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Neutrino Factory Phase-Rotation LIA: Pulsed Power System Optimization

I. Introduction

Concept of a LIA based phase rotating system for the Neutrino Factory (NF) has been introduced in [1], and then developed in [2]. Main idea of the system is to use time dependent electrical field of an induction nature to correct 200 MeV muon energy spread after energy-time correlation is developed in a drift space. The required electrical field time dependence has been calculated in [2] and is shown in Fig. 1 below for the 100-m LIA length.

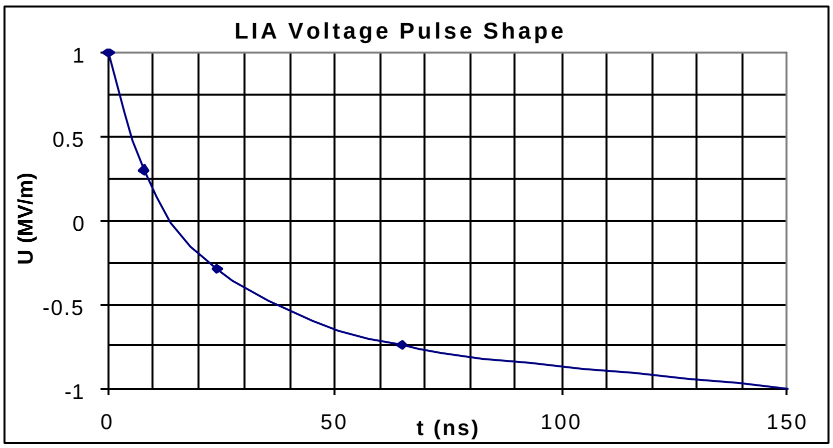

Fig. 1. Calculated Accelerating Field Strength at the Input into a Phase-Rotating Channel

At the input into the channel, head part of muon beam has higher energy, so electrical field must change from decelerating to accelerating during pulse duration, which is about 150 ns at the input into the system. Here and below there is an agreement that muons have a negative charge, so decelerating field corresponds to the positive (along the direction of movement) electrical field. It worth to note that because any section of the LIA is a combination of a drift channel and phase-rotating device, voltage shape and pulse duration changes while muons go along the phase rotation channel. Near the end of the channel, pulse length is to be close to 250 ns. It is also necessary to mention that muon beam current is quite low, about 10 A, and time dependence of the accelerating voltage pulse is very nonlinear. Each of these features is very unusual for an induction-type accelerating system that has by its nature high-current and high power capability. Muon decay time is about 1 μs, so the length of the channel must be reasonably short. Because accelerating rate of 1 MeV/m was demonstrated for LIA, this value has been taken as a base for future analysis. This corresponds to channel length of about 100 m.

Four pulses of muons following with head-to-head delay of 667 ns form one macro-pulse that must reappear every 66.7 ms (the basic operation frequency of the system is 15 Hz). The shape of the active part of each correction voltage pulse (Fig. 1) is not defined strictly: it allows certain displacement in vertical direction thus making accelerating (or decelerating) voltage higher or lower. This uncertainty has allowed three different approaches to pulse forming concept that can be found in [3], [4], and [5]. One of the approaches suggests a voltage pulse with active part that follows the chart in Fig. 1 [5];

the other uses higher decelerating voltage to reduce volt-seconds for the accelerating active part of the pulse in order to make the total pulse volt-seconds close to zero [3]; the third suggests using of a relatively long and low voltage reset pulse of decelerating polarity with active part of the pulse formed by the front of a pulse of accelerating polarity [4].

The goal of this work was to understand limits of our freedom of choosing system parameters, and to find a way of making some optimization in order to choose them using quantitative criteria.

II. Core power loss model

As it is well known, the key issue for developing a pulsed power system for LIA is energy loss in an induction core. Knowledge of how power loss in a core depends on core magnetization rate and excitation field allows us to make a realistic estimate of a power required to operate the phase rotation LIA with its unique voltage pulse shape. The model that describes energy loss for a core made of a metal ribbon was developed by R. Smith [6]. Depending of core magnetization rate, different processes are responsible for loss of energy. In our analysis we will take into account that magnetization rate is very high, so the saturation wave domain movement mechanism is responsible for the major part of power loss in a core. For example, for nanocrystalline alloy FT-1H, at magnetization rate of more than 10 T/μs, eddy currents described in terms of a saturation wave domain movement explain about 95% of total core power loss. Following [6], we can write for the magnetic field *H(t)* driving saturation wave domain:

$$H(t) = \left(\frac{d^2}{4r}\right) \cdot \left(\frac{\Delta B(t)}{2B_S}\right) \cdot \left(\frac{dB}{dt}\right) \qquad (1)$$

where $\Delta B(t) = B(t) - B_r$ is an efficient flux density swing, B_S is material saturation field, B_r is residual field, d is ribbon thickness, and r is a specific material resistance (all dimensions are in accordance with SI).

