ (**)
NA48, CERN [49]	$K^\pm \to \pi^\pm\pi^+\pi^-$ $K^\pm \to \pi^\pm\pi^0\pi^0$	$1.3 \cdot 10^9(K^+) + 0.75 \cdot 10^9(K^-)$ (**) $0.82 \cdot 10^8(K^+) + 0.47 \cdot 10^8(K^-)$ (**)	$\sim 1.7 \cdot 10^{-4}$ (**) $\sim 3 \cdot 10^{-4}$ (**)
OKA, IHEP [5]	$K^\pm \to \pi^\pm\pi^+\pi^-$ $K^\pm \to \pi^\pm\pi^0\pi^0$	$3.8 \cdot 10^9(K^+) + 2.1 \cdot 10^9(K^-)$ (**) $3.3 \cdot 10^8(K^+) + 1.9 \cdot 10^8(K^-)$ (**)	$\sim 1.0 \cdot 10^{-4}$ (**) $\sim 1.3 \cdot 10^{-4}$ (**)

Table 10 displays expected statistics and planned accuracy of the measurement of CP asymmetry in the decays $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$, $K^\pm \to \pi^0\pi^0\pi^\pm$ and $K^\pm \to \pi^\pm\pi^0\gamma$ in the CKM experiment. This Table corresponds to dedicated ran with periodically changed kaon polarity with total duration of 16 weeks for standard K^+ beam with intensity of $3 \cdot 10^7$ K^+/spill and K^- beam with intensity $1.5 \cdot 10^7$ K^-/spill (the same intensity of primary proton beam). Statistically optimal sharing of time is in sum 6.5 weeks with K^+ beam and 9.5 weeks with K^- beam). The expected statistical sensitivity of this measurement is $(\delta g)_\tau \leq 3.6 \cdot 10^{-5}$ and $(\delta g)_{\tau'} \leq 2 \cdot 10^{-4}$.

Table 10: Possible search for CP violation asymmetry in $K^\pm \to 3\pi$ and $K^\pm \to \pi^\pm\pi^0\gamma_{direct}$ in CKM

Decay	BR	Events/week	Number of decays $n(K^+)$ and $n(K^-)$	($\delta\Gamma$)	$(\delta g) = R(\delta\Gamma)$
$K^\pm \to \pi^\pm\pi^+\pi^-$	$(5.59 \pm 0.05) \cdot 10^{-2}$	$4.0 \cdot 10^9(K^+)$ $2.0 \cdot 10^9(K^-)$	$2.6 \cdot 10^{10}(K^+)$ $1.9 \cdot 10^{10}(K^-)$	$4.8 \cdot 10^{-6}$	$3.6 \cdot 10^{-5}$
$K^\pm \to \pi^\pm\pi^0\pi^0$	$(1.73 \pm 0.04) \cdot 10^{-2}$	$2.5 \cdot 10^7(K^+)$ $1.25 \cdot 10^7(K^-)$	$1.6 \cdot 10^8(K^+)$ $1.2 \cdot 10^8(K^-)$	$6.6 \cdot 10^{-5}$	$2.0 \cdot 10^{-4}$
$K^\pm \to \pi^\pm\pi^0\gamma_{direct}$	$(1.00 \pm 0.16) \cdot 10^{-5}$	$4.0 \cdot 10^4(K^+)$ $2.0 \cdot 10^4(K^-)$	$2.6 \cdot 10^5(K^+)$ $1.9 \cdot 10^5(K^-)$	$1.5 \cdot 10^{-3}$	
Notes:	a) Measurements for 16 weeks (dedicated run) b) Optimal time sharing between K^+ and K^- beams with $r = I(K^-)/I(K^+) = 1/2$ (the same intensity of protons) $t(K^+) = T\frac{\sqrt{r}}{1+\sqrt{r}} = 0.41T \simeq 6.5$ weeks($0.36 \cdot 10^{12}$ K^+decay/week - see Table 3) $t(K^-) = T\frac{\sqrt{r}}{1+\sqrt{r}} = 0.59T \simeq 9.5$ weeks($0.18 \cdot 10^{11}$ K^-decay/week - see Table 3) c) $(\delta\Gamma) = \frac{\sqrt{n(K^+)+n(K^-)}}{2\sqrt{n(K^+)\cdot n(K^-)}}$ d) For $K^+ \to \pi^+\pi^+\pi^-$ $R = 7.56$; for $K^+ \to \pi^+\pi^0\pi^0$ $R = 3.0$				

The main difficulty of the proposed experiments is to demonstrate that, indeed, there is no "dangerous" systematics at the level of $< 10^{-4}$. A possible strategy of the measurements is the change of the polarities of all the beam elements, say, every 6 hours. It's important then to change the polarity of the main spectrometer magnets at the same time. The possible sources of the systematics are:

1. Difference in the hadronic interactions between π^+ and π^-.
2. Different parameters of the positive and the negative beams. In particular, the tilt of the beams with respect to each other is potentially dangerous.
3. Effects of the mass resolution and background under the mass peaks.
4. Variation of the setup and the beam parameters with time, etc.

First attempts to investigate possible systematic uncertainties which where presented in [5] demonstrate that these uncertainties are smaller than 10^{-3}. It is clear that it is only first step and much more serious investigation of systematics must be performed in the future.

What is interesting to mention, is that there are several decays, where the $K^\pm$ asymmetry must be negligible. For example, in the decay $K \to e(\mu)\nu\pi^0$ the asymmetry must be very small because there is practically no final state interactions. The intense decay $K^+ \to \pi^+\pi^0$ is another example, here the expected effect is very small $[(\delta\Gamma) < 10^{-9}$ [50])], because the decay goes through one amplitude (S-wave, $\Delta T = 3/2$). These decays can be used to control the systematics.

Clearly a large amount of work is yet required to determine if the CKM apparatus is capable of this measurement, but it is at least encouraging that a reasonably robust experimental checking of the apparatus can be performed.

Let us briefly discuss the theoretical predictions for effects of direct CP violation in $K^\pm \to 3\pi$ decays. As a rule in the Standard Model these effects are very small and are beyond the possibilities of existing experimental technic. Some theoretical predictions for asymmetries $(\Delta\Gamma)$ and (Δg) in $K^\pm \to 3\pi$ and in some other decays are presented in Table 11.

In Table 11a are the results of SM calculations [45] based on the lowest order of CHPT approximation $O(p^2)$.

But in the work [51, 52] it was stated that asymmetry effects can be very strongly enhanced by the higher order contributions of CHPT (with enhancement factor $F > 10^2$ — see Table 11b) which leads to a large enough and possibly measurable asymmetry $(\delta g) \sim$ (several units) $\cdot 10^{-4}$. Unfortunately, this result is in contradiction with other works [45, 53] in which it was shown that although the asymmetry can be enhanced in higher orders of CHPT, this enhancement factor is moderate ($F < 10 \div 20$). Thus, in Table 11c the results of [53] are presented in which $O(p^4)$ calculations were performed and it seems that in the framework of SM it is difficult to expect the value of $(\Delta g)_{\tau;\tau'} > (2 \div 3) \cdot 10^{-5}$, which is still beyond the possibilities of current experiment.

