Thin-lens modeling of combined-function bend cooling lattices.

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Introduction

Muon Collaboration

- Make lattice cell as compact as possible: fewest possible magnets
- Need space for RF and absorbers
- Quadrupoles defocus in one dimension: simplest cell requires 2
- Combined-function bends can focus in both planes
  - Choose equal focusing
  - Ignoring moving closed orbit
- Desirable qualities of cooling lattice
  - Low β at absorber
  - Wide energy acceptance
  - These quantities compete.
- Thin lens approximations
  - Good until magnets intrude significantly into drifts

Single Magnet per Cell

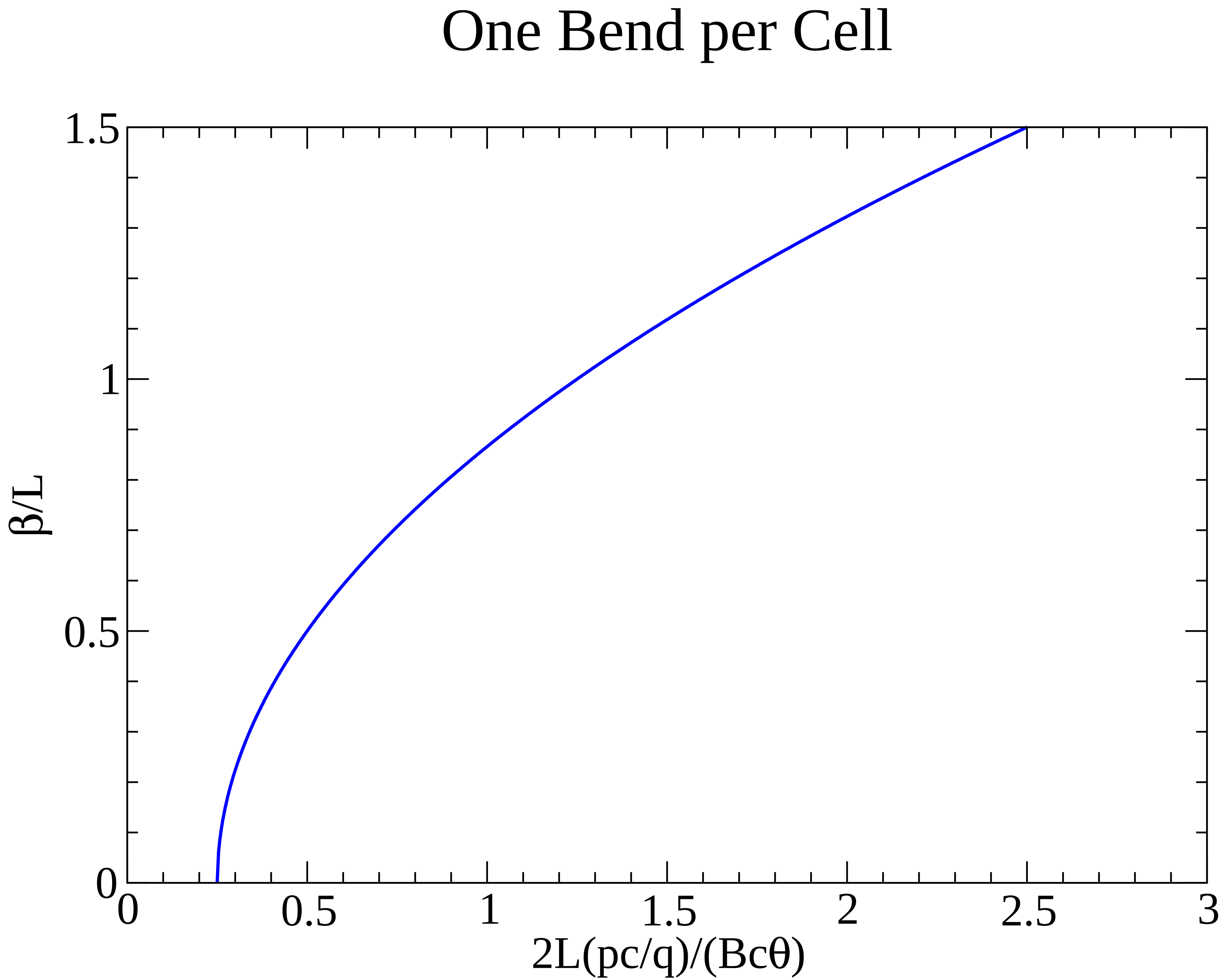
- Sinble combined function magnet with RF and wedges together in striaght
- Momentum passband from nonzero value to infinity
- Beta function zero at low energy, rises to infinity
- Larger momentum chunk required, β must be higher
  $(\Delta = (p_{\max} - p_{\min})/(p_{\max} + p_{\min}))$

