A New Look at the Cabibbo Angle

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- Why remeasure the Cabibbo angle?
- KTeV's Determination of $|V_{us}|$ (sin θ_c)
- Other Recent Measurements
- Conclusions

Unitarity Tests of CKM Matrix

Uncertainty on $\Sigma_j V_{ij}^2$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

0.2%

2.7%

30 %

For first row, PDG quotes 2.2 σ deviation from unitarity:

$$1-\left(\left|V_{ud}\right|^2+\left|V_{us}\right|^2+\left|V_{ub}\right|^2\right)=0.0043\pm0.0019 \quad \text{(PDG 2002)}$$

2002 PDG $|V_{ux}|$ Evaluations

$|V_{ud}| = 0.9734 \pm 0.0008$ from $0^+ \rightarrow 0^+$ nuclear β decays, neutron decay

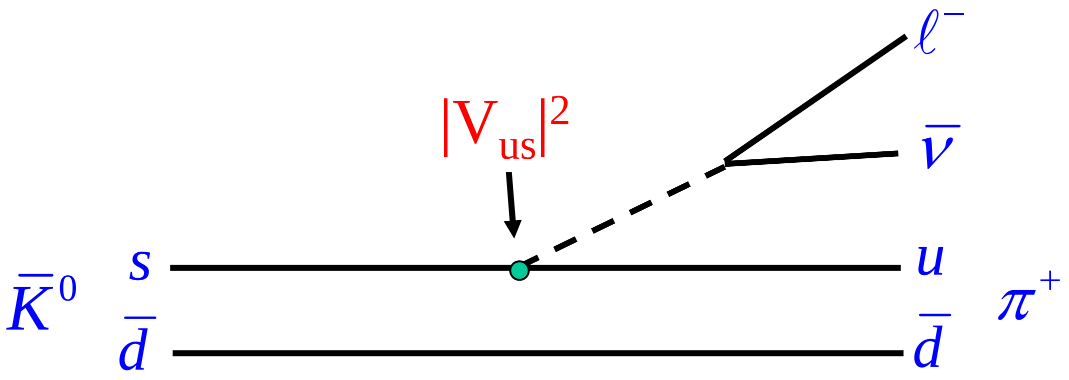
$|V_{us}| = 0.2196 \pm 0.0023$ from K^+, K^0 decays to πeν (πμν not used by PDG because of large uncertainties in form factor measurements).

$|V_{ub}| = (3.6 \pm 0.7) \times 10^{-3}$ from semileptonic B decay

2003 K^+ measurement from BNL E865 consistent with unitarity.

⇨ Interesting to revisit K^0 measurements (PDG fit values based on averages of many old experiments with large errors)

Determination of $|V_{us}|$ in Semileptonic K_L Decays



Experiment:
$B(K_L \rightarrow \pi e \nu)$ and
$B(K_L \rightarrow \pi \mu \nu)$, τ

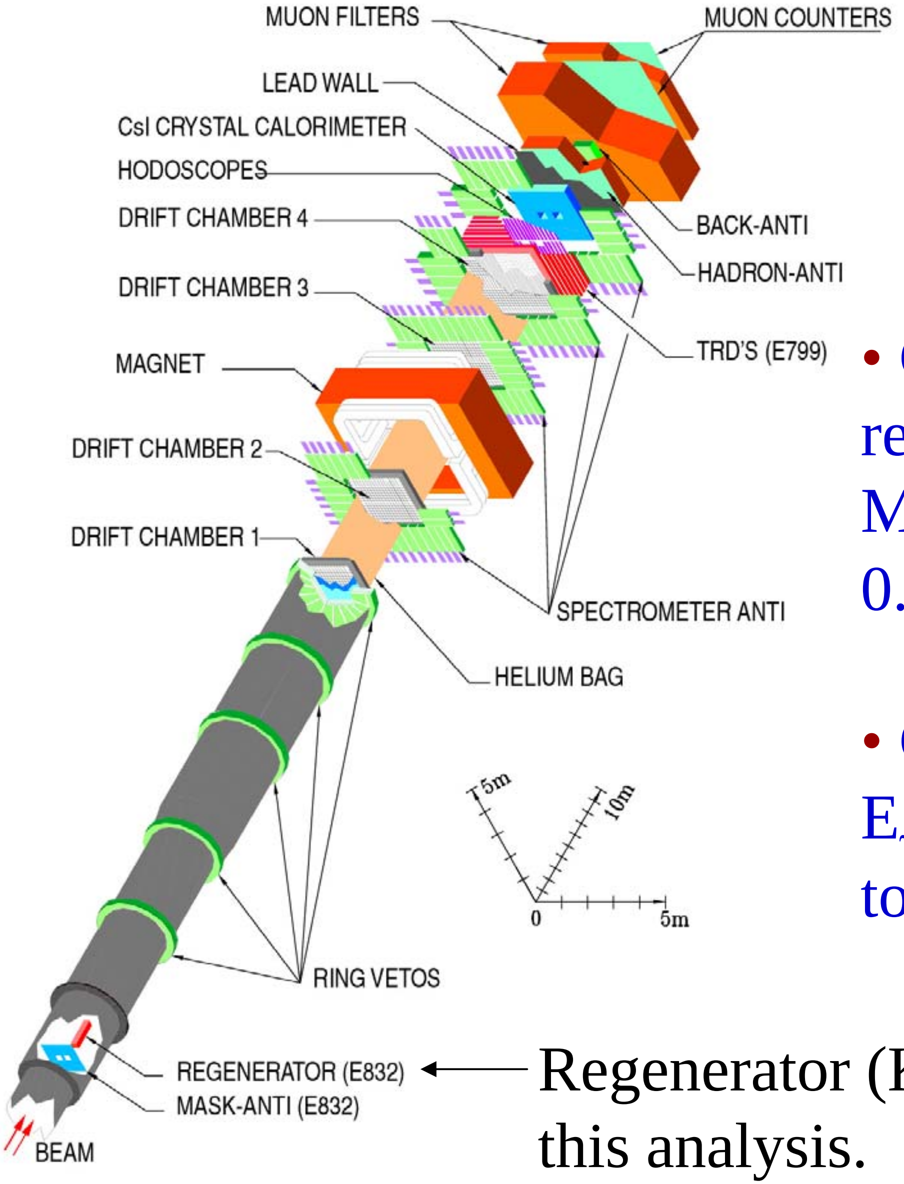
Experiment:
form factors needed
to calculate phase
space integrals

$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} \underbrace{S_{EW}(1+\delta_K^\ell)} \left|V_{us}\right|^2 f_+^2(0) I_K^\ell$$

Rad. Corrections
(theory)

Form factor
at t=0
(theory)

KTeV Detector



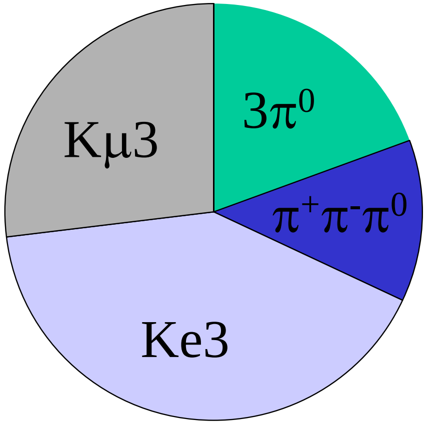
Experiment designed for measurement of ε'/ε

- Charged particle momentum resolution < 1% for p>8 GeV/c; Momentum scale known to 0.01% from $K \rightarrow \pi^+\pi^-$.