Power loss per unit of a magnetic material volume are described by the expression:

$$p(t) = H(t) \cdot \frac{dB(t)}{dt}, \qquad (2)$$

that gives after using (1)

$$p(t) = \left(\frac{d^2}{4r}\right) \cdot \left(\frac{B(t) + B_r}{2B_S}\right) \cdot \left(\frac{dB}{dt}\right)^2 \qquad (3)$$

We can find $\frac{dB}{dt}$ if we know required voltage pulse shape U(t):

$$\frac{dB}{dt} = \frac{U(t)}{S}, \qquad (4)$$

where S is an effective core cross-section per one meter of LIA length. To calculate power needed to support this voltage we need a model that describes mathematically LIA cell voltage. Active part of the voltage pulse shown in Fig. 1 does not describe completely pulse shape. It is necessary to take into account voltage rise and decay time in order to make a realistic estimate of pulse volt-seconds that define induction core volume. In this note it was accepted that the voltage pulse is described basically by exponential functions like it happens in real L-R or R-C circuits. The main assumption that was made at this stage was that there must be approximately equal volt-seconds for the positive and negative parts of the pulse. In this case, core reset process can be completed automatically by the end of the pulse, so fast chain of four (or more) pulses becomes just technical (not a principal one) problem that can be resolved after some amount of time (and money) is devoted to it.

There are many ways to describe LIA pulse shape. It was found convenient for this work to choose the next one:

$$U(t) = \begin{cases} \dfrac{U0}{1-e^{-2}} \cdot (1-e^{-\frac{t}{t1}}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 0 < t \le t1 \\ U0 - U1 \cdot (1-e^{-\frac{t-t1}{t0}}) - 3 \cdot 10^{12} \cdot (t - t1) \ldots\ldots t1 < t \le t1 + T_p \\ U(t1+T_p) \cdot e^{-\frac{t-(t1+T_p)}{t2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots t1 + T_p < t \end{cases} \quad (5)$$

Here $t1$ is pulse rising time, T_p is duration of active part of the pulse, $U0$ is the level of decelerating voltage in the very beginning of the pulse, $U1$, $t0$, $t1$, and $t2$ are the constants that define details of the pulse shape. With $U1$ = 1.55 MV and $t0$ = 15 ns, this expression fits nicely to the requested active part of the pulse (Fig. 1) with maximum deviation of not more than 7.5% except small region near the point t = 10 ns where the voltage curve is very steep. The moment where the front part of the pulse ends and active part of the part starts is chosen to be $t1 = 2 \cdot t1$. Then, $U(t1) \equiv U0$. With this agreement, volt-seconds for the front part of the pulse can be easily found:

$$VS_{FR} = \frac{1+e^{-2}}{1-e^{-2}} \cdot U0 \cdot t1 \quad (6)$$

Using this expression, it easy to adjust volt-seconds for the positive and negative parts of the voltage pulse by changing the parameter $t1$. Parameter $t2$ for the rear part of the pulse can be chosen arbitrary because after active part of the pulse ends, any increase of volt-seconds for the negative part of the pulse will lead to core saturation that will ensure a reproducible starting point for the core magnetization history.

The algorithm to fully describe voltage pulse is written down below:

- Choose maximum voltage level ***U*0** and find a root ***t*0** of the expression (5) (***t1*=0** can be accepted for this step);
- Find volt-seconds for the negative active part of the pulse by integrating ***U(t)*** starting from ***t*0** till the end of the active part of the pulse;
- Find volt-seconds of the positive active part of the pulse by integrating ***U(t)*** starting from the start of the active part of the pulse till ***t*0**;
- Using (6) define the front (positive) part of the pulse so that the total volt-seconds for the positive part of the pulse were equal to that of the negative active part of the pulse.
- Add tail part of the pulse using $t2 = t0$.