Upon the discovery of large value of $Re(\varepsilon'/\varepsilon)$, special attention was given to mechanisms of direct CP violation beyond the Standard Model. Effects of charge asymmetry in $K^\pm \to 3\pi$ decays were analyzed under the conditions of the spontaneous breakdown of CP invariance within the model involving a few Higgs doublets [54]. This analysis revealed that the possible value of charge asymmetry in the slopes g can significantly exceeds the Standard Model prediction, reaching a level of $\sim 10^{-4}$ (Table 11d). The value of $(\delta g)_\tau \sim 10^{-5}$ and even more at some special assumptions about the parameters of the model was obtained in the supersymmetry extension of SM [55] — see Table 11e.

Some predictions for CP violating asymmetries in $K^\pm \to \pi^\pm\pi^0\gamma$ and $K^\pm \to \pi^\pm\gamma\gamma$ decays were presented in [44, 56] — see Table 11p,g,h. All these effects seem to be unmeasurably small.

In connection with the currently prevalent situation, it seems that, despite the pessimistic predictions of the Standard Model, we need thorough measurements of charge asymmetry in $K^\pm \to 3\pi$ decays with the highest possible experimental precision (about 10^{-4}). This is motivated by the possible manifestations of new CP-violation mechanisms and by difficulties in performing CHPT calculations, especially in higher orders.

Certainly we should consider pioneering some of these measurements at levels of sensitivity above the current predications in order to learn to what measurements are possible with a more dedicated effort in the future. We are also well positioned to explore large charge asymmetries, as might be discovered in the

Table 11: Theoretical predictions for CP-violation asymmetries in $K^\pm \to 3\pi$, $K^\pm \to \pi^+\pi^0\gamma$, $K^\pm \to \pi^\pm\gamma\gamma$

The decays $K^+ \to 3\pi$			
a	[45]	$(\delta g)_\tau = -(2.3 \pm 0.6) \cdot 10^{-6}$; $(\delta\Gamma)_\tau = -(6.0 \pm 2.0) \cdot 10^{-8}$ $(\delta g)_{\tau'} = (1.3 \pm 0.4) \cdot 10^{-6}$; $(\delta\Gamma)_{\tau'} = (2.4 \pm 0.8) \cdot 10^{-7}$	SM Lower order in CHPT ($O(p^2)$)
b	[52]	$(\delta g)_\tau \simeq (2.5 \div 6.0) \cdot 10^{-4}$ $(\delta g)_{\tau'} \simeq (1.5 \div 3.4) \cdot 10^{-4}$	SM Large enhancement of asymmetry $(\delta g)_{\tau,\tau'}$ in higher orders of CHPT ($O(p^6)$) (the enhancement factor $F \gtrsim 10^2$)
c	[53] [31]	$(\delta g)_\tau \lesssim 3 \cdot 10^{-5} \sin\delta$; $(\delta\Gamma)_\tau \lesssim 2.5 \cdot 10^{-6} \sin\delta$ $(\sin\delta \lesssim 1)$	SM Calculation in ($O(p^4)$) approximation of CHPT. (the enhancement factor is rather limited $F \sim 10$)
d	[54]	$(\delta g)_\tau\vert_{SM+W} \lesssim 10^{-4}$	Model with CP violation in multihiggs mechanism of Weinberg type
e	[55]	$(\delta g)_\tau \sim 10^{-5}$ (in some assumption up to $\sim 10^{-4}$)	Supersymmetry extension of SM induced by SUSY chomemagnetic penguins
The decays $K^+ \to \pi^\pm\pi^0\gamma$			
f	[44]	$[\delta\Gamma(\pi^\pm\pi^0\gamma)] \lesssim 10^{-5}$; for direct emission radiation $[\delta\Gamma(\pi^\pm\pi^0\gamma)]_{DE} \lesssim 10^{-4}$	SM
g	[56]	$[\delta\Gamma(\pi^\pm\pi^0\gamma)] \lesssim 10^{-5}$; But for some special region of the Dalitz plot $[\delta\Gamma(\pi^\pm\pi^0\gamma)] \lesssim 10^{-4}$	Supersymmetry extension of SM
The decays $K^+ \to \pi^pm\gamma\gamma$			
h	[44]	$[\delta\Gamma(\pi^\pm\gamma\gamma)] \lesssim 10^{-4}$	SM

present Hyper CP experiment [48] and in other experiments [5, 49] which are in preparation now. Thus it would be more clear in the begin of CKM measurements if it would be reasonable to do a next and more precise step in further extension of these very difficult experiments on charge asymmetry in K^+ and K^- decays.

9 $K^+ \to \pi^+e^+e^-, K^+ \to \pi^+\mu^+\mu^-$ Form Factor and Branching Ratio Measurements

The precise study of the decays $K^+ \to \pi^+e^+e^-$ and $K^+ \to \pi^+\mu^+\mu^-$ opens new possibilities for the investigation of chiral perturbation theory (CHPT) in the $\triangle S = 1$ weak interactions and the search for anomalous couplings. Careful measurements of the corresponding branching ratios and Dalitz plot analyses to measure the form factors in these decay modes are tests of CHPT predictions.

The integral ratio of these two mode $R(\mu\mu/ee) = BR[K^+ \to \pi^+\mu^+\mu^-)]/BR[K^+ \to \pi^+e^+e^-]$ and the differential ratios as functions of kinematic variables $\rho(\mu\mu/ee)$ cancel many of the theoretical uncertainties and provide a focused comparison of CHPT predictions with these weak semileptonic interactions [57, 58]. In different models the predicted values of $R(\mu\mu/ee)$ vary in the range 0.2 – 0.3. Measurements with experimental accuracy of $\delta R = 2 - 3\%$ should give clean tests of the CHPT predictions to the 10% level. One early paper [59] suggested that the differential ratio $\rho(\mu\mu/ee)$ in the region if small π^+ energies is sensitive to the contribution of second order weak diagrams.

The existing data for these two decay mode are dominated by results from recent BNL K^+ experiments and first of all by detailed study of $K^+ \to \pi^+e^+e^-$ decay in E865 [60]. The large statistics obtained for this decay mode (10300 events) gives possibility to make following conclusions:

1. Experimental data are consistent with vector model for the decay process (which proceeds through one photon exchange) with amplitude $\frac{\alpha G_F}{4\pi} f_v(z) p^\mu \bar{u}_e \gamma_\mu u_e$ (here $p^\mu = p_K^\mu + p_\pi^\mu, q^\nu = p_K^\nu - p_\pi^\nu, z = q^2/m_K^2 = m_{e^+e^-}^2/m_K^2$). For 90% c.l. of most 2% of the decay width $\Gamma(K^+ \to \pi^+ e^+ e^-)$ could result for either scalar or tensor interactions (with amplitudes $G_F m_K f_S(z) \bar{u}_e u_e$ and $G_F f_T(z) \frac{p^\mu q^\nu}{m_K} \bar{u}_e \sigma_{\mu\nu} u_e$). This corresponds to upper limits $|f_S| < 6.6 \cdot 10^{-5}$ and $|f_T| < 3.7 \cdot 10^{-4}$.

2. Form factor $f_v(z)$ can be parameterized in the CHPT model [61] by expression $f_v(z) = a_+ + b_+ z + W^{\pi\pi}(z)$, where a_+, b_+ are parameters of the model and $W^{\pi\pi}(z)$ is contribution from a charged pion loop coupled through a virtual photon to e^+e^- transition. This form factor was measured in [60] by studying $d\Gamma/dz$ spectrum of e^+e^- pairs. The slope of form factor is significantly larger than CHPT pion calculations in $O(p^4)$ order are predicted (in this order $b_+ \simeq 0$ and from the experiment $a_+ = -0.587 \pm 0.010, b_+ = -0.655 \pm 0.044$). Although linear approximation for form factor is reasonable for $z \lesssim 0.3$ the experimental data clearly indicated the nonlinearity for larger values of $z(q^2)$ which is fit well by CHPT loop term (see Fig. 12). In [62] it was shown that the observed $f_v(z)$ is well explained also by combined effect of the pion and kaon form factors in vector meson dominance model and pion loop graph. After including form factor corrections the branching ratio $BR(K^+ \to \pi^+ e^+ e^-) = [2.94 \pm 0.05(\text{stat.}) \pm 0.13(\text{syst.}) \pm 0.05(\text{model})] \cdot 10^{-7}$ was obtained.