- CsI energy resolution < 1% for $E_\gamma > 3$ GeV; energy scale known to 0.1% from $K \rightarrow \pi e \nu$.

Regenerator (K_S) beam not used in this analysis.

To determine the semileptonic widths, KTeV measures the following 5 ratios:



$$\Gamma_{K\mu3}/\Gamma_{Ke3} = \Gamma(K_L \to \pi^\pm \mu^\mp \nu)/\Gamma(K_L \to \pi^\pm e^\mp \nu)$$

$$\Gamma_{+-0}/\Gamma_{Ke3} = \Gamma(K_L \to \pi^+ \pi^- \pi^0)/\Gamma(K_L \to \pi^\pm e^\mp \nu)$$

$$\Gamma_{000}/\Gamma_{Ke3} = \Gamma(K_L \to \pi^0 \pi^0 \pi^0)/\Gamma(K_L \to \pi^\pm e^\mp \nu)$$

$$\Gamma_{+-}/\Gamma_{Ke3} = \Gamma(K_L \to \pi^+ \pi^-)/\Gamma(K_L \to \pi^\pm e^\mp \nu)$$

$$\Gamma_{00}/\Gamma_{000} = \Gamma(K_L \to \pi^0 \pi^0)/\Gamma(K_L \to \pi^0 \pi^0 \pi^0)$$

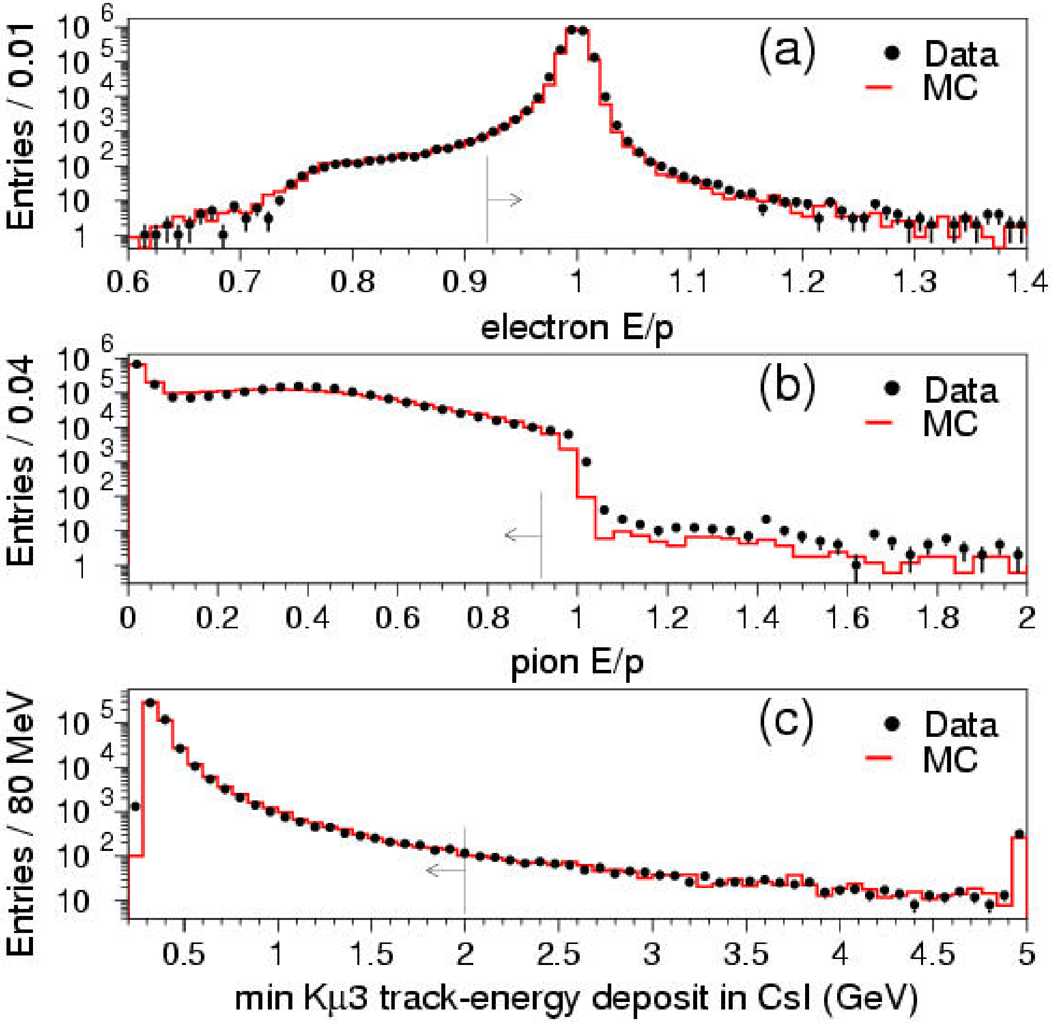
These six decay modes account for 99.93% of K_L decays, so ratios may be combined to determine branching fractions.

