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# Lepton flavory violating processes in $K^+$ decays in CKM

The main decay processes with lepton flavory violation (LFV) are summarized in Table 1 where upper limits for these decays obtained mainly in E865 BNL experiment [1, 2] are also presented. For completeness we included in this table also corresponding LFV results from $K_L^0$ decays [3, 4].

| $BR$ | Upper limit (90% c.l.) | Expected efficiency in CKM | Expected Statistical sensitivity in CKM (90% c.l. upper limits without background) |
|---|---|---|---|
| $BR(K^+ \to \pi^+\mu^+e^-)$ | $2.8 \cdot 10^{-11}$ [1] | 0.030 | $2.2 \cdot 10^{-12}$ |
| $BR(K^+ \to \pi^+\mu^-e^+)$ | $5.2 \cdot 10^{-10}$ [2] | 0.030 | $2.2 \cdot 10^{-12}$ |
| $BR(K^+ \to \pi^-\mu^+e^+)$ | $5.0 \cdot 10^{-10}$ [2] | 0.027 | $2.2 \cdot 10^{-12}$ |
| $BR(K^+ \to \pi^-\mu^+\mu^+)$ | $3.0 \cdot 10^{-9}$ [2] | 0.086 | $7.2 \cdot 10^{-13}$ |
| $BR(K^+ \to \pi^-e^+e^+)$ | $6.4 \cdot 10^{-10}$ [2] | 0.012 | $5.6 \cdot 10^{-12}$ |
| $BR(K_L^0 \to \mu e)$ | $4.7 \cdot 10^{-12}$ [3] | | |
| $BR(K_L^0 \to \pi^0\mu e)$ | $4.4 \cdot 10^{-10}$ [4] | | |

Table 1: Lepton flavory violation kaon decays

From the statistical consideration there is a clear opportunity for the CKM experiment [5] to perform (in parallel with the main $K^+ \to \pi^+\nu\bar{\nu}$ study) much more sensitive searches of all those $K^+$ LFV decays up to the level $BR \sim 10^{-12}$. These limits correspond to effective number of $K^+$ decays in 2 years of measurement $N(K^+\,\text{decays}) = (3.6 \cdot 10^{11}\,\text{decays/week}) \cdot (78\,\text{weeks}) = 2.81 \cdot 10^{13}\,(K^+\,\text{decays})$. These numbers include a safety factor 0.5 to take into account dead time effects and additional cuts. Certainly very careful investigations of possible background limitations for these searches must be done. Some of these works have already begun.

Even sensitivity of $10^{-12}$ for kaon LFV processes looks now modest to compare with ambitious new proposals for the search for $\mu \to e\gamma$ and $\mu^- + A \to e^- + A$ LFV processes on the level $BR < 10^{-14} \div 10^{-16}$ [6]. The difference in the sensitivities of future muon and kaon LFV experiments would be even larger due to additional factor $\tau_\mu/\tau_K \sim 10^2$ for the corresponding partial decay widths: $\Gamma(\text{kaon LFV})/\Gamma(\text{muon LFV}) = [BR(\text{kaon LFV})/BR(\text{muon LFV})] \cdot (\tau_\mu/\tau_K) \sim 10^4 \div 10^6$.

But it is very important to stress that the searches for LFV in kaon decays are complimentary to the $\mu \to e\gamma$ type of the experiments. Kaon decays are the only processes which are sensitive to $s \to d\mu\bar{e}$ interactions which may be exhibit of very exceptional properties and open an unique possibility to study lepton flavory violating decays with more favorable conditions for a hypothetical selection rule connected with a fundamental generation number conservation. In other words, for these decays a change of generation in the leptonic sector can be compensated by corresponding changing of generation in the quark sector ([7], see also [8–10]). Let us introduce the generation number $G_1 = 1$ for the first fundamental fermion generation and $G_2 = 2$ for the second one (for antifermions $G_1 = -1$; $G_2 = -2$). Then, as is seen from diagrams in figure 1, it is possible to classify different processes according to changing $\Delta G$, as it follows from Table 2. The strength of hypothetical $\Delta G$ selection rule is unclear now. Certainly, the new possibilities for the search for LFV processes in kaon decays will occure only if this strength is significant and every next order on $\Delta G$ will lead to significant suppression of corresponding process to compare with lower order. We nust bear in mind an unexpected difference between $K^+ \to \pi^+ e^- \mu^+$ and $K^+ \to \pi^+ e^+ \mu^-$ in the model with conserved generation number according to $\Delta G$ generation rule.