$$\beta_{\max} = 2L\sqrt{\frac{2\Delta}{1-\Delta}} \qquad \Delta = \frac{\beta_{\max}^2}{8L^2 + \beta_{\max}^2}.$$

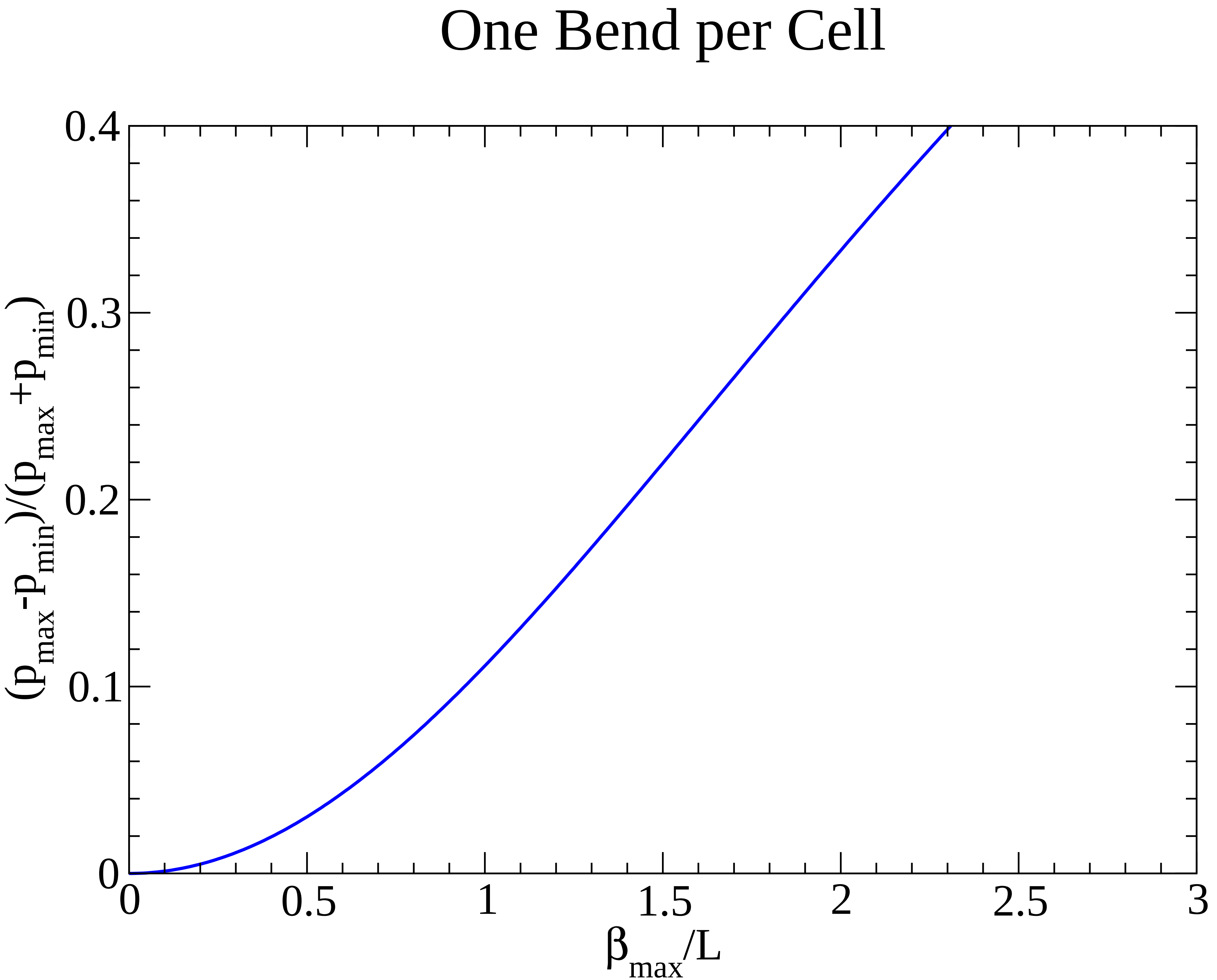
- No extra freedom: once β and L are chosen, energy spread is given to you.
- Field required:

$$B\theta = \frac{2}{c}\frac{p_{\mathrm{ref}}c}{q}\frac{8L}{8L^2 + \beta_{\max}^2}$$

  roughly independent of $\beta_{\max}$
- Shorter length, lower $\beta_{\max}$



One Bend per Cell
1.5
1
0.5
0
β/L
0
0.5
1
1.5
2
2.5
3
2L(pc/q)/(Bcθ)



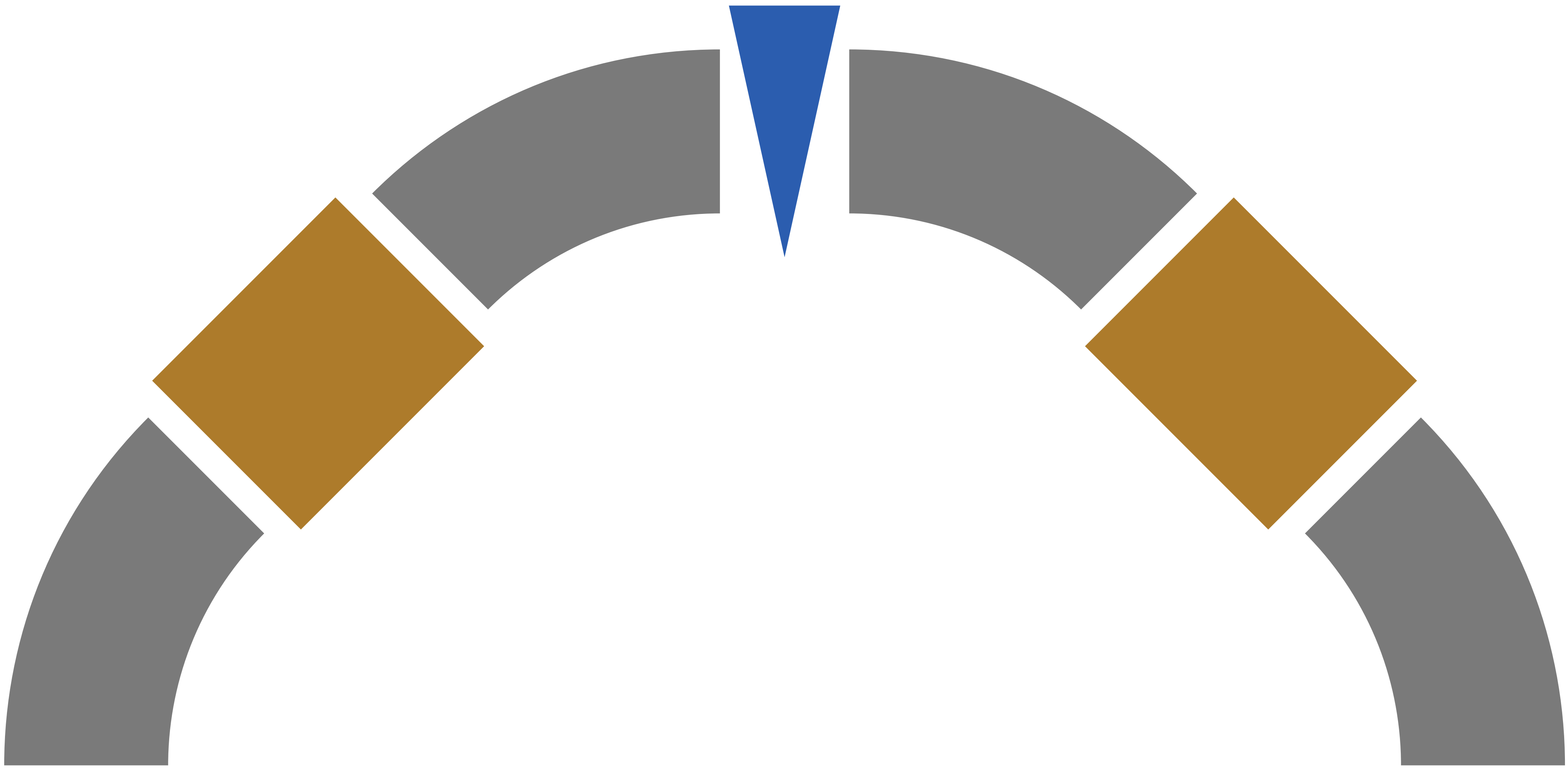
One Bend per Cell
0.4
0.3
0.2
0.1
0
(p_max-p_min)/(p_max+p_min)
0
0.5
1
1.5
2
2.5
3
β_max/L