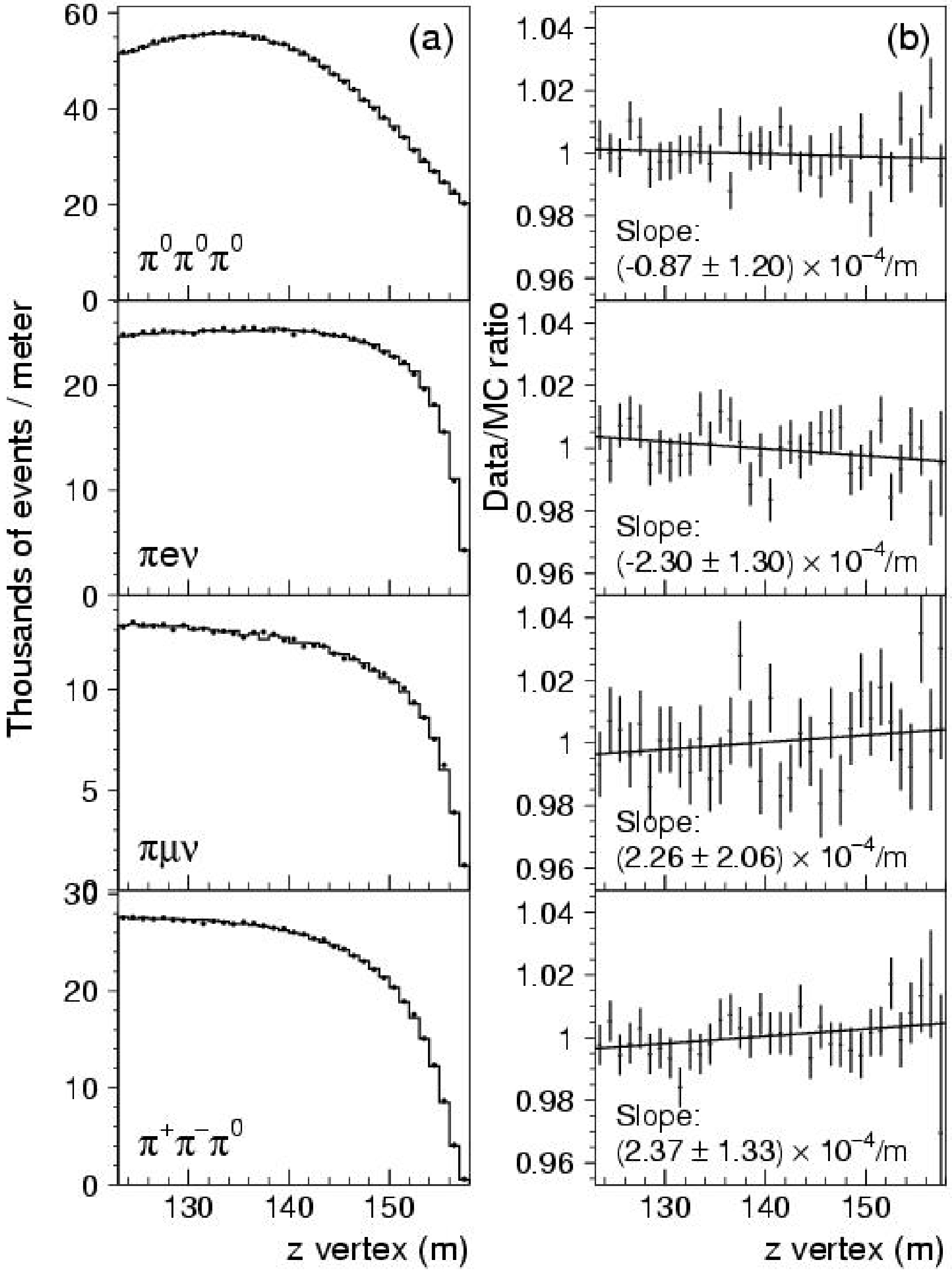
E.g.,

$$B_{Ke3} = \frac{0.9993}{1 + \frac{\Gamma_{K\mu3}}{\Gamma_{Ke3}} + \frac{\Gamma_{000}}{\Gamma_{Ke3}} + \frac{\Gamma_{+-0}}{\Gamma_{Ke3}} + \frac{\Gamma_{+-}}{\Gamma_{Ke3}} + \frac{\Gamma_{00}}{\Gamma_{Ke3}}}$$

KTeV Particle Identification

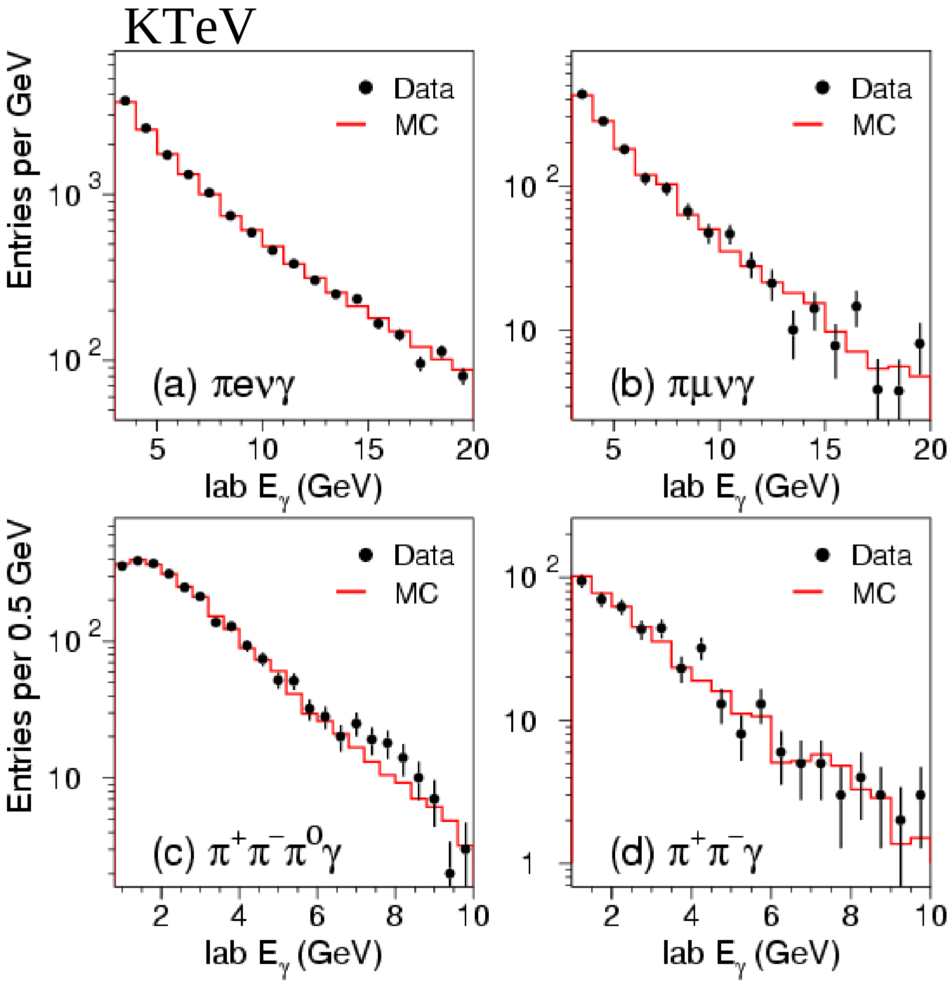
Simple event reconstruction and selection may be used to distinguish different decay modes with very little background (<0.1%).





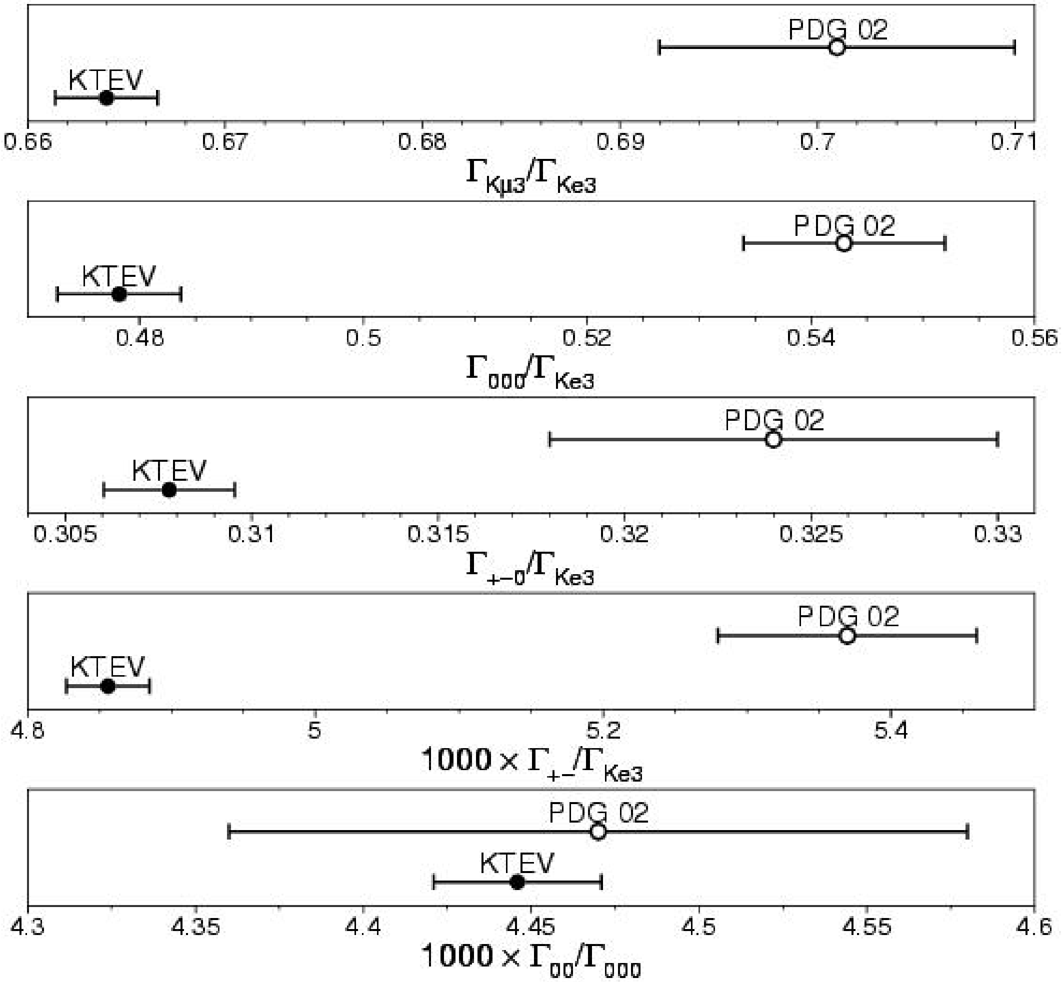
Comparison of data and Monte Carlo decay vertex distributions