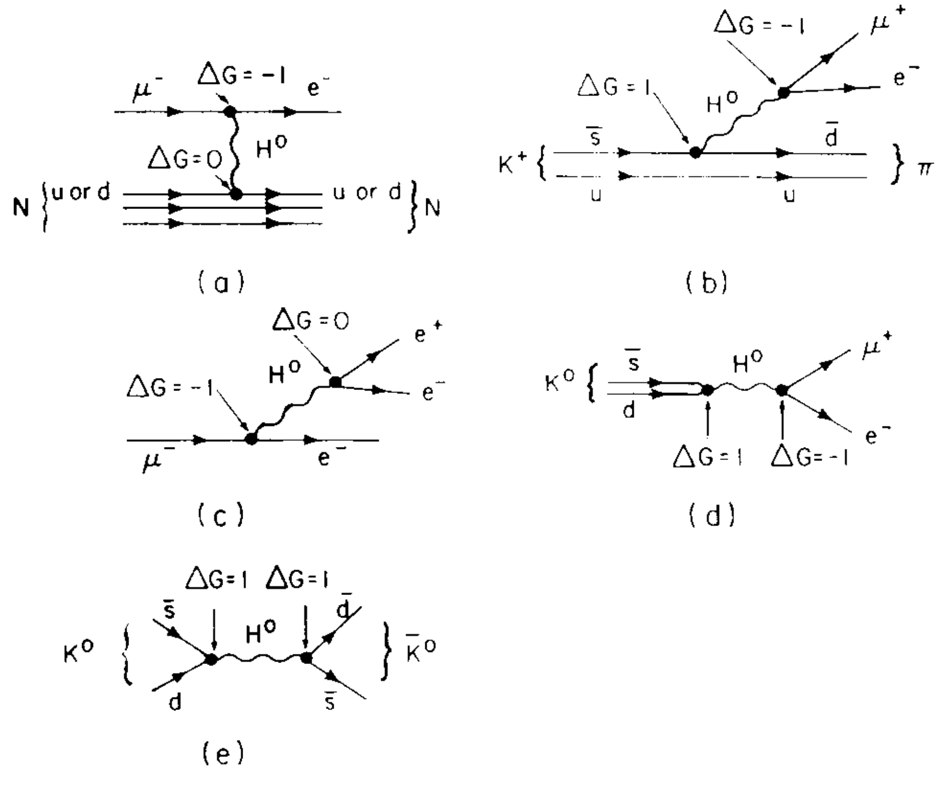


Figure 1: Diagrams with $\Delta G$ [7]

In the diagrams in figure 1 the decay processes are governed by exchange of "horisontal" intermediate boson $H^0$. But it is also possible to consider kaon decay LFV processes which are governed by exchange of leptoquarks $Y^*$ (see figure 2).

Amplitudes for $s \to d\mu\bar{e}$ process of pseudoscalar and axial-vector type (for $K_L^0 \to \mu\bar{e}$ decay) or with vector, scalar and tensor structure (for $K \to \pi\mu\bar{e}$ decay) with the "horizontal"

| | | |
|---|---|---|
| First order | $\Delta G = 0$ | $K^+ \to \pi^+\mu^+e^-$; $K_L^0 \to \mu^\mp e^\pm$;$K_L^0 \to \pi^0\mu^\mp e^\pm$ |
| Second order | $\vert\Delta G\vert = 1$ | $\mu \to 3e$; $\mu \to e\gamma$; $\mu^- N \to e^- N$; |
| Third order | $\vert\Delta G\vert = 2$ | $K^0 \rightleftarrows \bar{K}^0(\Delta m(K_1^0 - K_2^0))$; $\mu^- e^+ \to \mu^+ e^-$; $K^+ \to \pi^+\mu^-e^+$ |

Table 2: Classification of some rare generation changing processes according to $\Delta G$

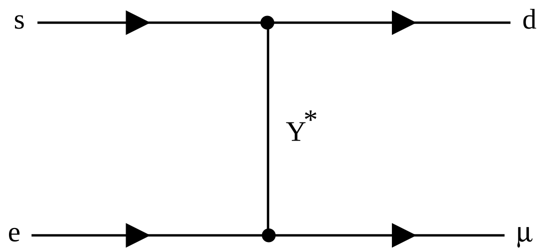


Figure 2: Leptoquark $Y^*$ exchange

$H^0$ boson exchange can be presented with the help of general generation number conserving operators [11]:

$$\left.\begin{array}{r}Q_{V,A} = \frac{g_H^2}{2M_H^2}\bar{d}\gamma_\mu[C_{Lq}P_L + C_{Rq}P_R]s \quad \cdot \quad \bar{\mu}\gamma^\mu[C_{Ll}P_L + C_{Rl}P_R]e + \text{h.c.} \\ Q_{S,P} = \frac{g_H^2}{2M_H^2}\bar{d}[C'_{Lq}P_L + C'_{Rq}P_R]s \quad \cdot \quad \bar{\mu}[C'_{Ll}P_L + C'_{Rl}P_R]e + \text{h.c.}\end{array}\right\} \qquad (1)$$

Here $P_L = \frac{1}{2}(1-\gamma_5)$; $P_R = \frac{1}{2}(1+\gamma_5)$; $C, C'$ – constants.
The results of calculation of branching ratios $K \to \mu e$ and $K \to \pi\mu e$ decays for interactions (1) are presented for illustration on figure 3 as function of the scale parameter $M_H$ (for $g_H = g$ – a weak constant; corresponding $C, C' = 1$).

There are other interesting models for $s \to d\mu e$ processes in the supersymmetry theories with R-parity nonconservation [12, 13], in left-right models with heavy Majorana neutrino [14] in leptoquark exchange model [15].

