Simulation of RFOFO Ring Cooler with Tilted Solenoids

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Abstract

Alternating solenoid focused RFOFO cooling ring proposed and described in [1] is investigated. Realistic magnetic field producing by tilted on 75 mrad solenoids is calculated and used. Developed in [2] methodic is applied to find closed orbits for any energy, dispersion, eigenvalues of transfer matrix, and other conventional accelerator characteristics. Cooling simulation is performed resulting the cooled beam parameters rather closed to presented in [1].

1 Introduction

RFOFO ring cooler for ionization cooling of muon beams was proposed and investigated for the first time in [1]. The ring consists of 12 periods each providing turn angle 30°. The period shown in Fig.1 includes 2 solenoid coils with opposite direction of currents, 5 pill box cavities, and liquid hydrogen wedge absorber. Period length is 275 cm along centerline of the coils that is a circle of radius $R_0 = 275 \times 12/(2\pi) \simeq 525.21$ cm. Another parameters of the equipment are taken from [1] and listed below.

Coils:

- Inner radius 77 cm;
- Outer radius 88 cm;
- Length 50 cm;
- Current density ± 95.27 A/mm^2;

Cavities:

- Frequency 201.25 MHz;
- Length 28.75 cm;
- Maximal voltage 4.6 MV/cavity;
- Energy gain 2.504 MeV/cavity.

Absorbers:

- Thickness at the center 28.5 cm;
- Energy loss at the center 12.52 MeV;
- Gradient of energy loss 0.6956 MeV/cm.

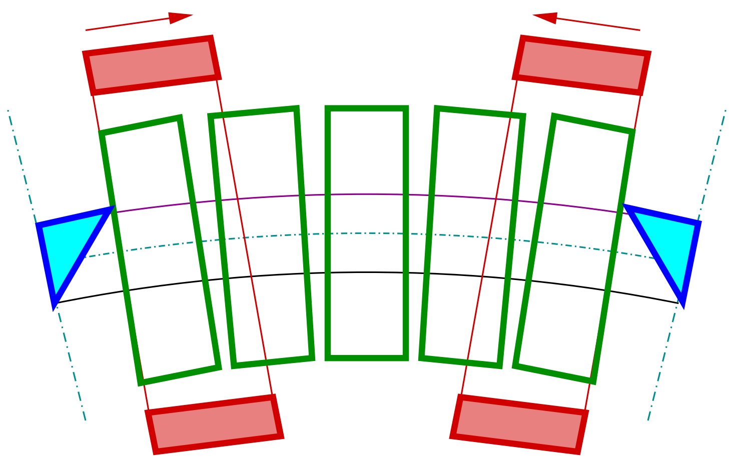

Figure 1: Basic layout of the cooling ring (one period, schematically).

Rectilinear cooling channel composed of similar cells has a region of transverse stability $186 < E < 213$ MeV where E is total energy of muon [1]. Therefore reference energy $E_{ref} = 220$ MeV is taken for the detailed investigation.

Vertical bending field in [1] was generated by a truncated Fourier decomposition of the field from a bent solenoid. It was presumed that corresponding coil position can be found later. In contrast with that, realistic field generated by vertically tilted coils is considered in this paper. Tilting angle is ± 52 rad for the coils with positive/negative current. As will be shown in Sec.3, such angle provides minimal horizontal and vertical swing of closed orbit at the reference energy, what is necessary condition to get high performances cooler [2].

2 Magnetic field

The tracking is performed in cylindrical frame $r\theta y$; however variables $x = r - R_0$, y, and $z = R_0\theta$ are actually used as more usual and convenient ($R_0 = 525.21$ cm). Magnetic field in this frame is obtained in several stages:
• Flat field map of single coil is generated in its natural frame $r_{coil}z_{coil}$ for the region $0 \leq z_{coil} \leq 500$ cm, $r_{coil} \leq 300$ cm using step of 2 cm in both directions. Field lines in a part of this region are plotted in Fig.2.
• Grid is prepared in the cylindrical xyz frame for the area: -36 cm $\leq x \leq$ 14 cm, -23 cm $\leq y \leq 27$ cm, -495 cm $\leq z \leq 495$ cm, at the step of 1 cm in any direction. The asymmetry of this area is explained by a shift of reference closed orbit from centerline what is explained in the next section.
• Position of each node of the grid is transformed to the frame of positive coil with its inclination taken into account. Field in this point is calculated in $r_{coil}z_{coil}$ frame using the coil field map and linear interpolation.
• Obtained field components are transformed to the cylindrical frame xyz resulting field map of single tilted coil with positive current centered at $x = y = z = 0$. Field of the coil with opposite current and inclination is generated using properties of symmetry.
• Centers of positive and negative coils are moved to $z = \mp 82.5$ cm correspondingly, and their fields are added resulting field of the coil pair in the region -412.5 cm $\leq z \leq 412.5$ cm measured from the pair center.

