Lepton Flavor Violation in K_S and K^-

- $K_S \rightarrow \pi^0 e\mu$
- $K_S \rightarrow e\mu$
- $K^- \rightarrow \pi^- e\mu$

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$K_S \to \pi^0 e\mu$

$$\Gamma\left(K_L \to \pi^0 e\mu\right) \propto \left| \frac{(1+\varepsilon)\left\langle \pi^0 e\mu \middle| K^0 \right\rangle}{\sqrt{2}\sqrt{1+|\varepsilon|^2}} + \frac{(1-\varepsilon)\left\langle \pi^0 e\mu \middle| \overline{K}^0 \right\rangle}{\sqrt{2}\sqrt{1+|\varepsilon|^2}} \right|^2$$

$$\left\langle \pi^0 e\mu \middle| K^0 \right\rangle = \left\langle \pi^+ e\mu \middle| K^+ \right\rangle / \sqrt{2}$$

$$\left\langle \pi^0 e\mu \middle| \overline{K}^0 \right\rangle = \Psi_3 \left\langle \pi^+ e\mu \middle| K^+ \right\rangle / \sqrt{2}$$

Ψ_3 parameterizes difference between s and s

Data on $K_L \to \pi^0 e\mu$ and $K^+ \to \pi^+ e\mu$ make it possible to constrain Ψ_3 to a circular locus in the complex plane:

$$\frac{\Gamma\left(K_L \to \pi^0 e\mu\right)}{\Gamma\left(K^+ \to \pi^+ e\mu\right)} = \frac{|1+\varepsilon|^2 + 2\Re\left[(1+\varepsilon)^*(1-\varepsilon)\Psi_3\right] + \left|\Psi_3(1-\varepsilon)\right|^2}{4\left(1+|\varepsilon|^2\right)}$$

- Center of the circle depends only on ε
- $R^2 = 0.960 \dfrac{Br\left(K_L \to \pi^0 e\mu\right)}{Br\left(K^+ \to \pi^+ e\mu\right)}$

There is another circle for

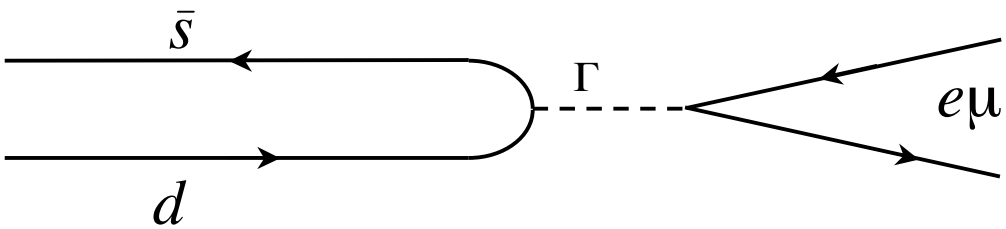
$$\frac{\Gamma\left(K_S \to \pi^0 e\mu\right)}{\Gamma\left(K^+ \to \pi^+ e\mu\right)} = \frac{|1+\varepsilon|^2 - 2\Re\left[(1+\varepsilon)^*(1-\varepsilon)\Psi_3\right] + \left|\Psi_3(1-\varepsilon)\right|^2}{4\left(1+|\varepsilon|^2\right)}$$

Find maximum distance between any point on Ψ_3 and the center of the second circle… that distance $\propto \sqrt{Br\left(K_S \to \pi^0 e\mu\right)}$

$$Br\left(K_S \to \pi^0 e\mu\right) = \left(\sqrt{Br\left(K_L \to \pi^0 e\mu\right)\frac{\tau_S}{\tau_L}} + \sqrt{\frac{Br\left(K^+ \to \pi^+ e\mu\right)}{\left(1+|\varepsilon|^2\right)|1-\varepsilon|^2}\frac{\tau_S}{\tau^+}\left\{\left(1-|\varepsilon|^2\right)^2 + \left(2\Im(\varepsilon)\right)^2\right\}}\right)^2$$