LFV processes are known to take place both in the minimal supersymmetric model (MSSM) and in its modifications which include violation of $R$-parity. As for the context of the MSSM, rare kaon decays allow one to correlate (s)quark and (s)lepton mixing, while in $R$-violating supersymmetry one can probe non-trivial products of Yukawa couplings.

Within framework of the MSSM the magnitudes of the predicted rates depend on the details of the masses and mixings of sparticles including the sneutrinos. Maximal contributions for kaon LFV decays are expected from the box diagrams involving chargino and neutralino exchange.

However, the universal GUT-scale parameters for sfermions and gauginos, when limited from the similar pure lepton decays $\mu \to e\gamma$ and $\mu \to e^+e^+e^-$, lead to a hopelessly low decay branchings in the order of $10^{-16} \div 10^{-18}$, and only a fine tuning of GUT parameters makes it of interest the search for lepton flavor violating $K$-decays.

But the situation can be much different in $R$-parity violating supersymmetry models as pointed out in [12, 13]. In this case the gauge symmetry of the SUSY model allows additional Yukawa coupling terms in the superpotential of the form $\lambda L_i L_j \bar{E}_k$, $\lambda' L_i Q_j \bar{D}_k$, $\lambda'' \bar{U}_i \bar{D}_j \bar{D}_k$.

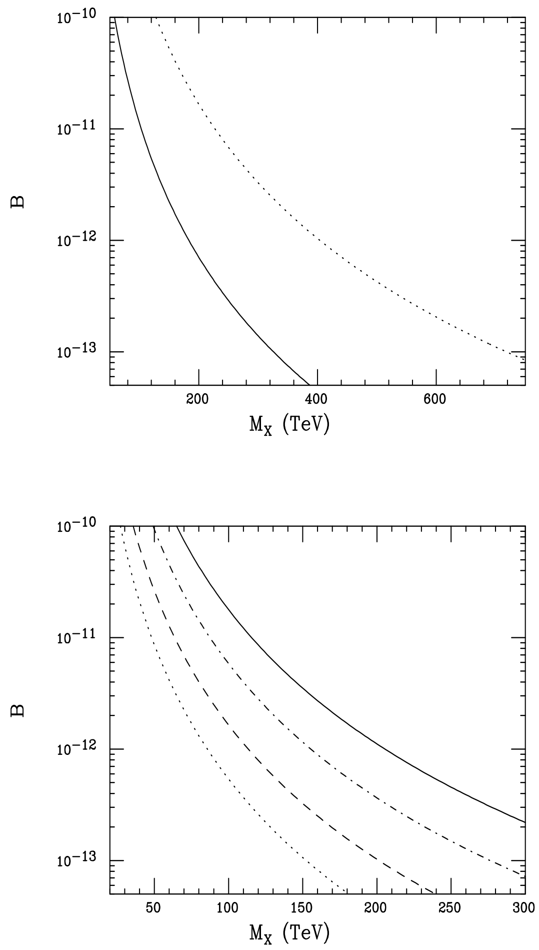


Figure 3: Branching fractions for $K_L \to e\mu$ (a) and $K \to \pi e\mu$ (b) as function of the scale parameter $M_X = M_H$ in (1) (assuming $g_X = g$ – weak constant). In (a) the solid (dotted) curves corresponds to A(P) exchange. In (b) the $K_L^0$ mode represented by the solid (S) and dashed (V) curves while the $K^+$ mode is represented dash-dotted (S) and dotted (V) curves (see [11]). Here V, A, S, P – vector, axial-vector,scalar and pseudoscalar interactions.

Here the $L(Q)$ are the left-handed lepton (quark) superfields and the $\bar{E}\,(\bar{D},\,\bar{U})$ are corresponding right-handed fields. Each of these terms give three Yukawa couplings. For example the first term is

$$\lambda_i j k(L_i L_j \tilde{E}_k + L_i \tilde{L}_j \bar{E}_k + \tilde{L}_i L_j \bar{E}_k)$$

It may be seen that this terms involve a single SUSY state and thus break the $R$-parity of MSSM. For electroweak SU(2) and color SU(3) symmetry it was shown that there are 45 $R$-violating couplings. Certainly, there are many experimental constrains on this set of constants (nonobservation of proton decay and some exotic processes, lepton universality, nonobservation of possible modification of different SM processes). In spite of these limitations it was stated in [12] that it is still possible that several $R$-violating operators may be large enough and useful limitations for some of them can be obtained from existing upper limits for LFV kaon decays and from their future searches. In this class of models, whilst $\mu \to e\gamma$ occurs at the one loop level, $\mu \to 3e$, $\mu \to e$ conversion, $K_L^0 \to \mu^\pm e^\mp$ and $K \to \pi\mu^\pm e^\mp$ decays may occur at the tree level via different combinations of couplings. Thus LFV kaon decays have the possibility to provide an important complimentary information for this $R$-parity violating processes.