All the main $K^+ \to \pi^+ l^+ l^-$ data [60, 63–66] are presented in Table 12. From this table it is clear that for $K^+ \to \pi^+ e^+ e^-$ decay there is a reasonable agreement between the experiments [60] and [64]. But the data for $K^+ \to \pi^+ \mu^+ \mu^-$ decay from E787 [66] and E865 [65] are in the disagreement by more than 3.6σ for reasons that are not yet understood. From the E865 data the ratio $R(\mu^+\mu^-/e^+e^-)_{exp} = BR(K^+ \to \pi^+\mu^+\mu^-)/BR(K^+ \to \pi^+ e^+ e^-) = 0.314 \pm 0.031$ which can be compared with $R(\mu^+\mu^-/e^+e^-)_{CHPT} \gtrsim 0.23$ [61].

In CKM we estimate sensitivity to this two decay modes of order 10^3 and $2 \cdot 10^3$ events per week for $K^+ \to \pi^+ e^+ e^-$ and $K^+ \to \pi^+ \mu^+ \mu^-$ respectively (in parallel with the main $K^+ \to \pi^+ \nu \bar{\nu}$ measurements). Thus it would be possible to obtain tremendous statistics which can be used for very careful form factor measurements and their comparison with very complicated future $O(p^6)$ predictions for CHPT, for real significant test of chiral approach and for further search for new types of interactions (scalar and tensor).

In the CKM it is possible to use additional identification of one of the muons in $K^+ \to \pi^+ \mu^+ \mu^-$ in pion RICH (by the charge of dropping the efficiency for this process by factor of 2) to suppress the background for this reaction from $K^+ \to \pi^+ \pi^+ \pi^-$ with two pion decays in flight. We have also very good identification of electrons in $K^+ \to \pi^+ e^+ e^-$.

10 Conclusion

We considered in this paper only several rare kaon decays to illustrate the possibility of the CKM experiment and to compare them with other measurements. Certainly the number of interesting processes can be significantly increased as is seen from Table 3.

It is clear from this paper that the CKM experiment will open many exciting possibilities for the search for new effects and for careful study of low energy hadron physics in favorable conditions of kaon decay processes. The sensitivity of the CKM experiment for the rare processes is on the level of 10^{-12} and exceeds all other experiments by one or 2 orders of magnitude. New advances in the searches for LEV in kaon decays, for direct CP violation effects, for new types of interactions and for hadronic dynamics studies are the highlights of the CKM which can be comparable for their importance with the main experimental program of studying FCNC effects and properties of CKM matrix in $K^+ \to \pi^+ \nu \bar{\nu}$ decays.

For further development of this experimental program we continue now a careful study of possibilities of the setup, it's trigger and DAQ requirements, background and systematics with processes under study. Further development of physical program is also under consideration.

I am greatly obliged to members of the CKM Collaboration for stimulating discussions and their help. My special gratitude for V. F. Obraztsov for some discussion of T-odd correlation in K decays and the

Table 12: The results from BNL experiments for studying $K^+ \to \pi^+ l^+ l^-$ decays and expected CKM statistics

	E777 BNL [63]	E851 BNL [64]	E787 BNL [66]	E865 BNL [60,65]	Expected CKM statistics
$N(K^+ \to \pi^+ e^+ e^-)$ events	~ 500	~ 800	-	~ 10300 [60]	$1.2 \cdot 10^5$
$N(K^+ \to \pi^+ \mu^+ \mu^-)$ events	-	-	196 ± 17	~ 430 [65]	$2.2 \cdot 10^5$
$BR(K^+ \to \pi^+ e^+ e^-)$	$(2.75 \pm 0.26) \cdot 10^{-7}$	$(2.81 \pm 0.20) \cdot 10^{-7}$	-	$(2.94 \pm 0.15) \cdot 10^{-7}$ [60]	
$BR(K^+ \to \pi^+ \mu^+ \mu^-)$	-	-	$(5.0 \pm 1.0) \cdot 10^{-8}$	$(9.22 \pm 0.77) \cdot 10^{-8}$ [65]	
$R(\mu^+\mu^-/e^+e^-)$	-	-	-	0.314 ± 0.031	
λ (for form factor parametrization $F(Z) = 1 + \lambda Z$)	1.31 ± 0.48	-	-	$2.14 \pm 0.20 (K^+ \to \pi^+ e^+ e^-)$ [60] $2.45^{+1.30}_{-0.95} (K^+ \to \pi^+ \mu^+ \mu^-)$ [65]	

search for different types of interactions and for D. V. Vavilov for the help with many calculations and with pictures.

References

[1] J. Frank et al., A Proposal for a Precision Measurement of the Decay $K^+ \to \pi^+ \nu\bar{\nu}$ and Other Rare K^+ Processes at Fermilab Using the Main Injector, Fermilab, April 2, 2001.

[2] S. Adler et al., Phys. Rev. Lett. **79**, 2204 (1997); S. Adler et al., Phys. Rev. Lett. **84**, 3768 (2000).

[3] B. Bassalleck et al. (BNL E949), "An experiment to measure $B(K^+ \to \pi^+ \nu\bar{\nu})$ at BNL", BNL 67247 (1999).

[4] B. Aubert et al. (BaBar Collab.), hep-ex/0107013; Phys. Rev. Lett. **87**, 091801 (2001);
K. Abe et al. (Belle Collab.), hep-ex/0107061; Phys. Rev. Lett. **87**, 091802 (2001).

[5] L. G. Landsberg, V. F. Obraztsov, Proceedings of the Workshop on K-Physics KAON-99, Chicago, 1999, p. 589.

[6] L. G. Landsberg, D. V. Vavilov, CKM Note 48, 2001.

[7] L. G. Landsberg et al., CKM Note 47, 2001.

[8] R. Appel et al., Phys. Rev. Lett. **85**, 2450 (2000).

[9] R. Appel et al., Phys. Rev. Lett. **85**, 2874 (2000).

[10] D. Ambrose et al., Phys. Rev. Lett. **81**, 5734 (1998).

[11] KTeV preliminary results, Conf. APS Division of Part. and Field, Columbus Ohio, August 9-12, 2000.

[12] A Alavi-Harati et al., hep-ex/0108037.

[13] L. M. Barkov et al., Research Proposal to PSI, 1999; http://www.icepp.s.utokyo.ac.jpmeg;
M. Bachman et al., MECO Proposal to BNL, 1997; http://meco.ps.uci.edu.

[14] R. N. Cahn, H. Harari, Nucl. Phys. **B176**, 135 (1980).

[15] J. Ritch, S. Wojcicki, Rev. Mod. Phys. **65**, 1149 (1992);
T. S. Kosmas et al., Preprint Univ. of Joannina IOA: 300/93, Ioannina, 1993;
J. Bordes et al., Phys. Rev. **D60**, 013005 (1999).