Data – MC Comparison for Radiative Photon Candidates

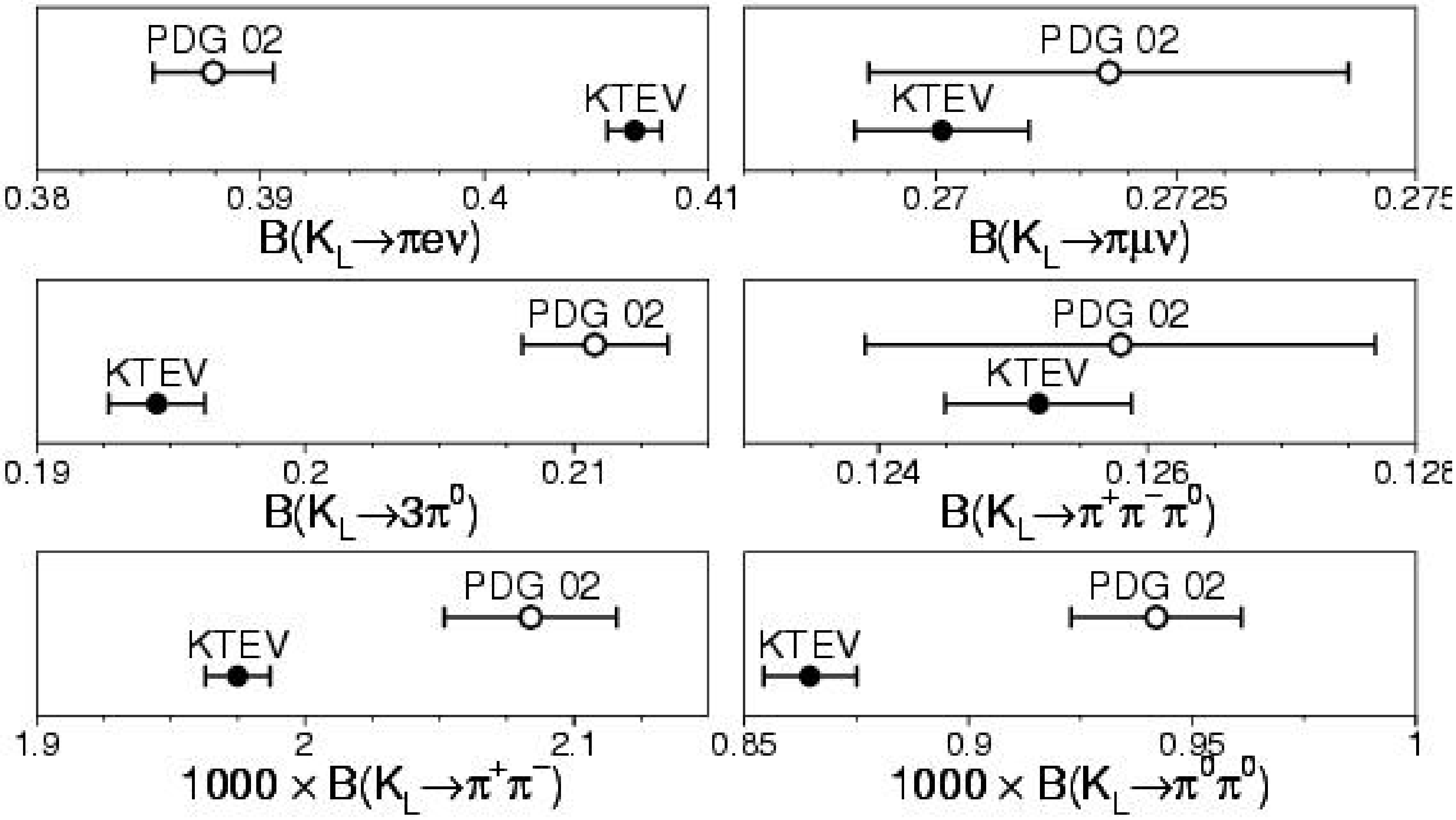


Radiation changes K_{e3} acceptance by 3%; effect on other modes is < 0.5%.

KTeV Measured Partial Width Ratios



Comparison of KTeV and PDG Branching Fractions

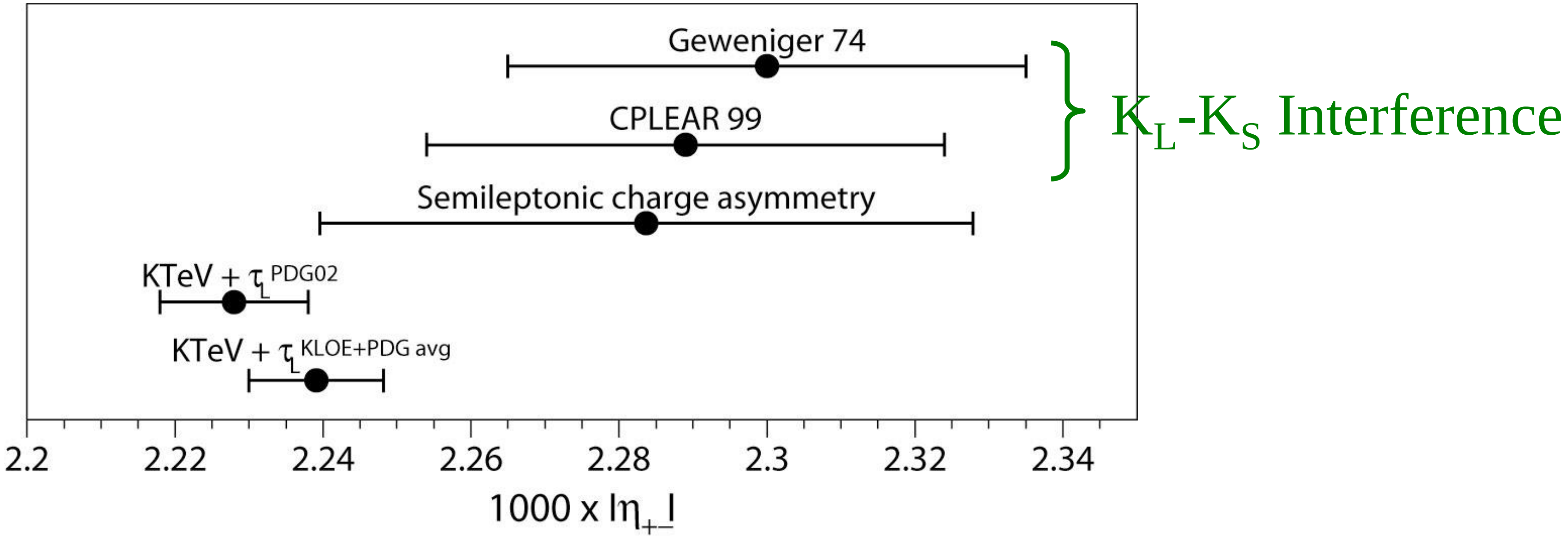


Determination of $|\eta_{+-}|$ Using $B(K_L \to \pi\pi)$

$$|\eta_{+-}|^2 = \frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)} = \frac{\tau_S}{\tau_L} \frac{B^L_{\pi^+\pi^-} + B^L_{\pi^0\pi^0}\left[1+6\,\mathrm{Re}(\varepsilon'/\varepsilon)\right]}{1-B^S_{\pi\ell\nu}}$$