For the illustration it is possible to consider $d\bar{s} \to l_j^- l_k^+$ interaction with effective Lagrangian written in terms of quark and lepton fields

$$\begin{aligned}\mathcal{L}_{d\bar{s}\to l_j^- l_k^+} &= \frac{1}{m_{\tilde{\nu}_i}^2}\left[\lambda_{ijk}^* \lambda'_{i12}(\bar{s}_R d_L(\bar{l}_{jL} l_{kR}) + \lambda_{ikj}\lambda'^*_{i12}(\bar{s}_L d_R)(\bar{l}_{jR} l_{kR})\right] - \\ &\quad -\frac{\lambda'^*_{ji1}\lambda'_{ki2}}{2m_{\tilde{u}_i}^2}(\bar{s}_R\gamma_\mu d_R)(\bar{l}_{jL}\gamma^\mu l_{kL}) \qquad (2)\end{aligned}$$

The tree diagrams for $K^0 \to l_j^- l_k^+$ and $K \to \pi l_j^- l_k^+$ decays are presented on figure 4.

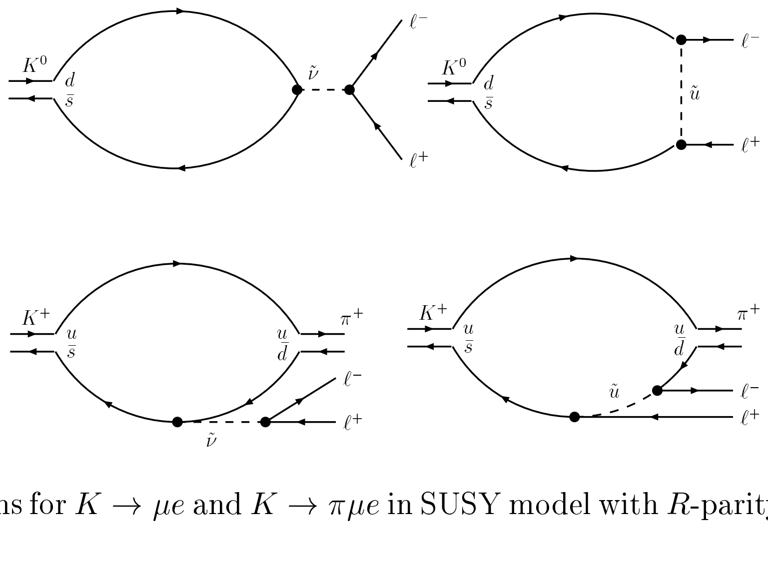


Figure 4: Diagrams for $K \to \mu e$ and $K \to \pi\mu e$ in SUSY model with $R$-parity nonconservation [12]

The calculations with (2) and upper limits for $K_L^0 \to \mu^\pm e^\mp$ decays (Table 1) give limitations for the values of $\lambda\lambda'/m_{\tilde\nu}$ and $\lambda\lambda'/m_{\tilde u}$ with sneutrino ($\tilde\nu$) and up squark ($\tilde u$) contributions

$$\left.\begin{array}{lll} \lambda_{i21}\lambda'_{i12}\left(\frac{100\,\mathrm{GeV}}{m_{\tilde\nu}}\right) & \leq & 6.2\cdot 10^{-9} \\ \lambda_{i21}\lambda'_{i12}\left(\frac{100\,\mathrm{GeV}}{m_{\tilde u}}\right) & \leq & 1.9\cdot 10^{-7} \end{array}\right\} \tag{3}$$

From $K \to \pi l_j^- l_k^+$ decays the upper limits for $\lambda\lambda'/m_{\tilde\nu}$ and $\lambda\lambda'/m_{\tilde u}$ are by 1-2 orders of magnitude larger than in (3).

It is also important to mention the unique properties of double muon decay process $K^+ \to \pi^-\mu^+\mu^+$ which were discussed [13, 16–18] in the model of cascade kaon decays with rather heavy Majorana neutrino (with mass $\sim 300\,\mathrm{MeV}$) $K^+ \to \nu_h + \mu^+$; $\nu_h \to \pi^-\mu^+$ and with small admixture of $\nu_h$ to the muon neutrino $\nu_\mu$ (this small admixture based for the well-known sea-saw mechanism). These is also possible to use $K^+ \to \pi^- e^+ e^+$ decay for the search of the some type Majorana neutrino in the processes $K^+ \to \nu_h e^+$; $\nu_h \to \pi^- e^+$ (due to small admixture of $\nu_h$ in $\nu_e$ based also on sea-saw mechanism – see [17]).

Other possibilities for double muon decay of kaons (with double charge Higgs boson exchange [19] and with heavy Majorana neutrino mechanism [18]) were also considered.

Thus, further searches for LFV kaon decays are very important since they can offer complementary information for possible theories with lepton flavory nonconservation which is not covered even with very sensitive new projects on the LEV searches in muon processes.

As was shown in Table 1 in the CKM experiment the expected statistical sensitivity for the search for LFV kaon decays is on the level of $BR \sim 10^{-12}$. But the real sensitivity can be reduced due to background for these searches. Here we consider only one example – the decay $K^+ \to \pi^-\mu^+\mu^+$ were the sensitivity of E865 was the worst one because of the background from $K^+ \to \pi^-\pi^+\pi^+$ with two pion decays in flight $\pi^+ \to \mu^+\nu$ (see figure 5). We studied this background for the CKM setup (see figure 6) with CKMGEANT MC simulation on the statistics of $10^7$ $K^+ \to \pi^-\pi^+\pi^+$ events with forced $\pi^+ \to \mu^+\nu$ decays for both $\pi^+$-s and with corresponding weights ($\sim 10^{-2}$) for proper normalization. We also performed MC simulation of the main LFV decay under study $K^+ \to \pi^-\mu^+\mu^+$.