[16] T. G. Rizzo, hep-ph/9809526, "Proceedings of the Workshop on CP Violation", University of Adelaide, Adelaide, Australia, 3-8 July, 1998.

[17] A. Belyaev et al., hep-ph/0008276; hep-ph/0107046.

[18] L. Littenberg, R. Shrok, Phys. Rev. Lett. **68**, 443 (1992); hep-ph/0010048.

[19] Z. Gagyi-Palffy et al., Nucl. Phys. **B513**, 517 (1998); Z. Gagyi-Palffy et al., Preprint RAL-090, Chilton, 1994.

[20] S. Davidson et al., Z. Phys. **C61**, 613 (1994).

[21] C. Did et al., Phys. Lett **B493** (2000) 82-87. K-meson neutrinoless double muon decay as a probe of neutrino masses and mixings, hep-ph/0006277.

[22] M. Schepkin, Private communication; CKM Note 43.

[23] K. Zuber, Phys. Lett. **B479** (2000) 33-36. New limits on effective Majorana neutrino masses from rare kaon decays, hep-ph/0003160.

[24] C. Picciotto, Phys. Rev. **D56** 1612 (1997).

[25] R. G. Sachs, The Physics of Time Reversal, Univ. of Chicago Press, 1987.

[26] G. C. Branko, L. Lavora, J. P. Silva, CP-violation, Clarendon Press, Oxford, 1999.

[27] I. J. Bigi, A. I. Sanda, CP-violation, Cambridge Univ. Press, 1999.

[28] J. J. Sakurai, Phys. Rev. **109**, 980 (1958).

[29] A. R. Zhitnitskii, Yad. Fiz, **31**, 1014 (1980);
V. P. Efrosinin et al., hep-ph/0008199 Aug, 2000.

[30] Y. Grossman, Nucl. Phys. **B426**, 355 (1994);
G. Belanger, C. Geng, Phys. Rev. **D44**, 2789 (1991);
R. Peccei, hep-ph/9909236, Overview of Kaon Physics to be published in the Proceedings of KAON99, Chicago, Illinois June 1999.

[31] E. P. Shabalin, UFN, **171**, 951 (2001).

[32] S. Blatt et al., Phys. Rev. **D27**, 1056 (1983).

[33] M. Abe et al., Phys. Rev. Lett. **83**, 4253 (1999).

[34] L. Littenberg, Proc. of Int. KEK Workshop, "Kaons, Muons, Neutrino Physics and Future", KEK, 31 October - 1 November 1997 (ed. Y. Kuno, T. Shinkava), KEK Proc. 97-24, P. 27.

[35] J. Gevais, J. Iliopoulos, J. Kaplan, Phys. Lett. **20**, 432 (1966).

[36] V. Bragita et al., hep-ph/0106147.

[37] J. Bignens et al., The Second DANE Phys. Handbook (ed. L. Maiani et al.) LNF, Frascati, 1995, P. 315.

[38] V. N. Bolotov et al., Yad. Fiz. **44**, 68 (1986).

[39] H. Y. Cheng, Phys. Rev. **D26**, 143 (1982).

[40] O. Shanker, Nucl. Phys. **B204**, 375 (1982); **B206**, 253 (1982).

[41] M. Finkemeier, The Second DANE Phys. Handbook (ed. L. Maiani et al.) LNF, Frascati, 1995, P. 389.

[42] S. S. Bulanov, Master Thesiso, MFTI, Moscow, 1999.

[43] D. E. Groum et al. (PDG), Eur. Phys. J. **C15**, 1 (2000).

[44] G. D'Ambrosio, G. Isidori, Int. Journ. of Mod. Phys. **A13**, 1 (1998).

[45] L. Maiani, and P. Paver, The Second DANE Phys. Handbook, **1**, 51 (1995).

[46] P. Franzini, The Second DANE Phys. Handbook (ed. L. Maiani et al.) LNF, Frascati, **1**, 15 (1992).

[47] V. V. Ammosov et al., Preprint IHEP 98-2, Protvino, 1998.

[48] N. Leros, Nucl. Phys. B (Proc. Suppl.) **99B**, 211 (2001).

[49] R. Batley et al., Proposal P253/CERN (SPSC); CERN/SPSC 2000-003.

[50] C. O. Dib, R. D. Peccei, Phys. Lett. **B249**, 325 (1990).

[51] A. A. Bel'kov et al., Phys. Lett. **B232**, 118 (1989); Phys. Lett. **B300**, 283 (1993).

[52] A. A. Bel'kov et al., hep-ph/0010142.

[53] E. P. Shabalin, Nucl. Phys. **B409**, 87 (1993); Yad. Fiz. **62**, 1552 (1999).

[54] E. P. Shabalin, Yad. Fiz. **62**, 1657 (1999).

[55] G. D'Ambrosio, hep-ph/9911522.

[56] G. Colangelo et al., hep-ph/9908415.

[57] G. Ecker et al., Nucl. Phys. **B291**, 692 (1987).

[58] L. Bergstrom, P. Singer, Phys. Rev. Lett. **55**, 2633 (1985); Phys. Rev. **D43**, 1568 (1991).

[59] E. Shabalin, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 117 (1975).

[60] R. Appel et al., Phys. Rev. Lett. **83**, 4482 (1999).

[61] G. D'Ambrosio et al., JHEP. **8**, 4 (1998).

[62] H. Burkhardt et al., hep-ph/0011345.

[63] A. L. Despande, Ph. D. thesis, Yale University, 1995.

[64] C. Alliegro et al., Phys. Rev. Lett. **68**, 278 (1992).

[65] H. Ma et al., Phys. Rev. Lett. **84**, 2580 (2000).

[66] S. Adler et al., Phys. Rev. Lett. **79**, 4756 (1997).