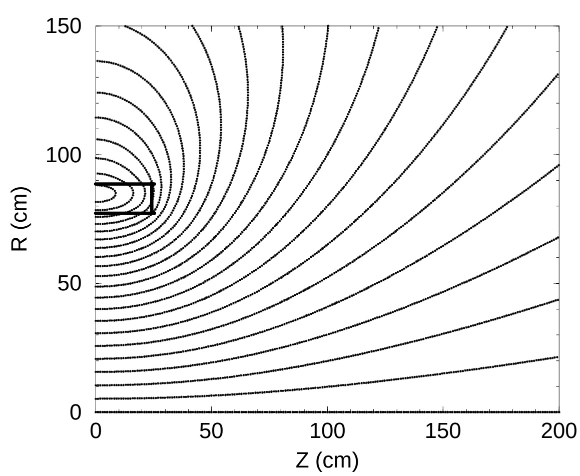

Figure 2: Field lines of single coil.

• Fields of central pair and 2 pairs shifted on $\Delta z = \pm 275$ cm are added resulting field map of the period at -137.5 cm $\leq z \leq 137.5$ cm. Including of more distant coils changes the field less of 1%; besides, they probably will be shielded by magnetic yokes (and other ferromagnetics) which are not included in this consideration.

Components of periodical magnetic field of the cell are plotted in Figs.3-4 against longitudinal coordinate at different transverse coordinates. It is seen that the longitudinal field almost does not depend on x and y, radial field depends mostly on x (and z), and vertical field – on y (and z).

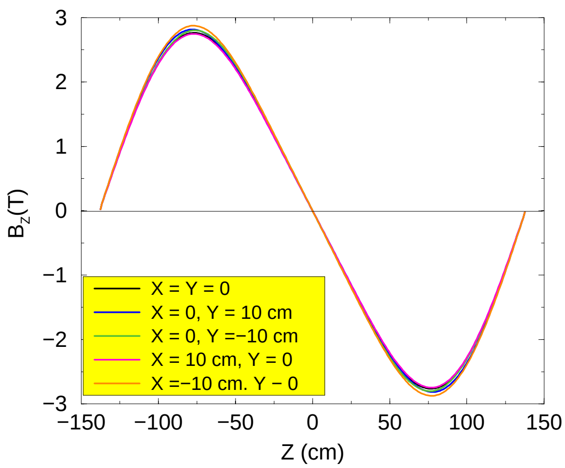

Figure 3: Longitudinal magnetic field at different transverse coordinates.

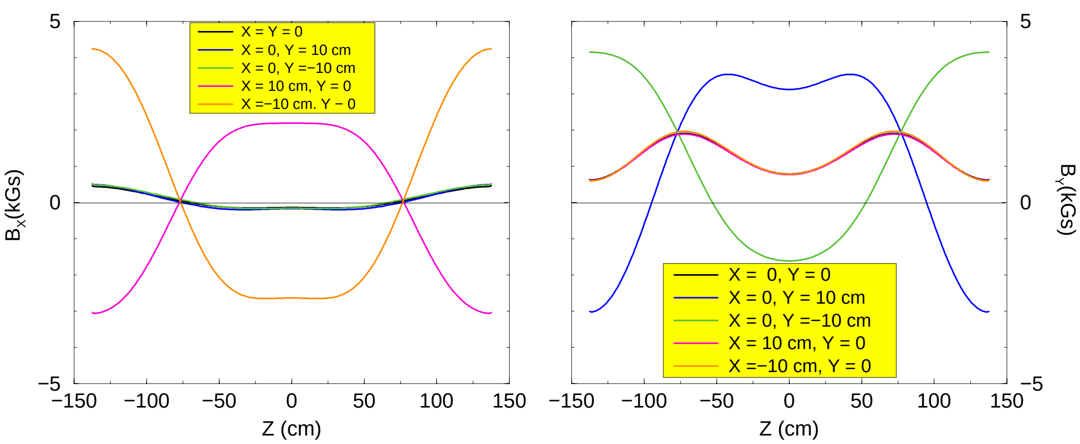

Figure 4: Transverse magnetic field at different transverse coordinates.