As a first step we used "standard cuts" (some reasonable cuts on $\chi^2$ values, secondary particle energies at calorimeters and a cut on total $P_T^2$) listed in the Table 3 below: These

| Cut | Description |
|---|---|
| ntrack=4.and.numstrk=1 .and.ndmstrk.eq.3 | $K^+$ decay happened, Number of reconstructed tracks in UMS/DMS is 1/3 correspondingly |
| findok=1 | Secondary $\pi$ hitted in FVS, secondary $\mu$-s hitted MVS |
| $E_{\{\pi,\mu,\mu\}} > 2$ GeV | Energy of every secondary |
| $\chi^2_{\mathrm{DMS}} < 80$ | $\chi^2$ of every track in DMS |
| $\chi^2_{\mathrm{UMS}} < 4.5$ | $\chi^2$ of UMS track |
| "$\chi^2_{vtx}$"$< 0.5$ cm$^2$ | Vertex "$\chi^2$" |
| $P_T^2 < 1.8\cdot 10^{-4}$ GeV$^2$ | Transverse momentum of secondaries compared to an incident $K^+$ |

Table 3: The description of "standard cuts" in CKMGEANT MC simulation

results are presented on figure 7a. They correspond to the expected sensitivity (controled by this background level) $BR(K^+ \to \pi^-\mu^+\mu^+) < 2.5 \cdot 10^{-9}$ (90% c.l.) what is the same as in E865 [2]. After this we used 2 additional cuts:

a) In the assumption that all 3 charged secondary particles are pions we calculate the effective mass $M_{3\pi}$ and reject the events with $M_{3\pi} < 0.52$ GeV (for the signal reaction assigning a pion mass to muon tracks produces a mass spectrum lying above this threshold). This cut reduces the secondary pion decays after DMS spectrometer. The results with this cut in addition are presented in figure 7b. The background is reduced significantly but in the dangerous region of $M(\pi^-\mu^+\mu^+)$ it practically does not change.

b) We performed a double measurement of the momentum of a fast muon (with $P_\mu \gtrsim 10$ GeV what is over the threshold for the Pion RICH counter) in the DMS and in the velocity RICH spectrometer and we selected events with

$$1.27 < R = \frac{P_\mu(\mathrm{DMS})}{P_\mu(\mathrm{RICH})} < 1.35$$

(in the existing program for Pion RICH $R = 1$ corresponds to a pion and $R = 1.33$ – to a muon; a ring fit quality cut $5 < nprch < 40$ was also applied)

The background simulation results after the cut (b) are presented in figure 7c and the results with both cuts (a) and (b) are presented in figure 7d. As is seen in these pictures the RICH cut reduced the background level drastically although it also reduced the efficiency of $K^+ \to \pi^-\mu^+\mu^+$ registration because of the implicitly applied momentum threshold $P_\mu \gtrsim 10$ GeV for a fast muon (from $\epsilon = 0.086$ to $\epsilon = 0.03$). Thus the possibility of using the RICH velocity spectrometer is very important for the search for LFV processes in the CKM experiment.

The distributions of "fast" ($P_\mu \gtrsim 10$ GeV) and "slow" muon tracks from $K^+ \to \pi^-\mu^+\mu^+$ in the MVS are presented in figure 8.

We have not enough MC statistics for the simulation of background level for the measurements with the RICH selection cut. From comparison of figures 5 (E865) and 7c,d in the region where we see some background (near the mass $M(\pi^-\mu^+\mu^+) = 0.46$ GeV) it is possible to conclude that expected background level of the CKM experiment would be $\sim 2$ orders of magnitude smaller than in E865 (that is around $\sim 10^{-11}$).