$$= 7.2 \times 10^{-12}$$

$K_S \to e\mu$

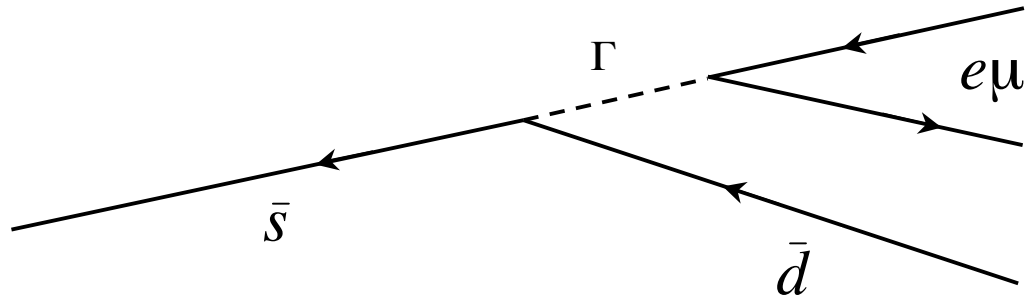


$$M_2 = \langle 0|\bar{s}\gamma^5 d|K\rangle \quad \left[\bar{u}(p_\mu)(a_S + a_P\gamma^5)v(p_e)\right]$$
$$+\langle 0|\bar{s}\gamma^5\gamma^\alpha d|K\rangle \quad \left[\bar{u}(p_\mu)(a_V + a_A\gamma^5)\gamma_\alpha v(p_e)\right]$$

For $\overline{K^0}$ in general one may have couplings $\bar{a}_i$

For K_S, the linear combo of a_i and $\bar{a}_i$ may redefine a_i

Get these redefined constants from $K_S \to \pi^0 e\mu$



$$M_3 = \langle \pi | \bar{s} d | K \rangle \quad \left[\bar{u}(p_\mu)(a_S + a_P \gamma^5) v(p_e) \right]$$
$$+ \langle \pi | \bar{s} \gamma^\alpha d | K \rangle \quad \left[\bar{u}(p_\mu)(a_V + a_A \gamma^5) \gamma_\alpha v(p_e) \right] \quad + \langle \pi | \bar{s} \sigma^{\alpha\beta} d | K \rangle \left[\bar{u}(p_\mu)(a_T) \sigma_{\alpha\beta} v(p_e) \right]$$

Have to neglect tensor terms to proceed - more on that later. General scheme is:

1. Use the previous $K_S \to \pi^0 e\mu$ result, constrain a_i
2. Use those constraints to limit $K_S \to e\mu$.
3. Use measured form factors and decay constants for V, A currents
4. Use covariant derivatives of V, A currents for the S, P currents

I used

- $\langle \pi | \bar{s}\gamma^\alpha d | K \rangle = \left(f_+ [p_K + p_\pi]^\alpha + f_- [p_K - p_\pi]^\alpha \right) = f_+ \left([p_K + p_\pi]^\alpha + \xi [p_K - p_\pi]^\alpha \right)$

  (a fairly standard form) with $f_\pm = f_\pm(0) \left[1 + \lambda_\pm \left(\frac{(p_K - p_\pi)^2}{m_\pi^2} \right) \right]$

  with $f_+(0) = 1$, $\lambda_+ = 0.030$, $f_-(0) = -0.0025$, and $\lambda_- = 0.0$

- $\langle 0 | \bar{s}\gamma^5\gamma^\alpha d | K_S \rangle = \frac{(1+\varepsilon)\langle 0 | \bar{s}\gamma^5\gamma^\alpha d | K^0 \rangle}{\sqrt{2}\sqrt{1+|\varepsilon|^2}} - \frac{(1-\varepsilon)\langle 0 | \bar{s}\gamma^5\gamma^\alpha d | \bar{K}^0 \rangle}{\sqrt{2}\sqrt{1+|\varepsilon|^2}} = \frac{\sqrt{2}\varepsilon i f_K}{\sqrt{1+|\varepsilon|^2}} [p_K]^\alpha$

  with $f_K = 160\ MeV$

- $\langle \pi | \bar{s} d | K \rangle = \frac{[p_\pi - p_K]_\alpha \langle \pi | \bar{s}\gamma^\alpha d | K \rangle}{m_s - m_d}$ and