$$|\eta_{+-}| = \left(2.239 \pm 0.005_{KTeV} \pm 0.008_{EXT}\right) \times 10^{-3}$$

(using new average τ_L)

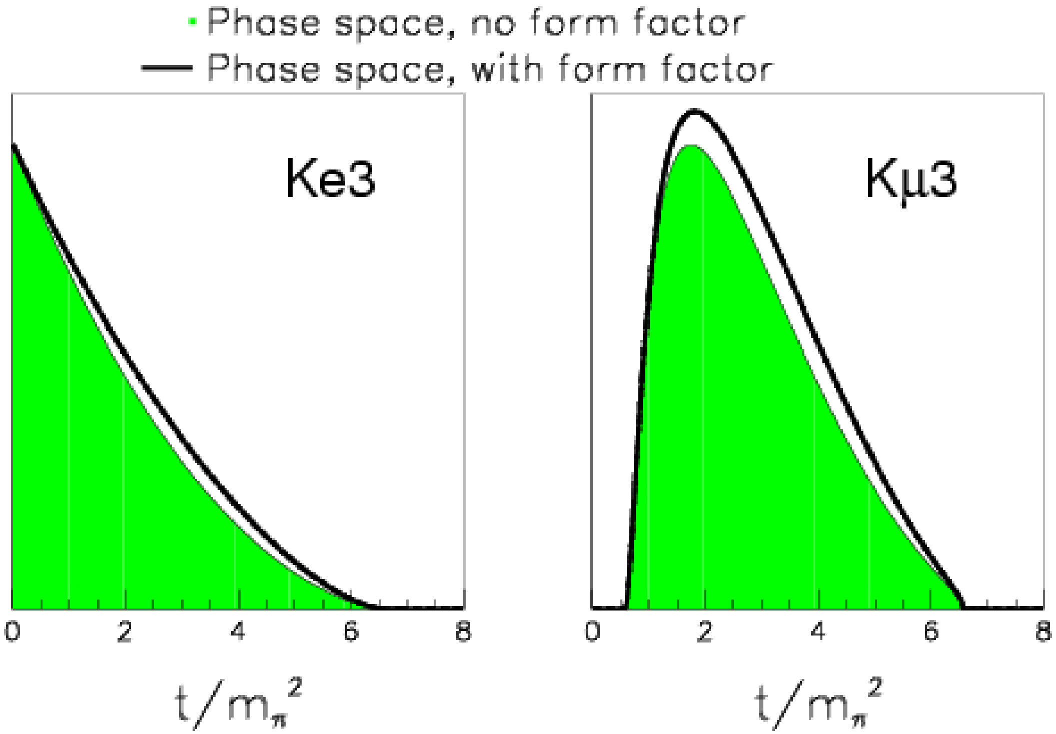


Semileptonic Form Factor Measurements

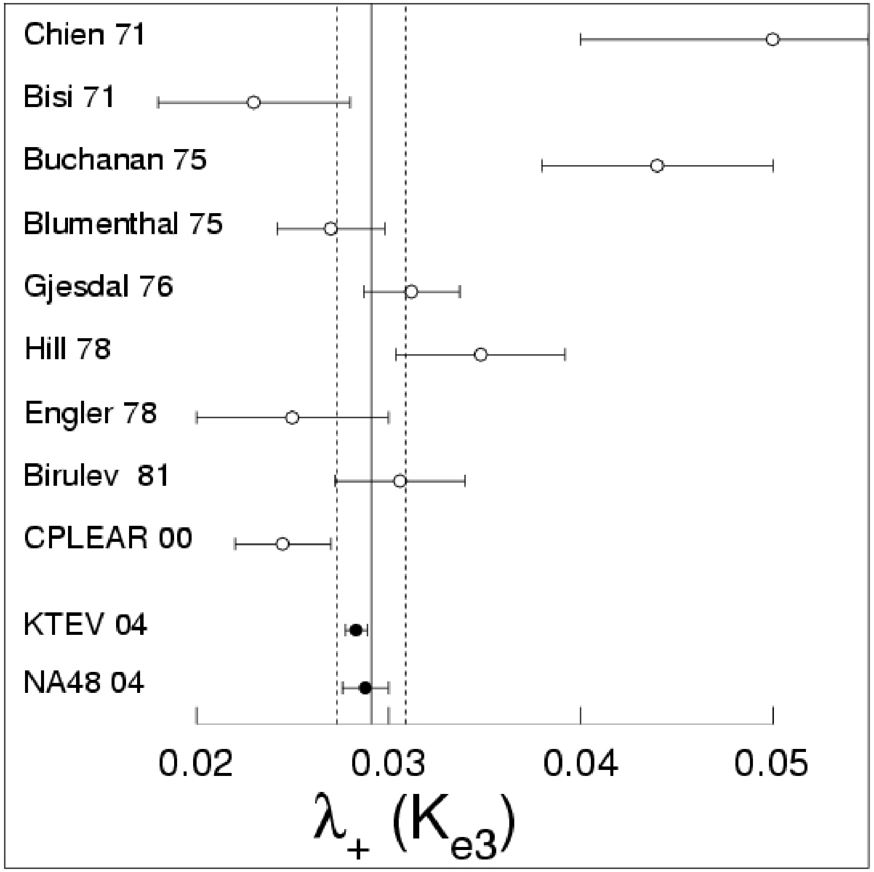
(to determine I_K integrals)

