Role of Charged Lepton Flavor Violation in Differentiating Viable Neutrino Models

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Present Oscillation Data and Unknowns

• Present data within $2\sigma$ accuracy

\[
\Delta m_{21}^2 = (7.3 - 8.5) \times 10^{-5} \text{ eV}^2
\]
\[
\Delta m_{31}^2 = (2.2 - 3.0) \times 10^{-3} \text{ eV}^2
\]
\[
\sin^2 \theta_{12} = 0.26 - 0.36
\]
\[
\sin^2 \theta_{23} = 0.38 - 0.63
\]
\[
\sin^2 \theta_{13} \leq 0.025
\]

• Data suggests the approximate tri-bimaximal mixing texture of Harrison, Perkins and Scott:

\[
U_{PMNS} = \begin{pmatrix}
2/\sqrt{6} & 1/\sqrt{3} & 0 \\
-1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\
-1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2}
\end{pmatrix}
\]

with $\sin^2 \theta_{23} = 0.5$, $\sin^2 \theta_{12} = 0.33$ and $\sin^2 \theta_{13} = 0$. 
• **Present unknowns**
  
  Hierarchy and absolute mass scales
  Whether neutrinos are Dirac or Majorana
  CP-violating phases of mixing matrix
  How close to zero is the reactor angle $\theta_{13}$?
  How near maximal is the atmospheric mixing angle?
  Is the approximate tri-bimaximal symmetry a softly-broken or accidental symmetry?
  How large is charged lepton flavor violation?
  
• **Scope of Survey**
  
  What do models say about $\theta_{13}$, hierarchy, and lepton flavor violation?
Models with Well-Defined Symmetry

• **Examples with Lepton Flavor Symmetry**
  - $\mu - \tau$ Interchange Symmetry
  - More restrictive $S_3$ or $A_4$ lepton flavor symmetry
  - SO(3) or SU(3) Flavor Symmetries
  - Texture Zeros

• **Examples involving GUT Models**
  - “Minimal” SO(10) Models with Higgs in 10, 126, (120, 45, 54)
  - “Lopsided” SO(10) Models with Higgs in 10, 16, 16(bar), 45
Survey of Predictions for $\theta_{13}$ and Hierarchy

- Survey made of 63 models in literature which give the LMA solution for the solar neutrino oscillations and firm and reasonably restrictive predictions for the reactor neutrino angle. (Cutoff date: May 2006)

- Most of models predict $10^{-4} < \sin^2 \theta_{13} < 0.04$

- Normal hierarchy is preferred 3 : 1

- Planned reactor experiments will reach $\sin^2 2\theta_{13} \sim 0.01$, so half of models will be eliminated if no $\bar{\nu}_e$ disappearance.

- Meanwhile MEG will probe $\mu \rightarrow e\gamma$ for LFV, so this may this may serve as even more immediate selector of models.
Lepton Flavor Violation in Radiative Decays

• In SM with 3 massive $N^c$'s, individual $L_e$, $L_\mu$, $L_\tau$ are not conserved. LFV arises in 1-loop where the neutrino insertion involves lepton flavor-changing Yukawa couplings.

$$BR_{21} \equiv \frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow \nu_\mu e\bar{\nu}_e)} = \frac{3\alpha}{32\pi} \left| \sum U_{\mu k}^* \frac{m_k^2}{M_W^2} U_{ke} \right|^2 \sim \frac{3\alpha}{128\pi} \left( \frac{\Delta m_{21}^2}{M_W^2} \right)^2 \sin^2 2\theta_{12} \sim 10^{-54}$$

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• In SUSY GUT models slepton - neutralino and sneutrino -
chargino loops contribute to radiative lepton decays.

• In the CMSSM version with universal soft masses and
trilinear couplings, LFV arises from evolution of Yukawa
couplings and soft parameters.