2.1 Closed orbits and dispersion

Because of symmetry of the magnetic field, radial and vertical deviations of closed orbit from centerline must be even function of z at any energy. It means that their derivatives are 0 on the ends of the period. This makes easier a search of closed orbits because a variation of 2 parameters x_0 and y_0 is only required. The results are presented in Fig.5 at total energy from 190 MeV to 270 MeV with step 10 MeV. When the energy increases, the orbit moves out the ring center to centerline of the coils, and its vertical swing increases also. It is minimal (and radial swing is rather small also) at E = 220 MeV what just explains the choice of the inclination angle of 52 mrad.

Three components of magnetic field on 220 MeV closed orbit are plotted in Fig.6. Vertical field is about 0.2 T in average what is significantly more of the field 0.12 T required to get closed orbit of radius $R_0 \simeq 5.25$ m at $E =$ 220 MeV. Therefore the reference orbit is shifted to the center approximately on 11 cm, where the particle is kept both by transverse and longitudinal fields. Note that decrease of the coil inclination only slightly moves the orbit from the center but increases the swing what is bad for stability of motion [2].

Dispersion at the absorber center, that is deviation of closed orbit from reference one, is plotted in Fig.7 against energy. The vertical dispersion is surprisely linear, however horizontal one is nonlinear at all. Therefore

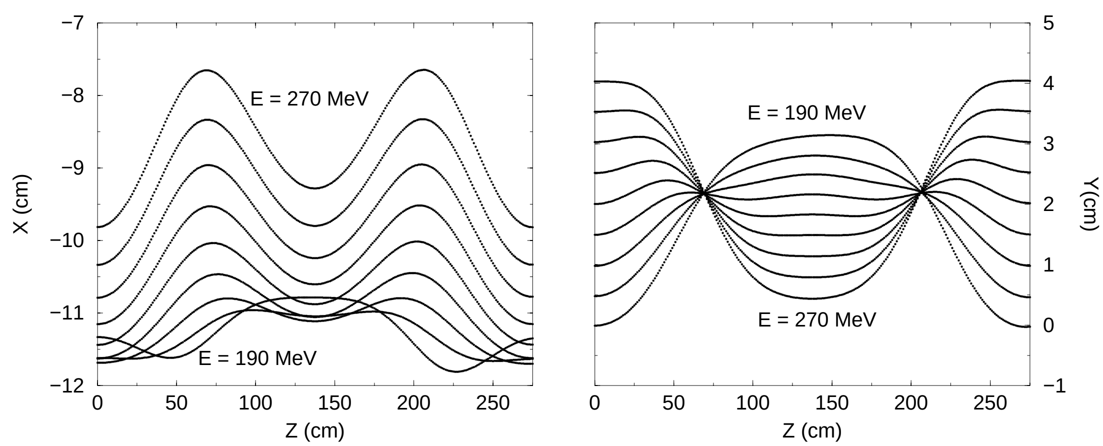

Figure 5: Closed orbit vs distance at different energy. Left – radial, right – vertical deviation from centerline.

only vertical wedge absorbers are used for emittance exchange at the cooling simulation (Sec.5).

Horizontal and vertical dispersion functions at the reference energy 220 MeV are plotted in Fig.8 against longitudinal coordinate z.

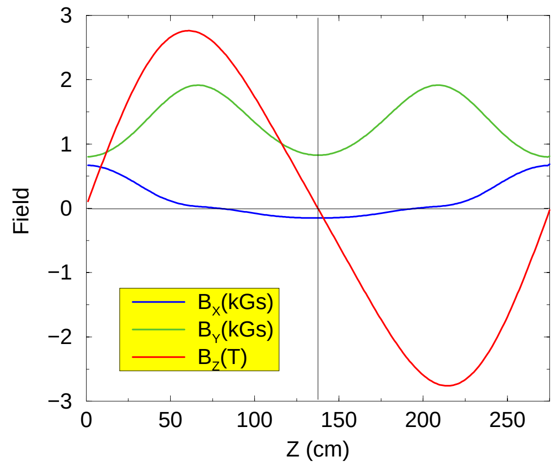

Figure 6: Magnetic field on 220 MeV closed orbit.

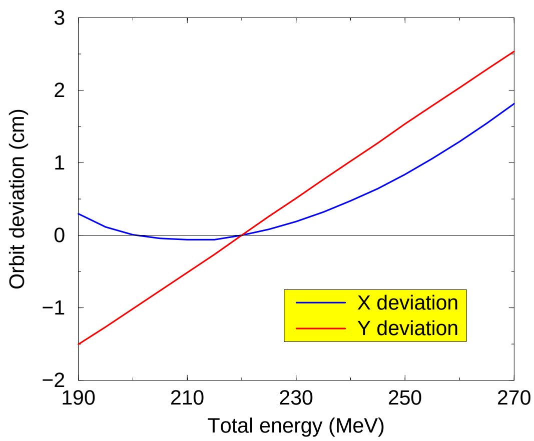

Figure 7: Dispersion vs energy at the absorber center.