  $\langle 0 | \bar{s}\gamma^5 d | K \rangle = \frac{[p_K]_\alpha \langle 0 | \bar{s}\gamma^5\gamma^\alpha d | K \rangle}{m_s + m_d}$

$$\Gamma_3(J=0) = \frac{1}{2m_k}\int d\Phi\, \overline{\left|M_3(J=0)\right|^2} = \left(8.56\times10^{13}\, MeV^7\right)\frac{\left|a_S\right|^2+\left|a_P\right|^2}{\left(m_s-m_d\right)^2}$$

$$\Gamma_3(J=1) = \frac{1}{2m_k}\int d\Phi\, \overline{\left|M_3(J=1)\right|^2} = \left(5.74\times10^{8}\, MeV^5\right)\left(\left|a_V\right|^2+\left|a_A\right|^2\right)$$

$$\Gamma_3(J=\otimes) = \frac{1}{2m_k}\int d\Phi\, \overline{\left|M_3(J=\otimes)\right|^2} = \left(2.51\times10^{11}\, MeV^6\right)\frac{\Re\left(a_A a_P^* - a_V a_S^*\right)}{\left(m_s-m_d\right)}$$

Σ is the previously determined $\Gamma(K_S \rightarrow \pi^0 e \mu)$

I found Eigenmath, at http://eigenhead.com, to be helpful in doing the matrix elements… it's not as powerful as Maple or Mathematica, but it's a lot cheaper!

Integration over phase space courtesy RAMBO, although the 2-body case is easy to do analytically

And for the 2-body decay,

$$\Gamma_2(J=0) = \frac{1}{2m_k}\int d\Phi\, \overline{\left|M_2(J=0)\right|^2} = \left(2.95\times10^{11}\,MeV^7\right)\frac{\left|a_S\right|^2+\left|a_P\right|^2}{\left(m_s+m_d\right)^2}$$

$$\Gamma_2(J=1) = \frac{1}{2m_k}\int d\Phi\, \overline{\left|M_2(J=1)\right|^2} = \left(5.37\times10^{4}\,MeV^5\right)\left(\left|a_V\right|^2+\left|a_A\right|^2\right)$$

$$\Gamma_2(J=\otimes) = \frac{1}{2m_k}\int d\Phi\, \overline{\left|M_2(J=\otimes)\right|^2} = \left(-2.52\times10^{8}\,MeV^6\right)\frac{\Re\left(a_A a_P^* - a_V a_S^*\right)}{\left(m_s+m_d\right)}$$

Notice minus sign

Basically, three variables… $m_s >> m_d$, reducing uncertainty in $(m_s - m_d)/(m_s + m_d) = 0.90 \pm 0.01$

For every combination of the three variables that gives the right $\Gamma(K_S \to \pi^0 e\mu)$, find $\Gamma(K_S \to e\mu)$; select the maximum thereof

And the result is… $Br(K_S \rightarrow e\mu) < 7.6 \times 10^{-12}$

If we assume that there is only a J=0 boson, and drop both the J=1 and cross-terms,

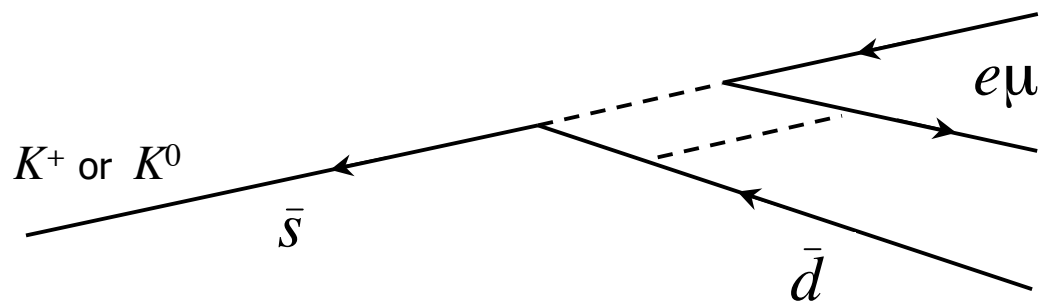
$$Br(K_S \rightarrow e\mu) < 5.6 \times 10^{-12}$$

For only a J=1 boson, helicity suppression is evident:

$$Br(K_S \rightarrow e\mu) < 1.9 \times 10^{-13}$$

What about the class of models that requires tensor terms?