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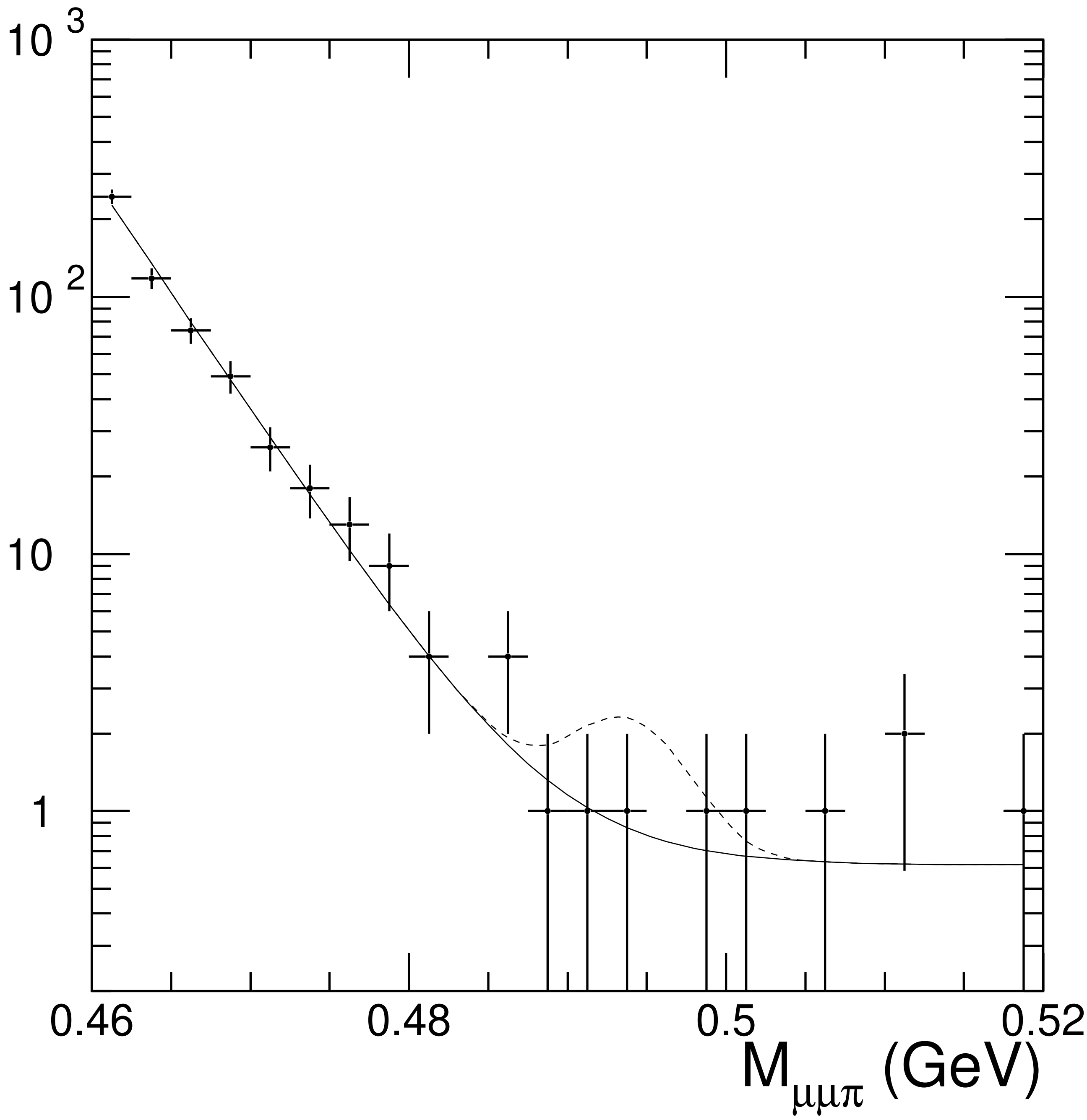


Figure 5: E865 data for $K^+ \to \pi^-\mu^+\mu^+$. The dashed curve corresponds to $BR(K^+ \to \pi^-\mu^+\mu^+) \simeq 3.0 \cdot 10^{-9}$