• With more comparable heavy masses in the loops and no
GIM mechanism, the LFV branching ratios can be much larger.
• In the LLA, largest contribution comes from the LL slepton mass matrix yielding

$$\text{BR}(\ell_j \rightarrow \ell_i \gamma) = \frac{\alpha^3}{G_F^2 m^8_S} |(m^2_{\tilde{\ell}})_{ji}|^2 \tan^2 \beta$$

where

$$(m^2_{\tilde{\ell}})_{ji} = -\frac{1}{8\pi^2} m_0^2 (3 + A_0^2/m_0^2) Y_{jk}^\dagger \log \frac{M_G}{M_k} Y_{ki}$$

• Full evolution effects are extremely well approximated by

$$m^8_S \simeq 0.5 m_0^2 M_{1/2}^2 (m_0^2 + 0.6 M_{1/2}^2)^2$$  \hspace{1cm} \text{Petcov et al.}$$

• MEG experiment only has a chance of seeing a positive signal from a SUSY GUT model. All other models considered here will give negative results.
Examples of Predictive SUSY GUT Models

- LFV has been studied in a number of papers in rather generic GUT models. Here we wish to differentiate between specific GUT models and draw some conclusions.

- SO(10) Models with indicated Flavor Symmetry and Higgs IRs
  1. AB (Albright-Barr): \( U(1) \times Z_2 \times Z_2 \) with \( 10, 16, \bar{16}, 45 \)
  2. CM (Chen - Mahanthappa): \( SU(2) \times (Z_2)^3 \) with \( 10, \bar{126} \)
  3. CY (Cai - Yu): \( S_4 \) with \( 10, \bar{126} \)
  4. DR (Dermisek - Raby): \( D_3 \) with \( 10, 45 \)
  5. GK (Grimus - Kuhbloc): \( Z_2 \) with \( 10, 120, \bar{126} \)
<table>
<thead>
<tr>
<th>Models</th>
<th>SO(10) IRs</th>
<th>Flavor Symmetry</th>
<th>$M_R$’s</th>
<th>$\tan\beta$</th>
<th>$\sin^2\theta_{13}$</th>
<th>Interesting Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>A - B</td>
<td>$10, 16, \bar{16}, 45$</td>
<td>$U(1) \times Z_2 \times Z_2$</td>
<td>$2.4 \times 10^{14}$</td>
<td>5</td>
<td>0.0020</td>
<td>Large $M_R$ hierarchy with lightest two nearly degenerate leads to resonant leptogenesis.</td>
</tr>
<tr>
<td>C - M</td>
<td>$10, \bar{126}$</td>
<td>$SU(2) \times (Z_2)^3$</td>
<td>$7.0 \times 10^{12}$</td>
<td>10</td>
<td>0.013</td>
<td>Large $M_R$ hierarchy with heaviest more than 3 orders of magnitude below GUT scale; large $\sin^2\theta_{13}$.</td>
</tr>
<tr>
<td>C - Y</td>
<td>$10, \bar{126}$</td>
<td>$S_4$</td>
<td>$2.6 \times 10^{12}$</td>
<td>10</td>
<td>0.0029</td>
<td>Degenerate $M_R$ spectrum 4 orders of magnitude below GUT scale.</td>
</tr>
<tr>
<td>D - R</td>
<td>$10, 45$</td>
<td>$D_3$</td>
<td>$5.5 \times 10^{13}$</td>
<td>50</td>
<td>0.0024</td>
<td>Mild $M_R$ hierarchy almost 3 orders of magnitude below GUT scale.</td>
</tr>
<tr>
<td>G - K</td>
<td>$10, 120, \bar{126}$</td>
<td>$Z_2$</td>
<td>$2.0 \times 10^{15}$</td>
<td>10</td>
<td>0.00059</td>
<td>Mild $M_R$ hierarchy just 1 order of magnitude below GUT scale; rather small $\sin^2\theta_{13}$.</td>
</tr>
</tbody>
</table>
Radiative Lepton Flavor Violation Predictions

• In CMSSM with universal soft parameters $m_0$, $M_{1/2}$, $A_0$, for given $\tan \beta$ and $\text{sgn}(\mu)$, a variety of plots are possible.

1) BR vs. $M_{1/2}$ for fixed $A_0 = 0$ and different choices of $m_0$.

2) $A_0/m_0$ vs. $M_{1/2}$ scatterplot with a color scheme to indicate branching ratio ranges.