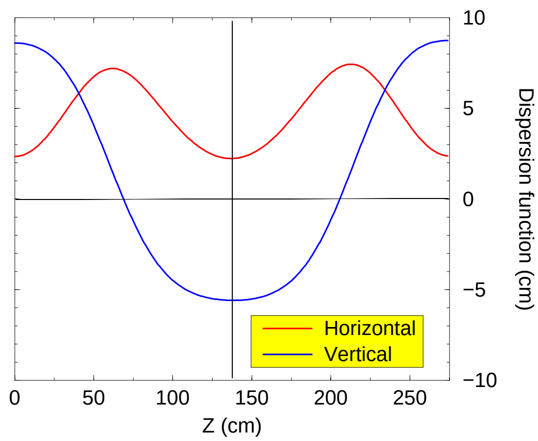

Figure 8: Dispersion function vs distance at $E = 220$ MeV.

3 Transfer matrix and eigenvalues

Transfer matrix for the vector $V = \{x, dx/dz, y, dy/dz\}$ is calculated for small transverse deviations of a particle with arbitrary energy from corresponding closed orbit. The matrix at energy 220 MeV is given below as an example (units – cm):

$-.51483$	-31.479	$+.01572$	$-.39690$
$+.02341$	$-.50781$	$+.00049$	$+.01837$
$-.00173$	$-.45216$	$-.50952$	-31.178
$-.00025$	$-.00564$	$+.02375$	$-.51046$

One can see that numbers at diagonal blocks of the matrix are several tens times more of non-diagonal ones. It allows to surmise that the coupling of x and y - directions produced by the bend of the channel is not crucial, and to neglect this at the estimation of beta-function. Dependence of beta-function on total energy obtained by this assumption is presented in Fig.9. The plot is very similar to given in [1], and both of them show that region of stability is located at 186 MeV $< E <$ 263 MeV between very strong π and 2π-resonances.

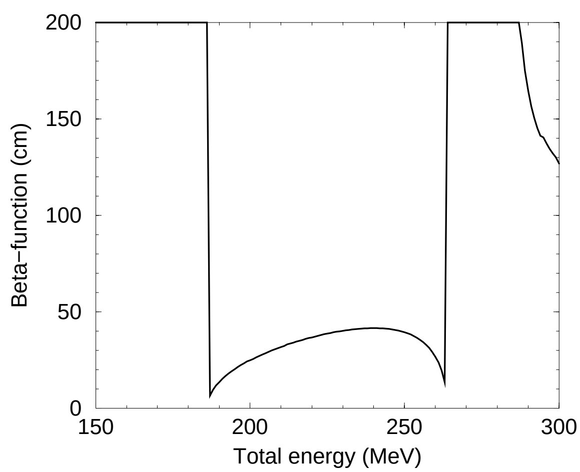

Figure 9: Beta-function of rectilinear channel vs total energy.

Dependence of complex eigenvalues of the 4D matrix on total energy is plotted in Fig.10. General view is shown in left part where red color marks magnitude of the eigenvalues, blue one – phase divided on π. The magnitudes are very close to 1, and both phases are almost identical at the energy interval marked above. Left plot gives a zoom of this region where red lines mean now $(mag - 1)$, and blue line represents difference of the phases. One can see that in a broad area the phases exactly coincide, and 2 of 4 eigenvalues (including conjugated) slightly more of 1 in magnitude what means a weak instability of the system. This fact cannot be explained by errors of calculations because in such case some fluctuations around 0 would be observed everywhere. Therefore the instability should be interpreted as a linear difference coupling resonance exited by the bend of the channel. This instability is very weak and should be suppressed by cooling; however its existing deserves an attention itself. In particular, it means that plot in Fig.9 can be used only for estimation of semi-axes of phase ellipsoid but not to determine its orientation in 4D space. Another words, slow rotation of the phase ellipsoid at the cooling should be expected what really exists as it is shown in the next section.