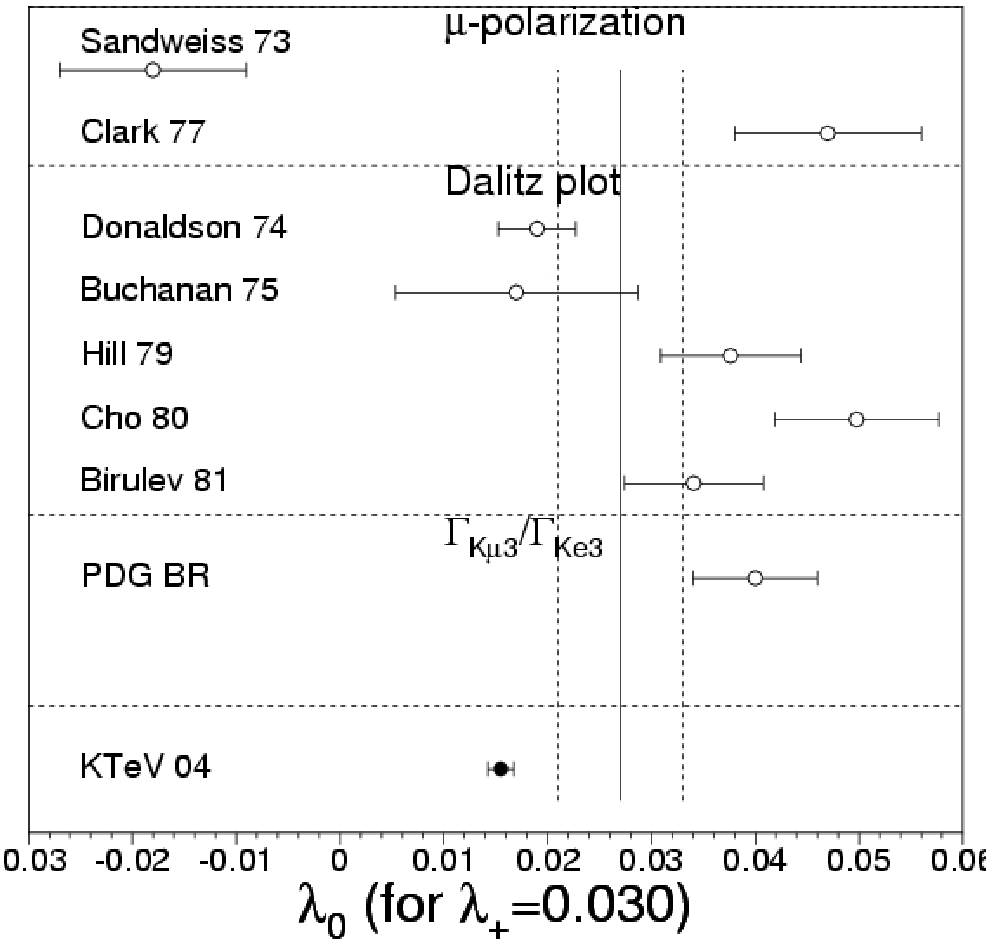
$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW} (1+\delta_K^\ell) |V_{us}|^2 \ f_+^2(0) I_K^\ell$$

I_K depends on the two independent semileptonic FFs: $f_+(t)$, $f\_(t)$



K_L Form Factor Results





Consistency of Branching Fraction and Form Factor Results with Lepton Universality

Compare $\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW} (1+\delta_K^\ell) \left|V_{us}\right|^2 f_+^2(0) I_K^\ell$ for K_{e3} and $K_{\mu 3}$

$$\left[\frac{\Gamma_{K\mu 3}}{\Gamma_{Ke3}}\right]_{PRED} = \left(\frac{1+\delta_K^\mu}{1+\delta_K^e}\right)\left(\frac{I_K^\mu}{I_K^e}\right)$$

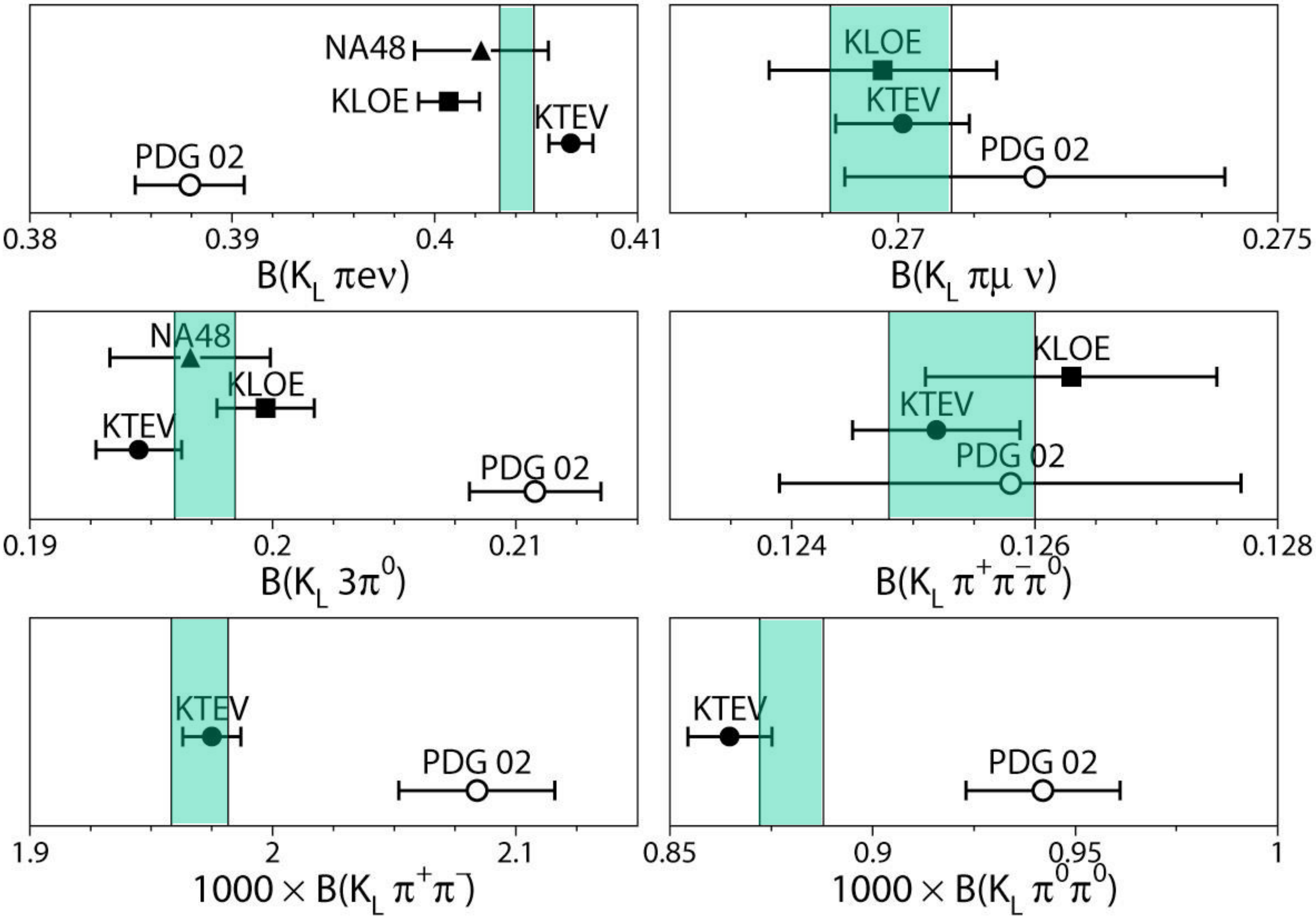
1.0058(10)
from Andre

0.6622(18) from KTeV

$$\boxed{\left[\frac{\Gamma_{K\mu 3}}{\Gamma_{Ke3}}\right]_{MEAS} \Bigg/ \left[\frac{\Gamma_{K\mu 3}}{\Gamma_{Ke3}}\right]_{PRED} = 0.9969 \pm 0.0048 = \left(\frac{G_F^\mu}{G_F^e}\right)^2}$$