This procedure fully describes the pulse that meets the requirement of self-resetting, so after the end of the pulse the system is fully ready to accept another one.

III. Pulse shape optimization

Three cases were investigated that correspond to U0 = 1.0 MV, U0 = 1.25 MV, and U0=1.5 MV. Figures 2, 3, and 4 show correction voltages for a 1-m section of a 100-m LIA for each of this cases.

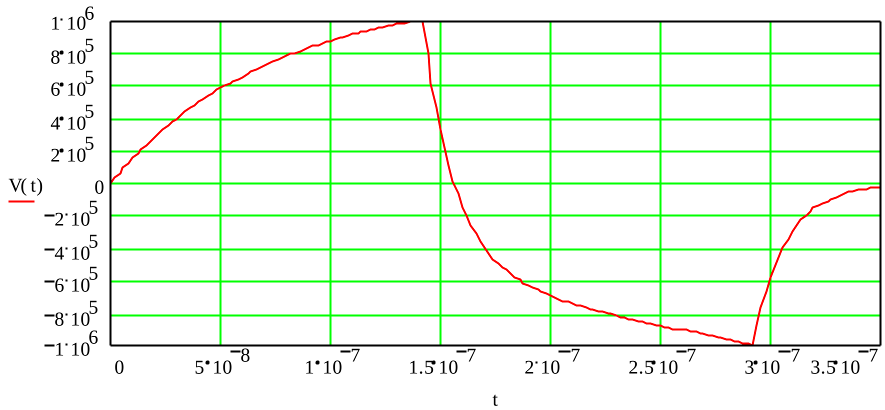

Fig. 2. Pulse shape corresponding to U0 = 1 MV

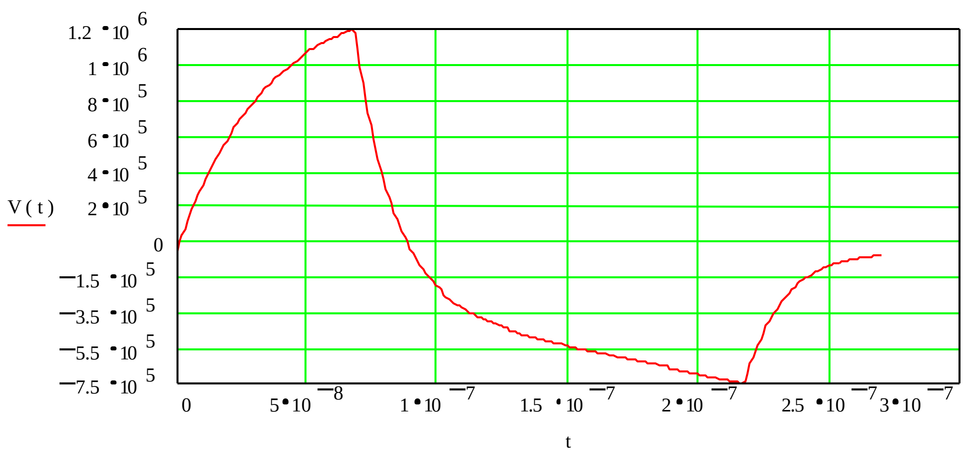

Fig. 3. Pulse shape corresponding to U0 = 1.25 MV

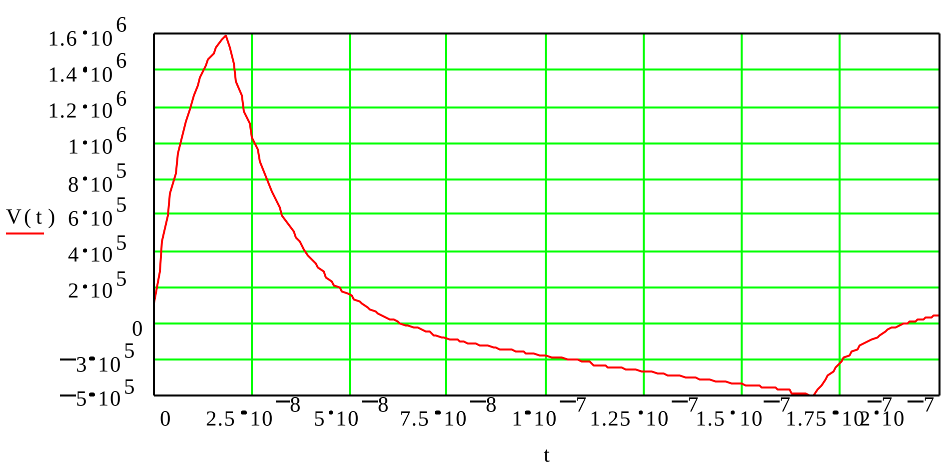

Fig. 4. Pulse shape corresponding to U0 = 1.5 MV

For each case, for 1-m accelerating section, minimal induction core cross-section was calculated that can support the voltage pulse, so that core magnetic field changes from $-B_r$ to $+B_r$ during the positive part of the pulse, and goes back to $-B_r$ when active part of the pulse ends.

$$S_{\min} = \frac{\int_{0}^{t0} U(t)dt}{2B_r} \equiv \frac{\int_{t0}^{t0+T_p} U(t)dt}{2B_r} \qquad (5)$$

Then, using this cross-section, (4), and (3), for any moment, we can find power needed to be delivered to a LIA section to provide the requested pulse shape. Here and below we will accept section length of 1 meter, and average core radius of 0.5 m. Preliminary section layouts [3, 4] does not contradict to this choice; needed corrections can be made as a second iteration. The calculations assumed that cores were fabricated from METGLAS 2605SC amorphous alloy ribbon with thickness of 25 μm. Specific resistance of this alloy is $1.3 \cdot 10^{-6}$ Ohm·m. Table 1 below summarizes system reaction to different cell voltage pulse shapes corresponding to Fig. 2, 3, and 4 above.

Table 1.

	S_{min} (m^2)	$P1_{max}$ (GW/m)	W1 (J/m)	P_{av} (MW)	ρ (Ohm)
U0 = 1.00 MV	0.045	5.54	460	2.75	9.0
U0 = 1.25 MV	0.030	11.65	440	2.64	5.4
U0 = 1.50 MV	0.016	18.9	360	2.14	4.0

For the three cases in study, this table shows minimal 1-m section core cross-section S_{min}, maximum power loss $P1_{max}$ per one meter of LIA (that corresponds, obviously, to the moment ***t = t*1**), full energy loss W1 per one cycle and per one meter of the channel length, and average 100-m LIA power for the four-pulse regime and 15 Hz operation frequency. It is possible to see from this table that minimal core