Two Magnets per Cell

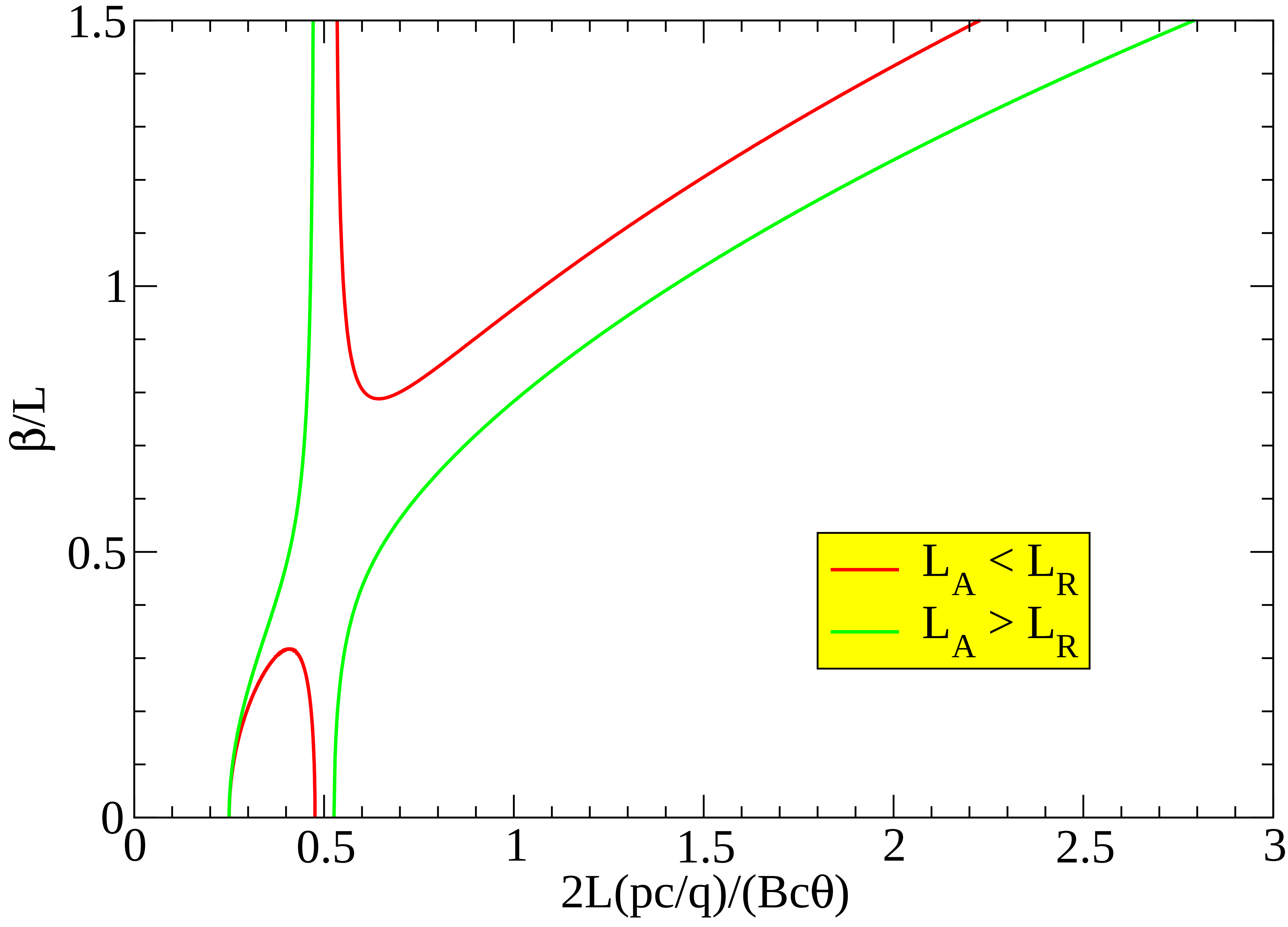
- Two identical bends
- Two straights: absorber (length L_A) and RF (length L_R)
- Generally minimum length for RF straight
- Momentum range has stopband, so two passbands
  - Deppending on relationship between L_A and L_R, lowest stopband has zeros on both ends or one end
  - Lowest β_{max}: zeros on both ends. Requires $L_A < L_R$.
- Relevant quantities:

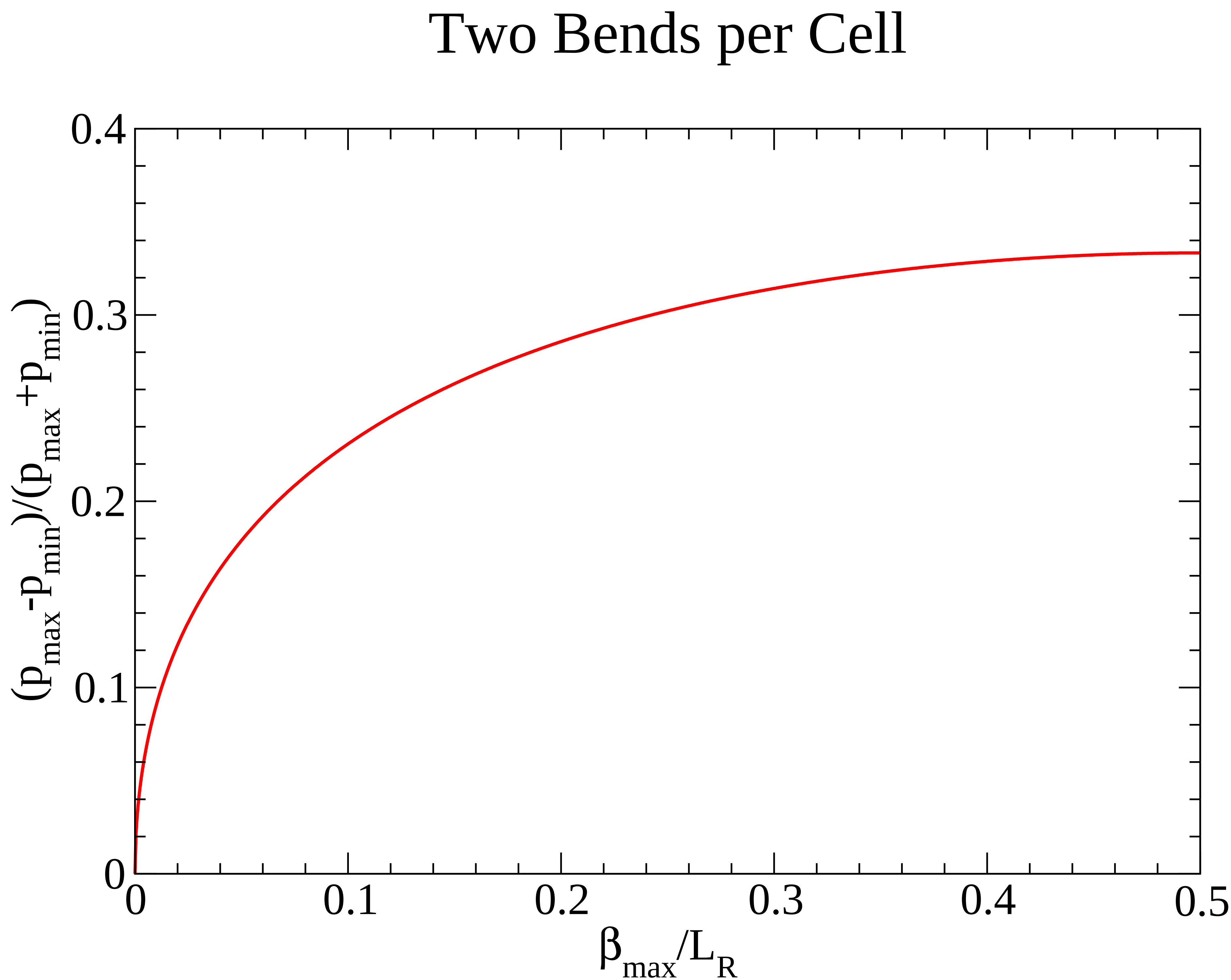
$$\Delta = \frac{\sqrt{\beta_{\text{max}}(L_R - \beta_{\text{max}})}}{L_R + \sqrt{\beta_{\text{max}}(L_R - \beta_{\text{max}})}} \qquad L_A = 2\sqrt{\beta_{\text{max}}(L_R - \beta_{\text{max}})}$$