List of Tables

List of Figures

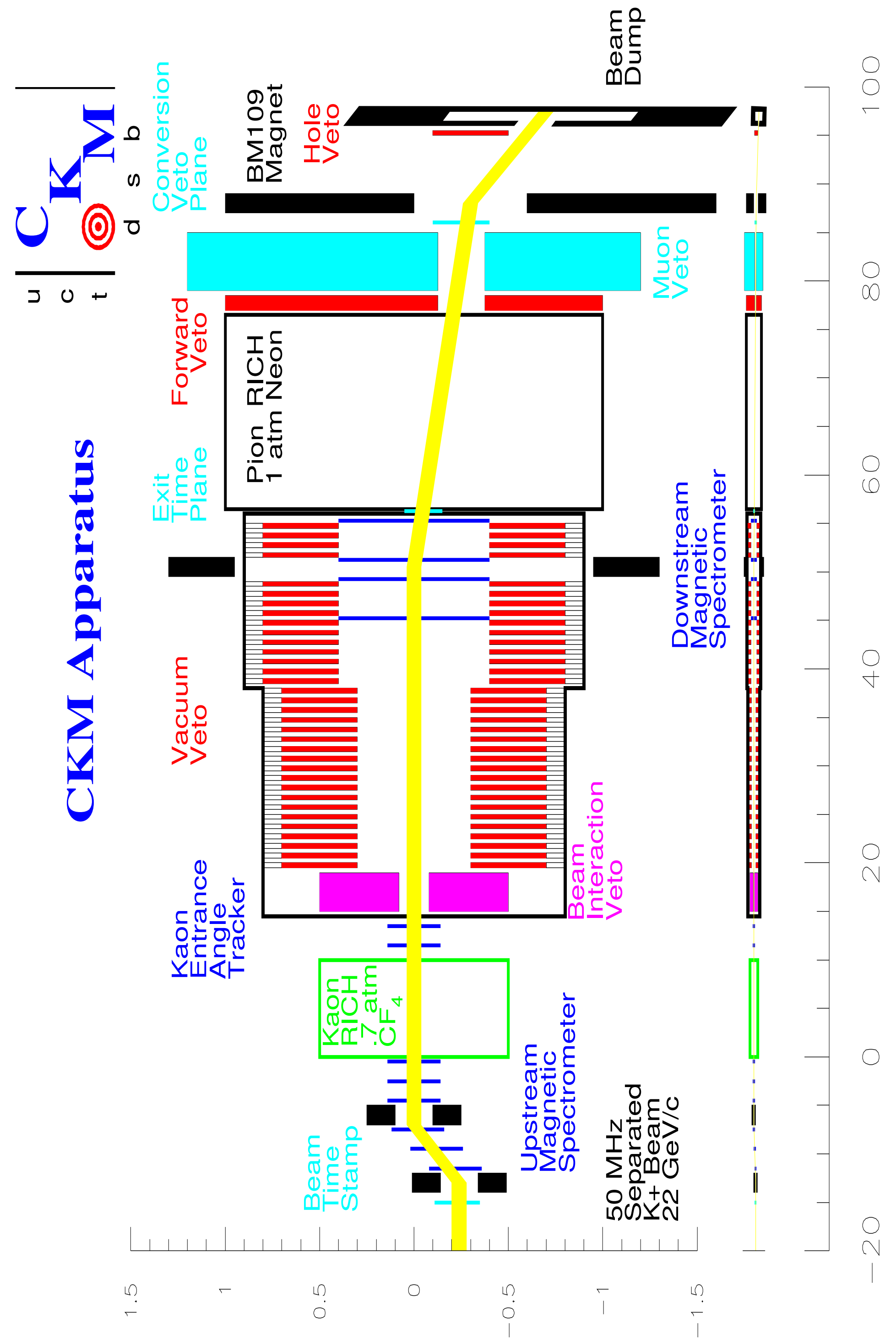

Figure 1: The CKM Detector [1]. The lower section show the true proportions of the apparatus. The main components of the setup are presented in this figure (see also text).

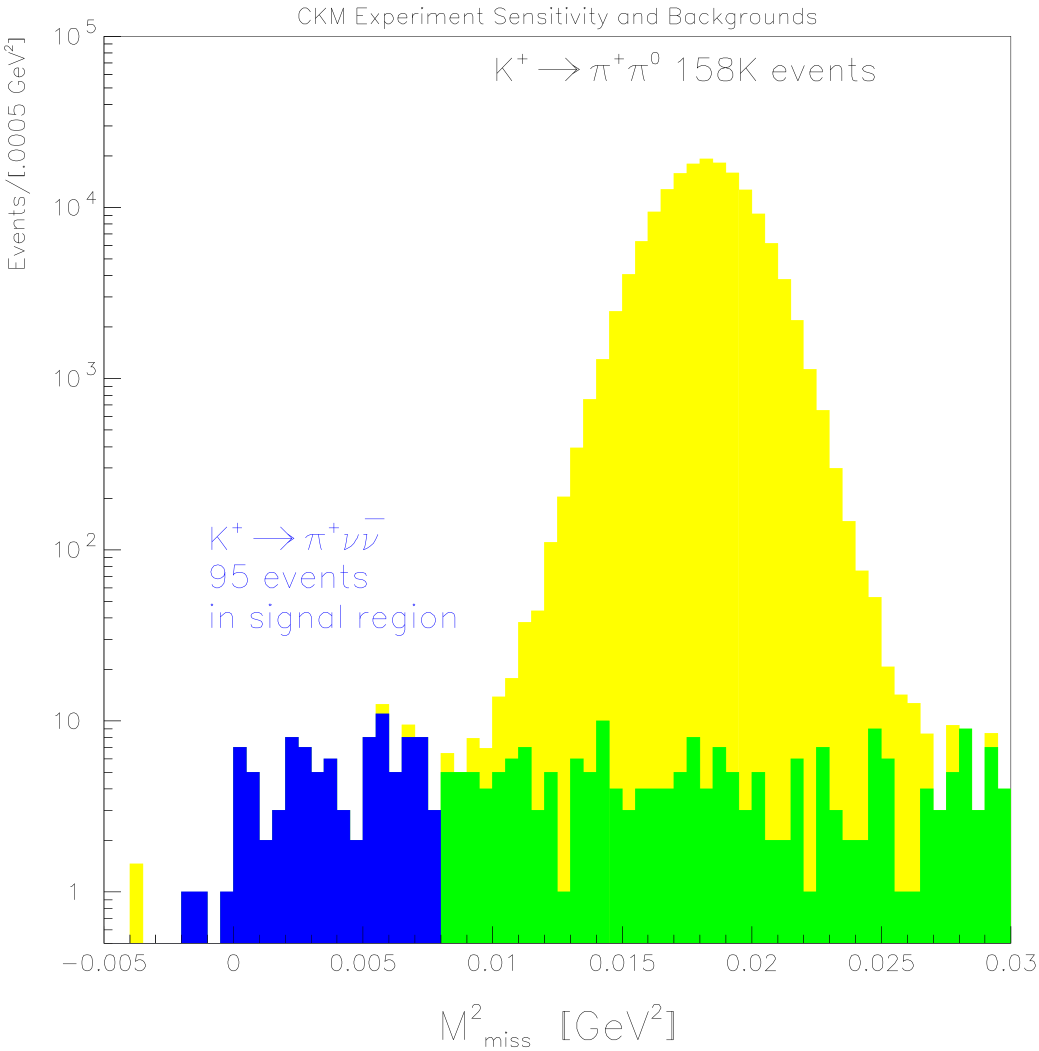

Figure 2: The combined measured missing mass squared ($M^2_{\nu\bar{\nu}}$) of the two spectrometers reconstructed in CKMGEANT. The signal region is shaded [1].