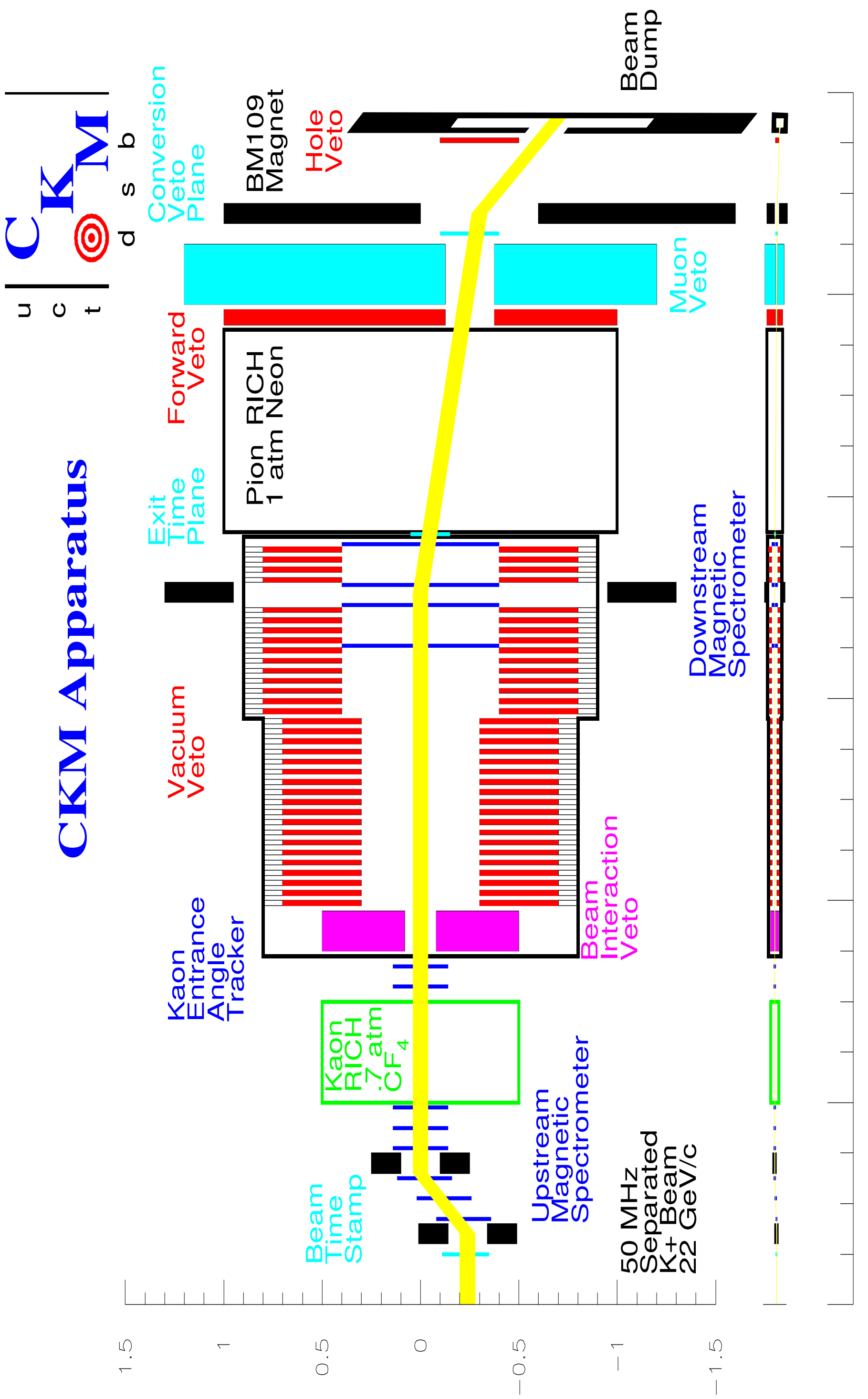


Figure 6: The CKM Detector. The lower section show the true proportions of the apparatus.