If the new physics involves things like box diagrams, as is the case in SUSY or LQ models,



then neglecting the tensor terms is probably a bad approximation. They could interfere destructively with potentially large scalar and vector terms in three body decays and those scalar and vector terms could create two body decays at a large rate. However…

$\Gamma(K_L \rightarrow e\mu) < 6.0 \times 10^{-26}\ MeV$, from BNL E871

$$\frac{\Gamma(K_S \to e\mu)}{\Gamma(K_L \to e\mu)} = \frac{\left|(1+\varepsilon)\langle e\mu|K^0\rangle - (1-\varepsilon)\langle e\mu|\overline{K}^0\rangle\right|^2}{\left|(1+\varepsilon)\langle e\mu|K^0\rangle + (1-\varepsilon)\langle e\mu|\overline{K}^0\rangle\right|^2}$$

A large $\Gamma(K_S \to e\mu)$ / $\Gamma(K_L \to e\mu)$ ratio requires that the denominator has a zero:

$$\left|(1+\varepsilon)\langle e\mu|K^0\rangle + (1-\varepsilon)\langle e\mu|\overline{K}^0\rangle\right|^2 \approx 0$$

$$\Rightarrow$$

$$\Psi_2 = \langle e\mu|\overline{K}^0\rangle / \langle e\mu|K^0\rangle \approx (\varepsilon+1)/(\varepsilon-1)$$

Qualitatively, it seems unlikely that the new short-range physics will give Ψ_2 that matches the value of ε, which is heavily influenced by long-range QCD effects.

$K^- \to \pi^- e\mu$

$$\Gamma\left(K_L \to \pi^0 e\mu\right) \propto \left| \frac{(1+\varepsilon)\left\langle \pi^0 e\mu \middle| K^0 \right\rangle + (1-\varepsilon)\left\langle \pi^0 e\mu \middle| \overline{K}^0 \right\rangle}{\sqrt{2}\sqrt{1+|\varepsilon|^2}} \right|^2$$

$$\left\langle \pi^0 e\mu \middle| K^0 \right\rangle = \left\langle \pi^+ e\mu \middle| K^+ \right\rangle / \sqrt{2} \qquad \left\langle \pi^0 e\mu \middle| \overline{K}^0 \right\rangle = \left\langle \pi^- e\mu \middle| K^- \right\rangle / \sqrt{2}$$

Max of K^- term is when it destructively interferes with K^+ term. Assign that relative phase to $\langle \pi^+ e\mu | K^+ \rangle$ and $\langle \pi^- e\mu | K^- \rangle$ and solve for $|\langle \pi^- e\mu | K^- \rangle|^2 \propto \Gamma(K^- \to \pi^- e\mu)$

$e^+\mu^-$:	$\Gamma = 8.79 \times 10^{-23}$ MeV	$Br = 1.65 \times 10^{-9}$
$e^-\mu^+$:	$\Gamma = 2.76 \times 10^{-23}$ MeV	$Br = 5.16 \times 10^{-10}$

This assumes that there is interference of the form
$K^0 \to \pi^0 e\mu$ with $K^0 \to \overline{K}^0 \to \pi^0 e\mu$

Such need not be the case if generation number is conserved - that case is like $K_{\mu 3}$ decay of the K_L where the partial width of s-quark decay is added directly to the (equal) partial width of s-antiquark decay

Dropping the interference term results in
$\Gamma = 1.68 \times 10^{-23}\ MeV$ $\qquad$ $Br = 3.17 \times 10^{-10}$
regardless of the polarity of the leptons

(the maximum in K^- width occurs when we say none of the K^+ width generates K_L width)

Conclusions

- $K^- \to \pi^- e\mu$:
  - $Br(K^- \to \pi^- e^+ \mu^-) < 1.7 \times 10^{-9}$ and $Br(K^- \to \pi^- e^- \mu^+) < 5.2 \times 10^{-10}$
  - Unless there is conservation of generation number, in which case $Br(K^- \to \pi^- e\mu) < 3.2 \times 10^{-10}$ (either charge combination)
- $K_S \to \pi^0 e\mu$:
  - Model independent analysis gives $Br(K_S \to \pi^0 e\mu) < 7.2 \times 10^{-12}$
- $K_S \to e\mu$:
  - In a broad class of models, $Br(K_S \to e\mu) < 7.6 \times 10^{-12}$
  - For $J=1$ intermediate bosons, an additional factor of ~ 30 helicity suppression can be applied to this result
  - Outside this class, $K_S \to e\mu$ at a high rate is qualitatively unlikely