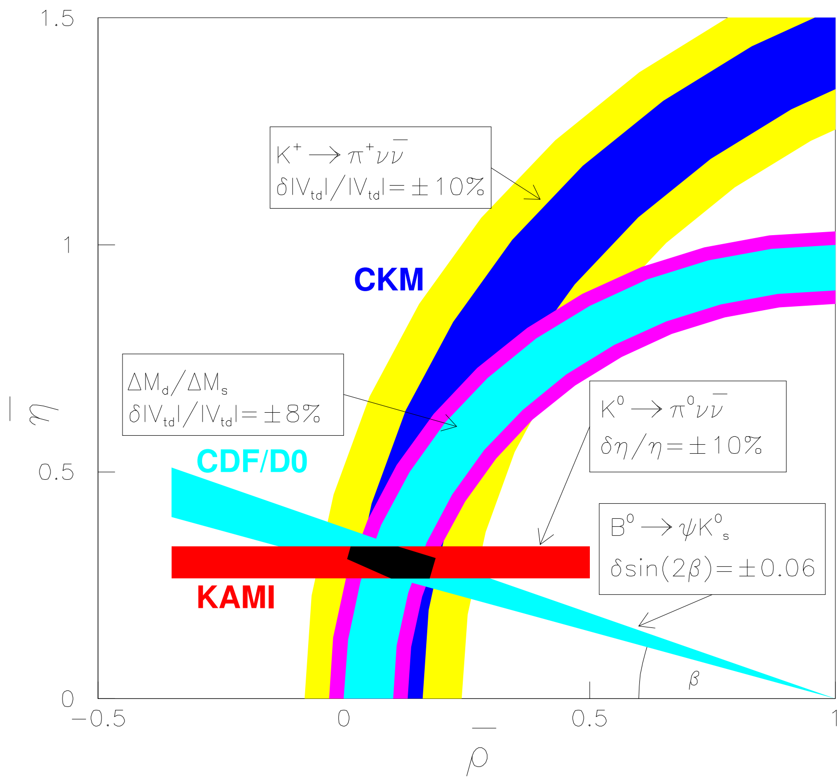

Figure 3: A possible outcome for parameters of unitary triangle $\bar{\eta}$, $\bar{\rho}$ and β obtained from future K-meson and B-meson experiments: $K^+ \to \pi^+\nu\bar{\nu}$ (CKM); $K^0_L \to \pi^0\nu\bar{\nu}$ (KAMI/K0PI0) and $\triangle m_{B_d}/\triangle m_{B_s}$; $B^0(\bar{B}^0) \to J/\psi K^0_S$. The light region includes possible theoretical uncertainties in $BR(K^+ \to \pi^+\nu\bar{\nu})$ connected with the influence of c-quarks [1].

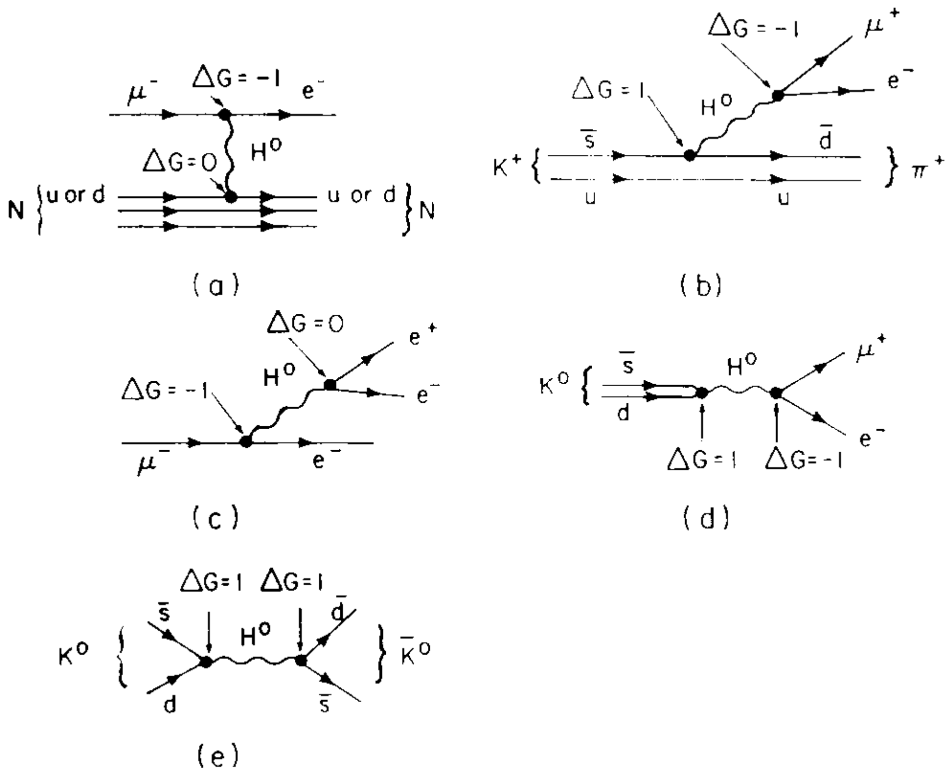

Figure 4: Diagrams for lepton flavor violation processes in the model [14] with generation number selection rule (see text and Table 6)

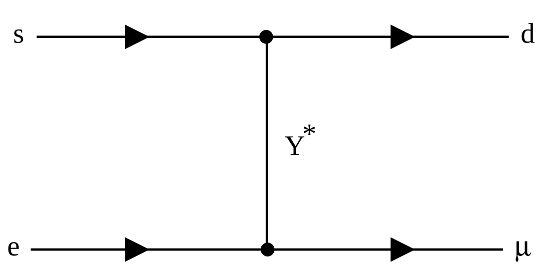

Figure 5: Diagram for lepton flavor violation in K decay with leptoquark Y^* exchange mechanism

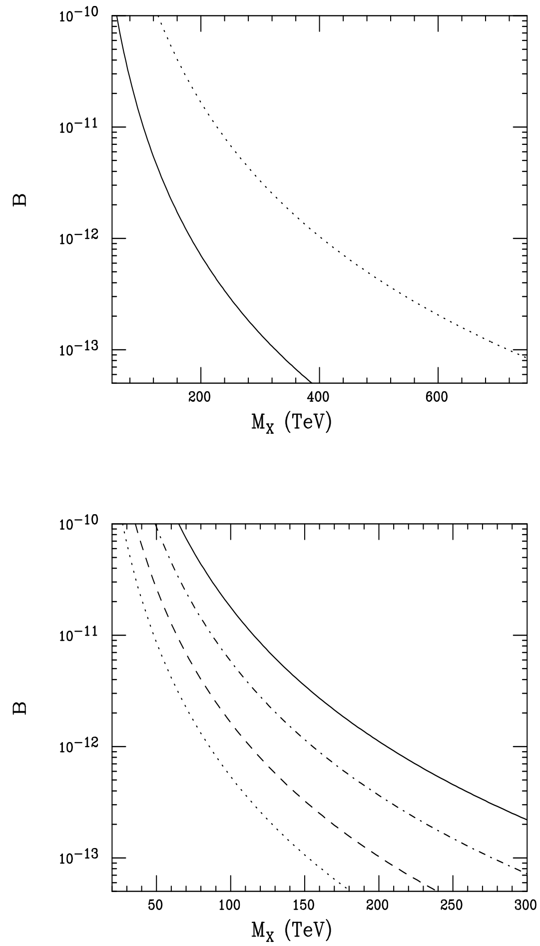

Figure 6: Branching fractions for $K_L \to e\mu$ (a) and $K \to \pi e\mu$ (b) as function of the scale parameter $M_X = M_H$ in (1) (assuming $g_H = g$ – weak constant). In (a) the solid (dotted) curves corresponds to A(P) exchange. In (b) the K_L^0 mode represented by the solid (S) and dashed (V) curves while the K^+ mode is represented dash-dotted (S) and dotted (V) curves (see [16]). Here V, A, S, P – vector, axial-vector,scalar and pseudoscalar interactions.