$$B\theta = \frac{2}{c}\frac{p_{\text{ref}}c}{q}\frac{L_R + 2\sqrt{\beta_{\text{max}}(L_R - \beta_{\text{max}})}}{\sqrt{\beta_{\text{max}}(L_R - \beta_{\text{max}})}(L_R + \sqrt{\beta_{\text{max}}(L_R - \beta_{\text{max}})})}$$



Two Bends per Cell




Two Bends per Cell
0
0.1
0.2
0.3
0.4
0
0.1
0.2
0.3
0.4
0.5
β_{max}/L_R
$(p_{max}-p_{min})/(p_{max}+p_{min})$

Two Magnets per Cell (cont.)

- Momentum acceptance doesn't fall too rapidly until $\beta_{\mathrm{max}} \ll L_R$
  - Much better than single magnet per cell
- To reduce β_{max} at absorber, shorten absorber straight
- Reduce β_{max} by reducing absorber straight
- Field increases with increasing β_{max}
- Maximum Δ: 1/3; maximum β_{max}: $L_R/2$
- Example: 1 m RF straight, 250 MeV/c central momentum, 0.25 m β_{max} at absorber:
  - $\Delta = 0.302$, $L_A = 0.867$, $B\theta = 5.02$
  - Strong fields required: 3.2 T for 90° bends (two cells per turn!)
    - Magnet lengths require significant correction
  - Fields would be weaker if ran in higher passband, but β_{max} higher
  - Poor dynamic aperture: combined function magnets have nonlinearities
    - Replace with edge focusing: still have ends