Same test with PDG widths and FF gives 1.0270±0.0182

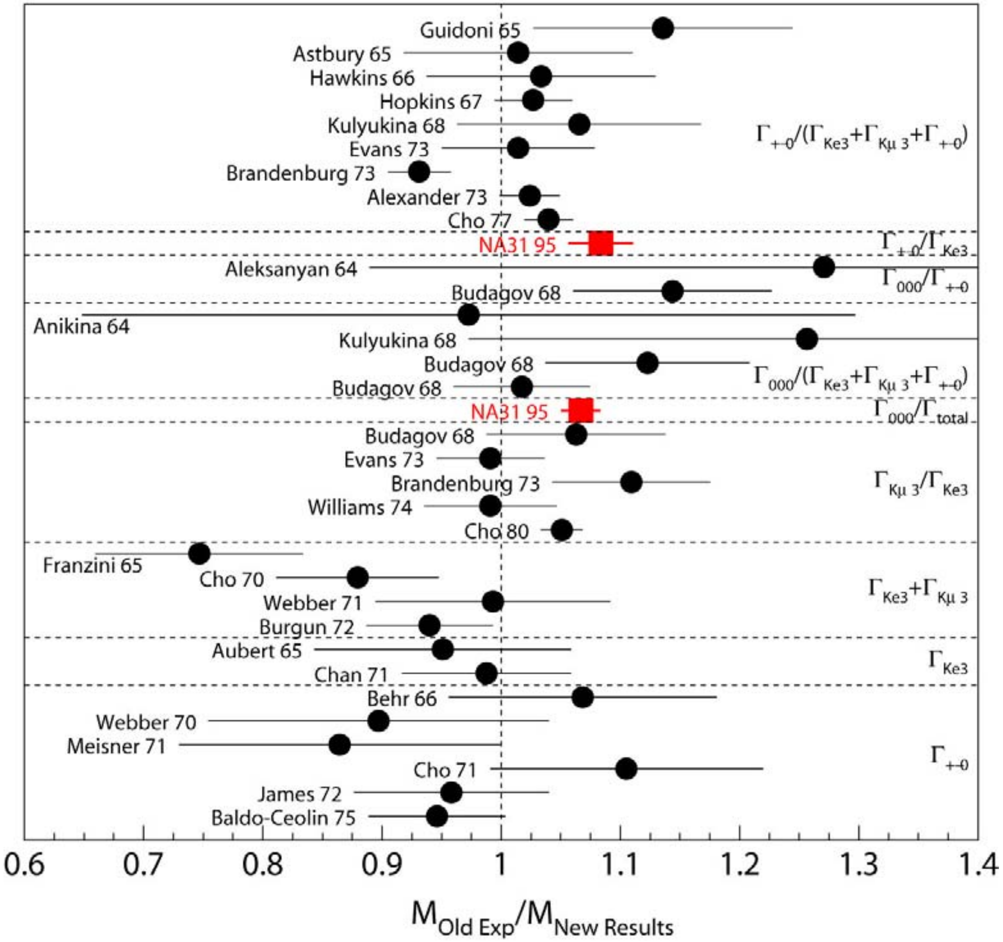
Comparison of KTeV, NA48, KLOE, PDG K_L Branching Fractions



🟩 Value based on PDG-style fit to all new measurements (KTeV, KLOE, NA48)

How could PDG averages be so far off?

Comparison with Individual Experiments



- PDG fit combined different width ratios from many (~40 experiments with constraint that $\Sigma\Gamma_i=1/\tau$.
- It’s likely that many (most?) experiments did not treat radiation adequately, particularly for electron modes.
- These potentially large correlated systematic errors were not taken into account in the PDG fit.

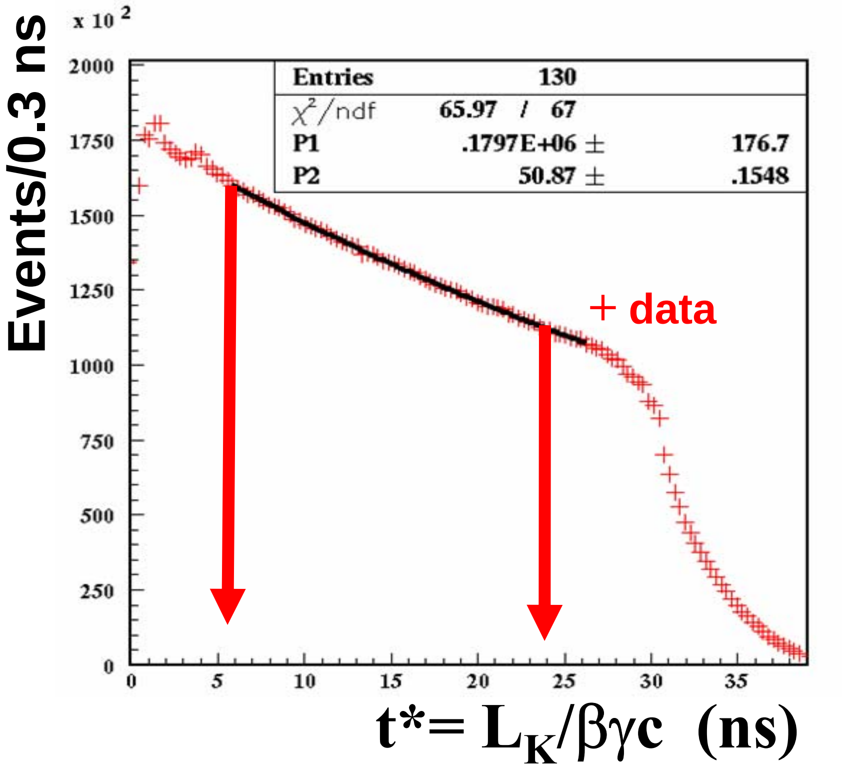
KLOE K_L Lifetime Measurements

1. “Indirect method” – from branching fraction measurement.

   Detector acceptance depends on τ_L. Comparison of $\Sigma B(K_L \rightarrow i) + \Delta_{small}$ with 1 can be used to determine

   τ_L= (50.72±0.14±0.33) ns

2. “Direct method” using using $K_L \rightarrow \pi^0\pi^0\pi^0$



τ_L=(50.87±0.16 ±0.26) ns

Combining both KLOE results:
τ_L=(50.81±0.23)ns

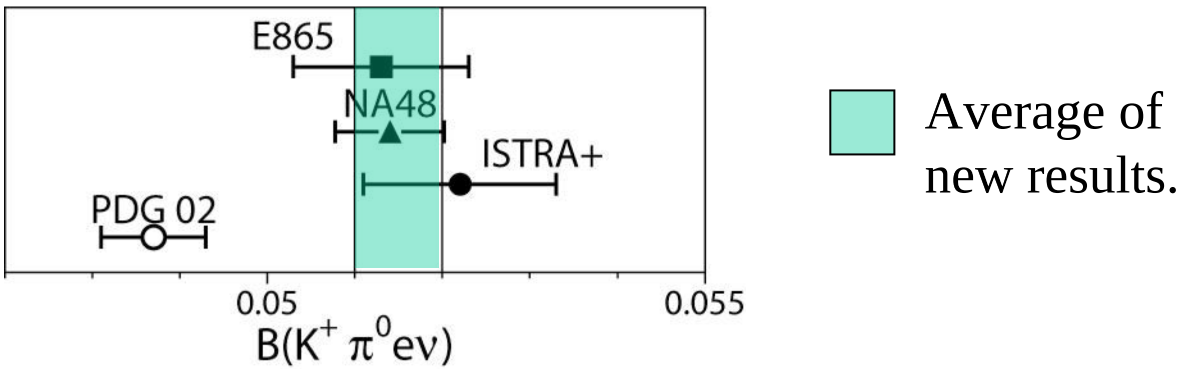
PDG Average: τ_L=(51.5±0.4)ns

New average: τ_L=(50.98±0.21)ns

Charged Kaon Decays

- New measurement of semileptonic form factors from ISTRA+

- New measurements of $B(K^{\pm}\to\pi^0 e^{\pm}\nu)$:



… but, decay modes used as normalization for $K^{\pm}\to\pi^0 e^{\pm}\nu$ have not been remeasured. ⇨ KLOE, NA48

- Also, new measurement of K^+ lifetime is needed. ⇨ KLOE

Input to Calculate “Recent” $|V_{us}|$ (on next page)

B(K_Le3): KTeV, KLOE, NA48
B($K_L\mu$3): KTeV, KLOE
B(K_Se3): KLOE
B(K^+e3): E865, NA48, ISTRA+
τ_L: KLOE+PDG average
τ_S: KTeV, NA48 average
τ_+: PDG

K^0: KTeV quadratic FF
(including 0.7% model dep.)