cross-section, full energy loss, and average power are minimal for the case with 1.5 MV maximum decelerating voltage although maximum pulse power is maximal in this case. The last column in the table shows needed distribution line impedance to assure proper section cell operation for 50 kV of an induction cell voltage. Very low values of the impedance, although quite realistic, reflect the fact that pulsed power needed to feed the system is very high. So some optimization of system parameters is required to reduce the power.

IV. System power optimization.

As it follows from (2) and (3), the most obvious way to reduce energy loss in LIA section is to increase core cross-section. Although it increases capital cost of LIA construction, it can save money in long-term spending that includes power cost. So, we can introduce an effective cost, which is a sum of a capital cost and energy cost for 5 years of system operation, as an optimization parameter. Table 2 shows maximum power consumption $\boldsymbol{P1_{max}}$, energy ***W1*** per one pulse per one meter of the channel length, and average LIA power P_{av} depending on the 1-m section core cross-section $\boldsymbol{S_{core}}$. Only the case with maximum decelerating voltage of 1.5 MV per 1-m LIA section was considered here because it gave minimal energy loss as we have learned recently. It is possible to see that while cross-section grows from its minimal value of 0.016 m^2 by factor of 3, maximum power, energy loss, and average power drop by factor of 9.5.

Table 2

Acore (m^2)	0.016	0.02	0.03	0.04	0.05
Pmax (GW/m)	18.9	11.25	5.47	3.1	1.97
W1 (J/m)	360	230	103	58	37.2
Pav (MW)	2.14	1.4	0.62	0.35	0.223

So, it is clear that there must be an optimal cross-section that corresponds to minimal effective cost. This optimal value in the simplest case depends only on the energy cost and on the induction cell core cost. After the first approach to pulsed power system design is made, it would be useful to include into account the cost of this system also to make the optimization more realistic. Nevertheless, even this simple approach will help us to choose a proper set of parameters for a preliminary design. Fig. 5 shows the results of this optimization that are based on the energy cost of $0.05 per 1 kW-h and on core cost of $20 per 1 kg of an assembled core [7, 8].