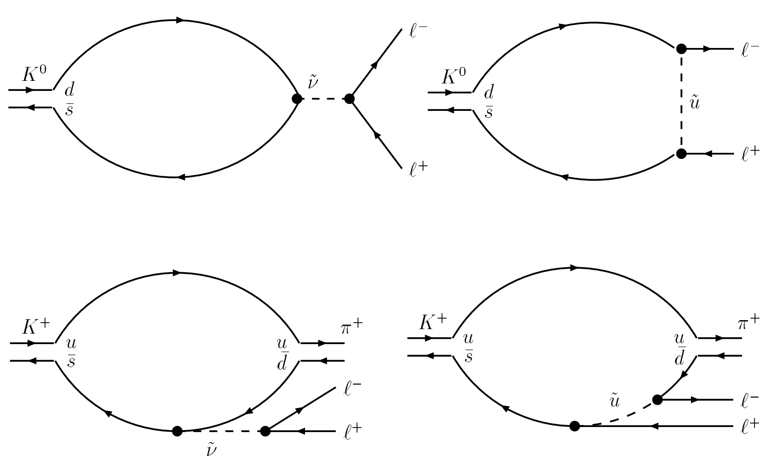

Figure 7: Diagrams for $K \to \mu e$ and $K \to \pi\mu e$ in SUSY model with R-parity nonconservation [17]

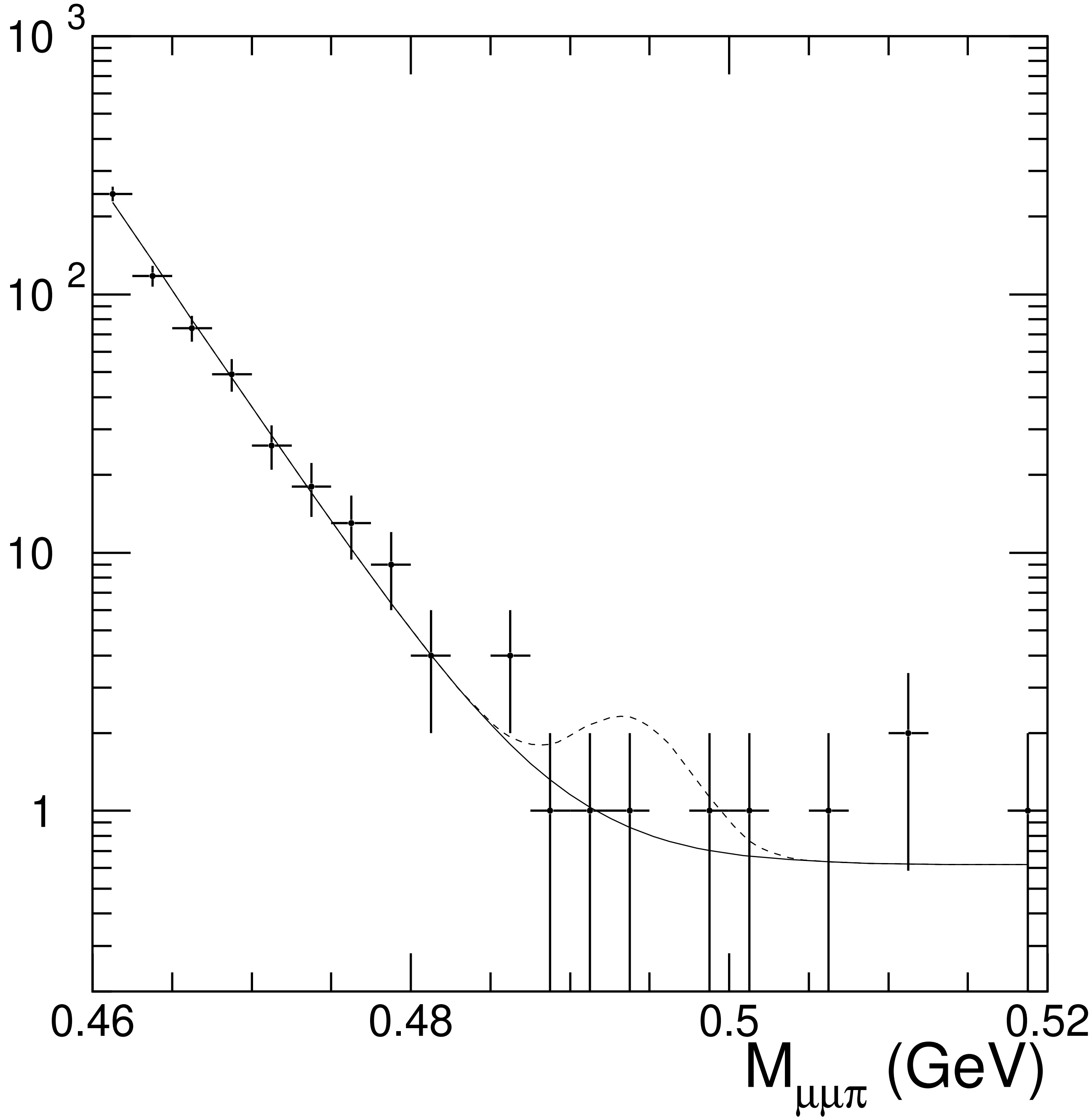

Figure 8: E865 data for $K^+ \to \pi^- \mu^+ \mu^+$. The dashed curve corresponds to $BR(K^+ \to \pi^- \mu^+ \mu^+) \simeq 3.0 \cdot 10^{-9}$