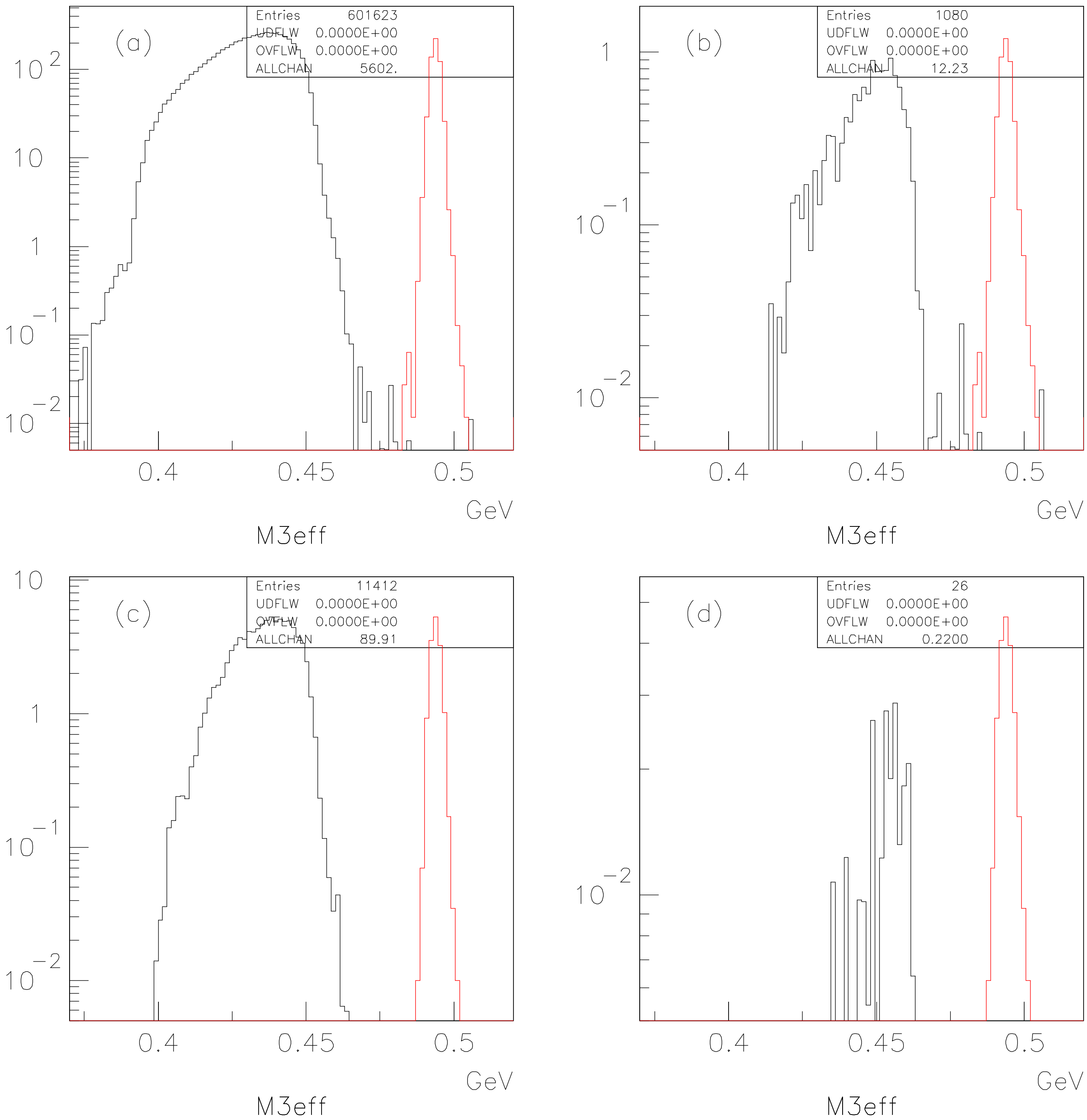


Figure 7: $K^+ \to \pi^-\pi^+\pi^+$ background (black histogram) after: (a) "standard cuts"; (b) $M_{3\pi}$ cut; (c) RICH cut; (d) both (b) and (c). Red peaks correspond to the signal but the vertical scale is different.

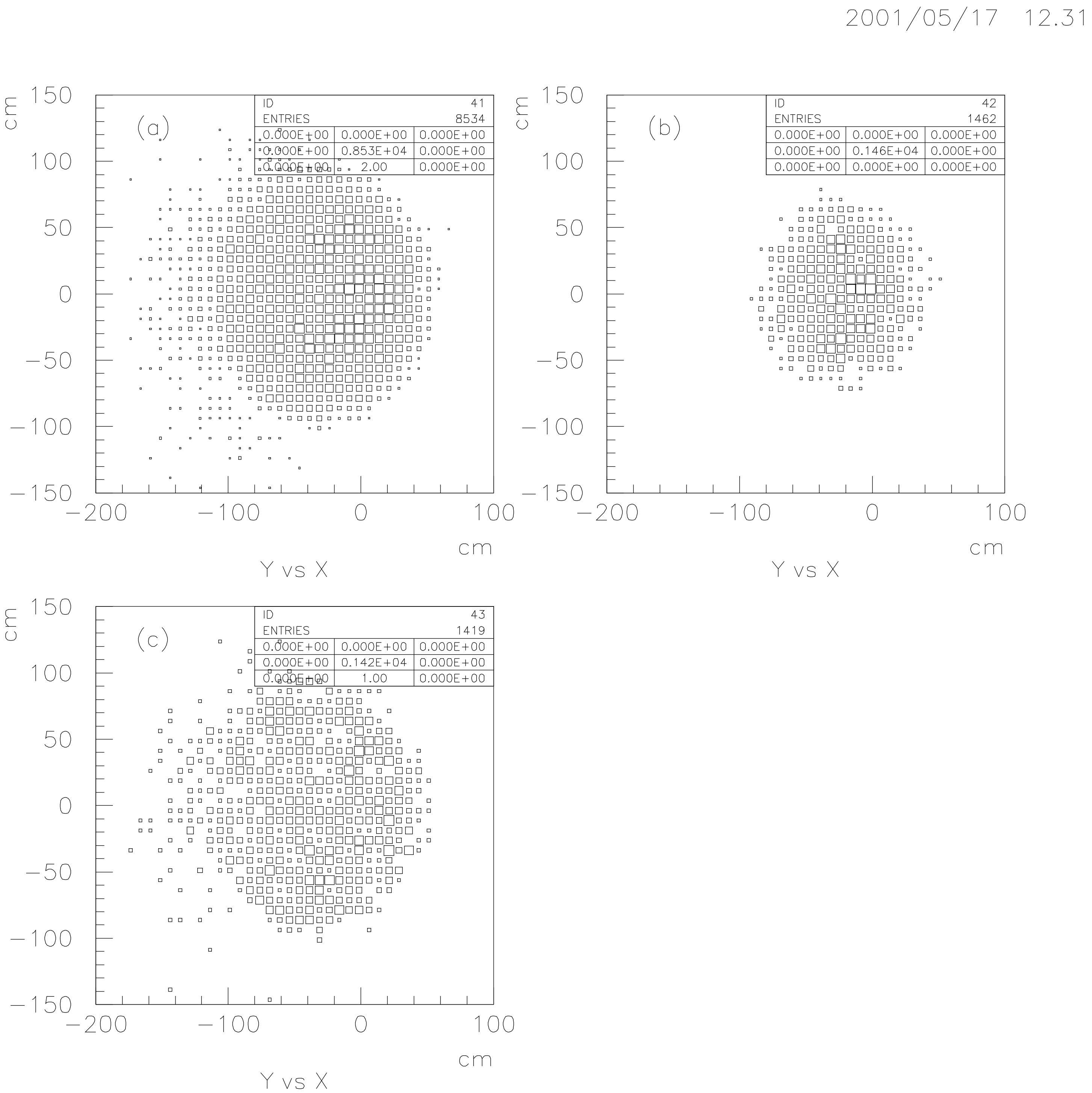


Figure 8: Distributions of coordinates of muons from $K^+ \to \pi^-\mu^+\mu^+)$ in MVS: a) for both $\mu$-s after all cuts except RICH one; b) for "fast" ($P_\mu \gtrsim 10$ GeV) muon after all cuts; c) for "slow" (($P_\mu \lesssim 10$ GeV) muon after all cuts.