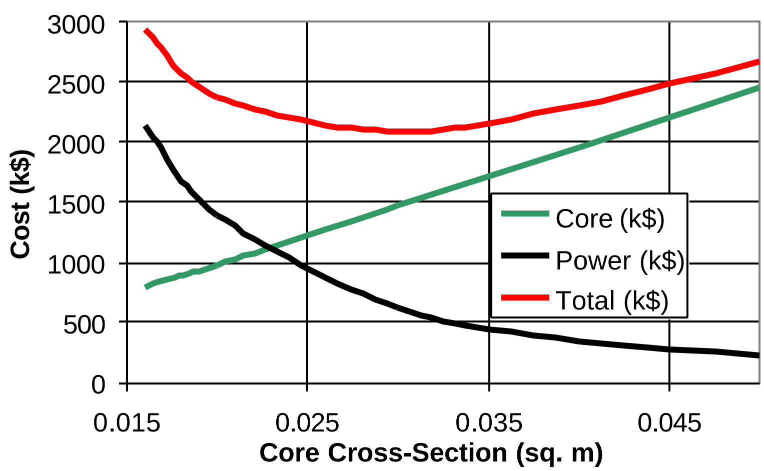

Fig. 5. Core cross-section optimization chart

It is possible to see that with the assumptions made above, optimal core cross-section is about 0.03 m^2, which is about 2 times larger than the minimal one.

Another possibility to reduce the system power is to increase the length of LIA. As we have seen from the section III above, optimal pulse shape requires increasing of decelerating voltage at the beginning of the pulse to the level of 1.5 MV or even higher. Hardware development will show where the limit to this increase can be, but from the point of view of system reliability, it is always useful to keep this voltage at "as low as it is possible" level. If the length is increased by factor of 1.5, according to (2) we will have about $(1.5)^2$ times less power consumption per 1 meter of the LIA. With this, the total pulse energy per LIA will drop by factor of 1.5. So, if we increase LIA length, we can:

- gain in maximum power, that can simplify significantly pulsed power system,
- increase system reliability (voltage breakdowns),
- save in operation cost.

Core volume increases only by about 6% as it can be seen from the table below that presents main parameters of the LIA with optimal core cross-section for cases of 100-m and 150-m length of the channel.

Table 3

LIA channel length (m)	100	150
Maximum electrical field gradient (MV/m)	1.5	1.0
Optimal 1-m section core cross-section (m^2)	0.03	0.022
Core radial thickness with compaction factor of 0.35 (m)	0.086	0.063
Amount of metglas alloy needed for LIA channel (kg)	75,000	80,000
Maximum pulsed power required for 1-m LIA section (GW/m)	5.5	3
Energy loss per one pulse for 1-m LIA section (Jouls)	100	57
Average power requirement for the LIA channel (kW)	620	512
50-kV cell distribution line impedance (Ohm)	13.6	16.67

V. Conclusion

The attempt of the Neutrino Factory Phase Rotation Channel LIA system optimization has been made. Mathematical model of the LIA voltage pulse shape has been suggested that allowed the fast pulse train regime. Based on the known theoretical model of power loss in laminated magnetic materials, the shape optimization was done with the goal to reduce the system power consumption. It was also found the possibility to optimize the amount of magnetic material used to build the LIA cell. For the METGLAS 2605SA material, this optimization gives the optimal mass of metglas cores for100 m LIA of about 75 T (only for the LIA channel). It was shown that the increase of the LIA length will further reduce required power for the LIA operation.

It would be useful in future to get answers to the questions from the following "what next to do " list:

1. It would be useful to optimize voltage pulse positioning and the pulse length with respect to muon beam at the input of the phase-rotating channel so that to lower required accelerating field as much as it is reasonably possible.
2. It will be necessary to investigate another materials for core fabrication: crystalline alloys and cobalt-based amorphous alloys.
3. It will be necessary to come out as soon as possible with any hardware solution that can generate needed shape of cell voltage pulse.
4. It will be necessary to get a direct answer to the core transverse magnetization problem. The positive answer is expected, but we need full understanding here to be able to make an optimal design.

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