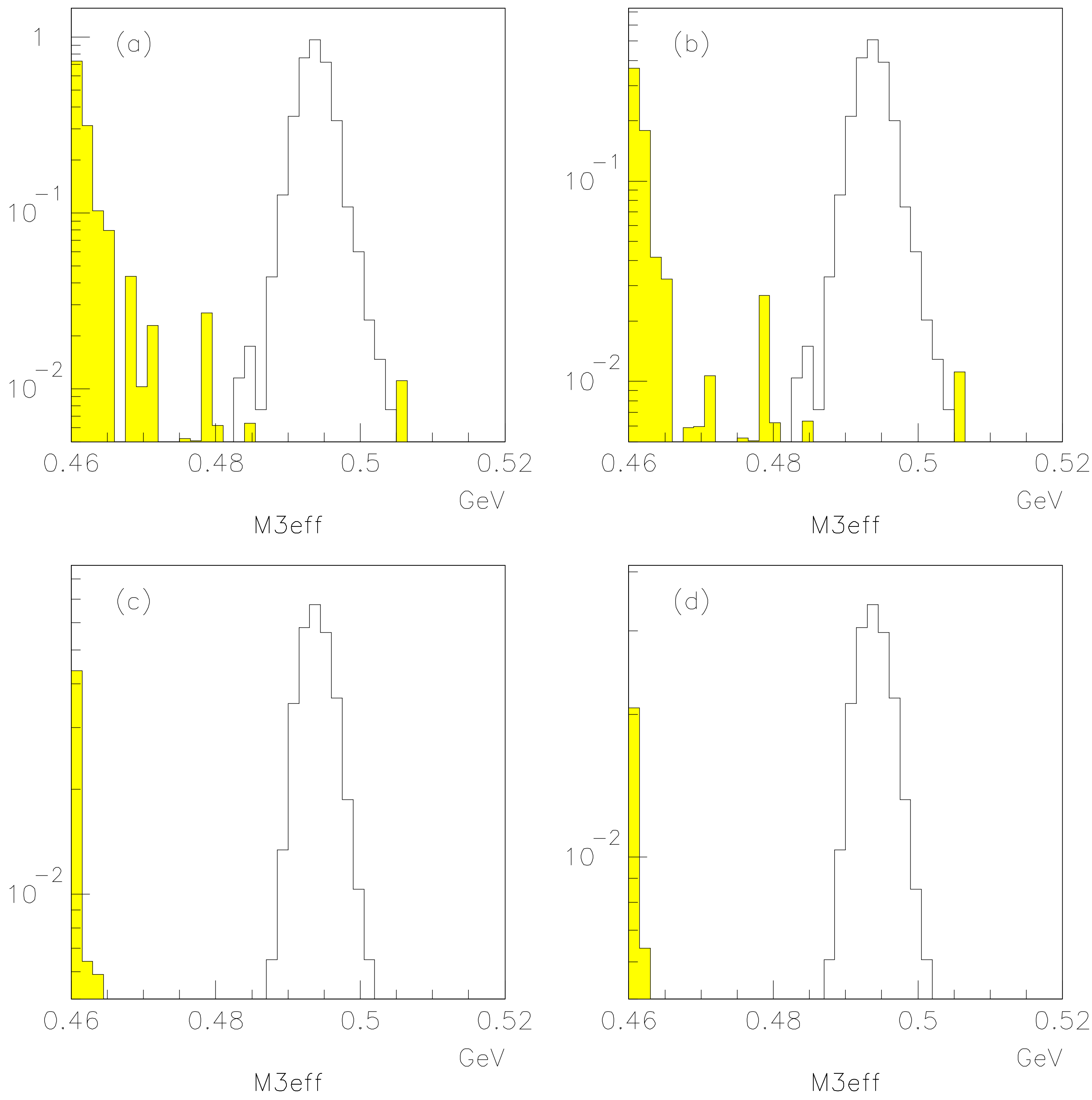

Figure 9: $K^+ \to \pi^- \pi^+ \pi^+$ background (filled histogram) after: (a) "standard cuts"; (b) $M_{3\pi}$ cut; (c) RICH cut; (d) both (b) and (c). Peaks in the middle correspond to the signal but the vertical scale is different.

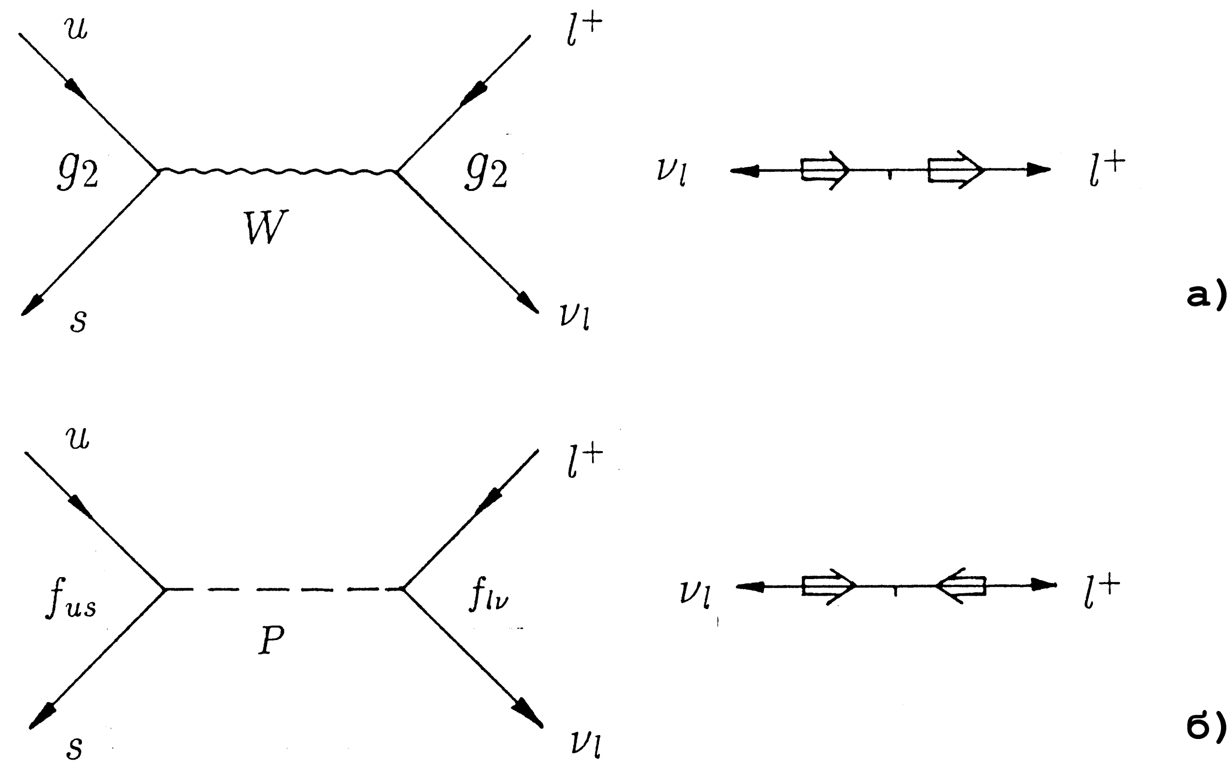

Figure 10: (a) Diagram for decay $K^+ \to l^+\nu_l$ within the Standard Model ($V - A$ interaction) for the case where the decay process proceeds through the exchange of a vector W boson. The Fermi coupling constant is $G_F = g_2^2/4\sqrt{2}M_W^2$. In this model the helicities of leptons and antileptons are, respectively, left- and right-handed. The decay $K^+ \to l^+\nu_l$ is suppressed by helicity conservation (the K^+-meson spin is zero); the helicity-flip amplitude is proportional to the mass m_l. (b) Diagram for the decay $K^+ \to l^+\nu_l$ within the model involving anomalous pseudoscalar weak interaction which coupling constant is $(G_{US}^{l\nu})_{PS} = f_{l\nu} \cdot f_{US}/4\sqrt{2}M_P^2$. In the case of P interaction the decay is no longer suppressed by helicity conservation.

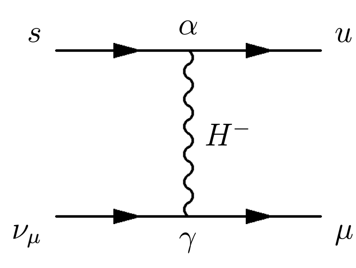

Figure 11: Diagram for $\mathrm{Im}\, g_S$ from H^- Higgs boson exchange; α, γ – Higgs interaction constants, M_{K^-} and M_{H^-} – masses of kaon and Higgs. $\mathrm{Im}\, g_S = \mathrm{Im}\, \alpha^*\gamma(M_{K^-}/M_{H^-})^2$.

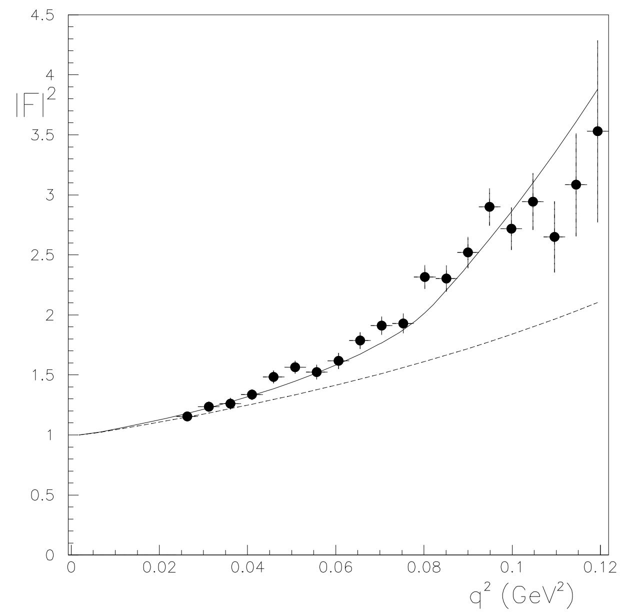

Figure 12: Formfactor squared $|F(q^2)|^2$ for $K^+ \to \pi^+ e^+ e^-$ as function of $q^2 = m_{ee}^2$ from [60]. The dash curve shows the approximation without the charge pion loop term. The solid curve includes the loop term. The theoretical curves are normalized to 1 at $q^2 = 0$ [62].