3) Ratio of the branching ratios, $BR_{32}/BR_{21}$ on log - log plot:

$$\log BR_{32} = \log BR_{21} + \log \left| \frac{(Y_{\nu}^+LY_{\nu})_{32}}{(Y_{\nu}^+LY_{\nu})_{21}} \right|^2$$

with unit slope and intercept the second term on right. Length of straight line depends on the soft parameters.
• Soft Parameter constraints imposed

For $\tan \beta = 5, 10$:

$m_0 : \quad 50 \rightarrow 400$ GeV

$M_{1/2} : \quad 200 \rightarrow 1000$ GeV

$A_0 : \quad -4000 \rightarrow 4000$ GeV

For $\tan \beta = 50$:

$m_0 : \quad 500 \rightarrow 4000$ GeV

$M_{1/2} : \quad 200 \rightarrow 1500$ GeV

$A_0 : \quad -50 \rightarrow 50$ TeV

• WMAP DM constraints in coannihilation regions

\[
m_0 = c_0 + c_1 M_{1/2} + c_2 M_{1/2}^2
\]
\[
c_i = c_i(A_0, \tan \beta) \quad \text{Stark, Hafliger, Biland, Pauss}
\]

If $M_{1/2}$ is too small, $m_h > 114$ GeV is violated.

If $M_{1/2}$ is too large, $\tilde{\chi}^0$ relic density is too large.
BR(\mu \rightarrow e + \gamma)

BR(\tau \rightarrow \mu + \gamma)

AB model  \quad (A_0 = 0)

CM model

CY model

DR model

GK model

$10^{-19}$  $10^{-18}$  $10^{-17}$  $10^{-16}$  $10^{-15}$  $10^{-14}$  $10^{-13}$  $10^{-12}$  $10^{-11}$  $10^{-10}$  $10^{-9}$  $10^{-8}$

$10^{-7}$  $10^{-6}$  $10^{-5}$  $10^{-4}$  $10^{-3}$  $10^{-2}$  $10^{-1}$  $10^{0}$
(A_0 = 0)
Lepton Flavor Violation in $\mu \rightarrow e$ Conversion

- One-loop diagrams involving gamma, Z, Higgs penguins and boxes all contribute, but in the CMSSM the gamma penguin dominates: 

\[ \mu^- \rightarrow \tilde{\ell}^+ \tilde{\chi}^- \rightarrow e^- \gamma \]

where the effects of the virtual $N^c$ and $\tilde{N}^c$ with their Yukawa couplings appear in $\tilde{\ell}$ loops, eg.

- The $\mu \rightarrow e$ conversion rate (relative to the capture rate) on Ti vs. BR21($\mu \rightarrow e\gamma$) is plotted for the 5 GUT models, where the tighter WMAP DM constraints have been imposed.
BR(\mu \rightarrow e + \gamma)

AB model
CM model \quad (A_0 = 0)
CY model
DR model
GK model

RATIOmueconv21.agr
Conclusions

Tried to differentiate models based on neutrino mass hierarchy, $\sin^2 \theta_{13}$, and charged lepton flavor violation predictions.

- Study initially based on 60+ models in literature (< 6/06)
  - Normal hierarchy preferred 3 : 1
  - Double CHOOZ and Daya Bay reactors will be able to eliminate roughly half of the 63 neutrino models surveyed, if their sensitivity reaches $\sin^2 2\theta_{13} \simeq 0.01$ as planned.
  - Of the order of 5 models have similar values for $\sin^2 \theta_{13}$ in the interval 0.001 - 0.08.
  - If the MEG experiment sees positive signals for $\mu \rightarrow e\gamma$, all non-SUSY models or non-NP models will be ruled out.
• Study narrowed to 5 predictive SO(10) SUSY GUT models

- All 5 models have type I seesaws implying normal hierarchy.
- \( \sin^2 2\theta_{13} \) predictions:
  
  CM (~ 0.05); AB, CY, DR (~ 0.01); GK (~ 0.001)

- Previous studies of generic SO(10) models have concluded that the LFV branching ratios depend critically on \( \theta_{13} \) and \( M_{R3} \). Here we find that \( M_{R3} \) appears to be more important.

- Branching ratio plots given for \( A_0 = 0 \) represent lower limits with higher predictions obtained for \( |A_0/m_0| > 0 \).

- If the MEG experiment can reach an upper bound of \( \text{BR}(\mu \rightarrow e\gamma) < 10^{-13} \), it will rule out the GK and AB models.

- If \( \mu \rightarrow e \) conversion can be performed and reach a branching ratio limit of \( 10^{-18} \) as originally anticipated, it can potentially rule out all 5 models considered.