K^+: ISTRA+ quadratic FF
(including 0.7% model dep.)

$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW}(1+\delta_K^\ell+\delta_{SU2})C^2\left|V_{us}\right|^2 f_+^2(0) I_K^\ell$$

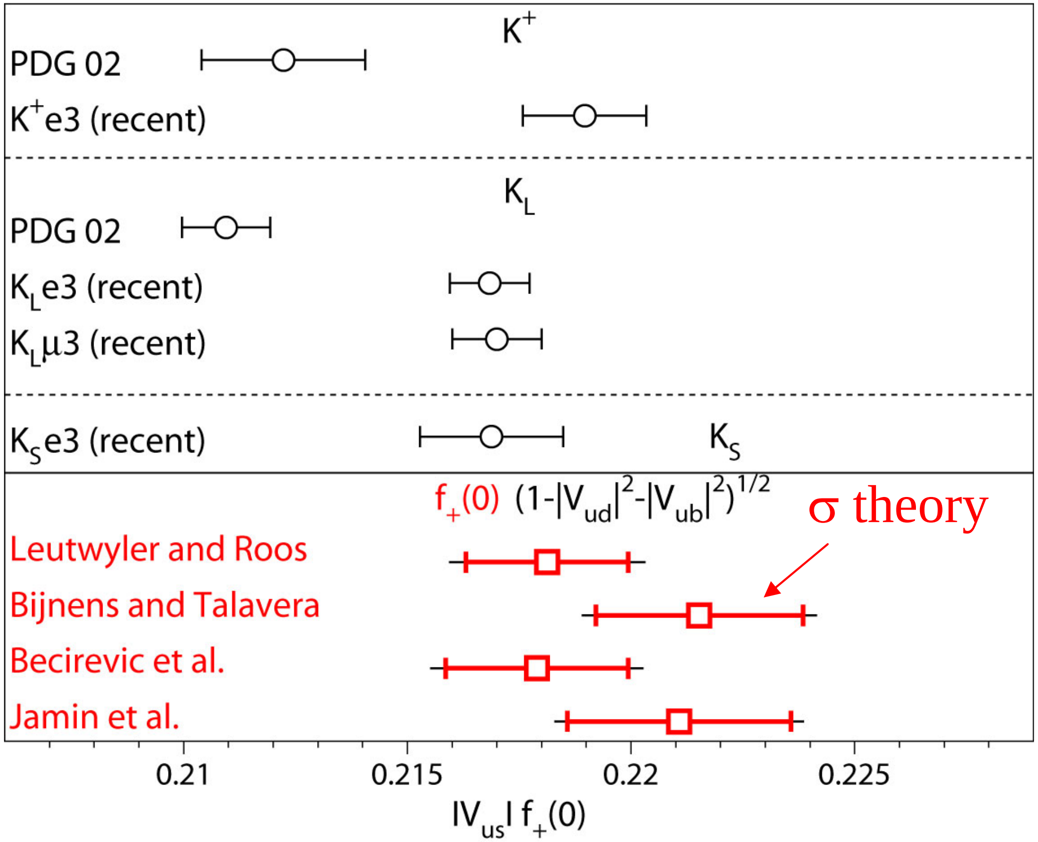
SEW (short-distance rad. corr) = 1.023 (Sirlin)

Long-distance radiative corrections: (Andre, Cirigliano et al.)
δ^e=0.0104± 0.002 (was ~2% from Ginsberg)
δ^μ=0.019 ± 0.003
δ^e_+=0.0006 ± 0.002

δ_{SU2}=0.046±0.04 (Cirigliano)

$f_+(0)$= 0.961 ± 0.008 (Leutwyler – Roos) + recent calculations

Comparison with Unitarity



Average of all "recent" results accounting for correlations:

$$\boxed{|V_{us}|\, f_+^{K^0\pi^-}(0) = 0.2173 \pm 0.0008}$$

Uses updated |V$_{ud}$| = 0.9739±0.0003 (Hardy, Towner; Marciano,Sirlin -- Kaon 2005)

Using f$_+$(0)= 0.961 ± 0.008 (Leutwyler – Roos),

$$\boxed{|V_{us}| = 0.2261 \pm 0.0021}$$

$$(\text{KTeV}: |V_{us}| = 0.2263 \pm 0.0022)$$

$$1 - \left(|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \right) = 0.0004 \pm 0.0011$$

Conclusions

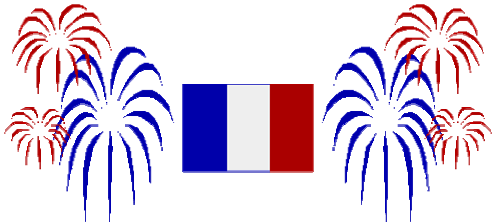
- New $K_{\ell 3}$ measurements result in +3% shift in $|V_{us}|$ compared to PDG (~5σ shift), and are consistent with CKM unitarity (depending on $f_+(0)$):

$$1-\left(\left|V_{ud}\right|^2+\left|V_{us}\right|^2+\left|V_{ub}\right|^2\right)=0.0004\pm 0.0011.$$

- Other methods ($K_{\mu 2}/\pi_{\mu 2}$, τ) give somewhat lower $|V_{us}|$; new measurements of $0^+ \rightarrow 0^+$ nuclear β decays in progress.

- 5-8% shifts observed in main K_L branching fractions.

  Value in repeating old measurements with modern, high statistics experiments!



EXTRA SLIDES

$|V_{us}|$ from $K^+\rightarrow\mu^+\nu$ and f_K/f_π

$$\frac{\Gamma(K^+\rightarrow\mu^+\nu)}{\Gamma(\pi^+\rightarrow\mu^+\nu)}=\frac{V_{us}^2}{V_{ud}^2}\frac{f_K^2}{f_\pi^2}\frac{M_K^2-M_\mu^2}{M_\pi^2-M_\mu^2}\left[1-\frac{\alpha}{\pi}\left(C_\pi-C_K\right)\right]$$

Marciano, 2004

Using $B(K^+\rightarrow\mu^+\nu)=0.6366\pm0.009\pm0.00015$ (KLOE)

$F_K/f_\pi = 1.210 \pm0.014$ (MILC)

$C_\pi - C_K = 3.0\pm0.75$ (Finkemeier; Knecht et al., Cirigliano et al.)

$|V_{us}|=0.2223 \pm 0.0026$

➡ $1-\left(|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2\right)=0.0023\pm0.0015$