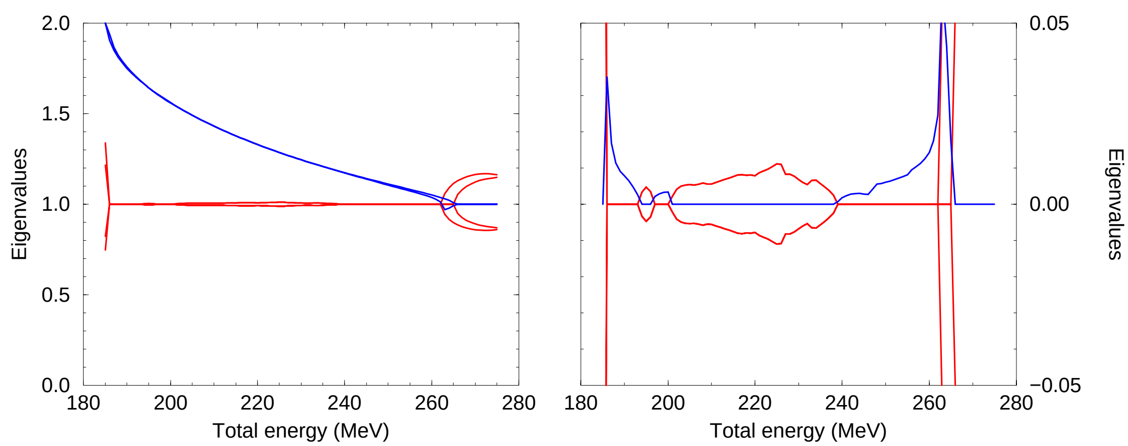

Figure 10: Eigenvalues of the transfer matrix vs total energy. Left – general view, right – zoom. Red – magnitude (L) or $(mag-1)$ (R). Blue – phase/π or $(ph_1 - ph_2)/\pi$.

4 Cooling simulation

2000 particles are used for the cooling simulation at Gaussian distribution with the following parameters:

- $\sigma_x = \sigma_y = 4.25$ cm;
- $\sigma_{p_x} = \sigma_{p_y} = 30$ MeV/c.
- $\sigma_{cT} = 8$ cm;
- $\sigma_{\Delta E} = 20$ MeV.
- Normalized transverse emittance 1.2 cm;
- Longitudinal emittance 1.5 cm;

After random generation, the following correlations have been applied to take into account linear dispersion:

$$x = x_{random} + D_x(\Delta p/p)_{random}$$

and similar expression for y where $D_x = 2.35$ cm and $D_y = 8.60$ cm. Then a correlation due to dependence of revolution frequency on transverse momentum is included by the formula:

$$E_{total} = \Delta E_{random} + E_{ref}\sqrt{1 + (p_{t,random}/mc)^2}.$$

(Note that all the correlations are excluded by inverse transformation before calculation of the beam emittance at the cooling.) The beam is injected in the absorber center at transverse coordinates: $x_c = -11.63$ cm, $y_c = 1.51$ cm corresponding to closed orbit at reference energy 220 MeV.

Evolution of the beam emittance and transmission are shown in Fig.11. There is very nice cooling in all directions at good transmission. Cooling/merit factors are wonderfully close to presented in [1] in spite of very different initial conditions, (input in [1]is taken from precooling part of feasibility Study 2 simulation). The comparison is done in the following table where ratio of emittances and some other parameters are given for both cases in the beginning and after 275 m cooling.

Parameter	This simulation	[1]
Average transverse emittance (cm)	1.20/0.220 = 5.4	1.08/0.206 = 5.2
Longitudinal emittance (cm)	1.58/0.476 = 3.3	5.11/1.37 = 3.7
6D emittance (cm^3)	2.28/0.023 = 99	5.9/0.059 = 100
Transmission without/with decay (%)	70 / 56	61 / 50
Merit factor	55	50

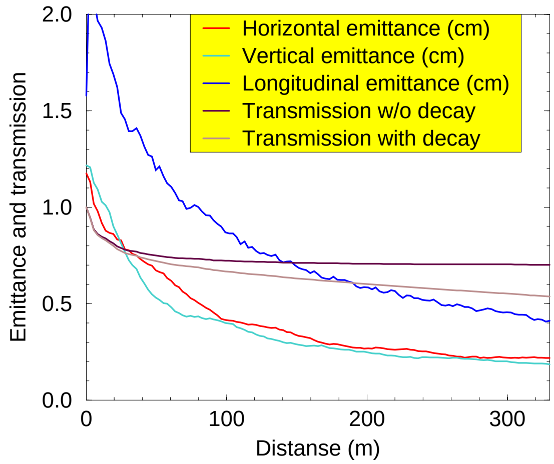

Figure 11: Evolution of the beam parameters at the cooling.

An important thing is that phase ellipsoid is tilted in transverse space at the cooling, resulting $x - p_y$ and $y - p_x$-correlation in the cooled beam. One of them is shown in Fig.12, second is very similar, and there are no another

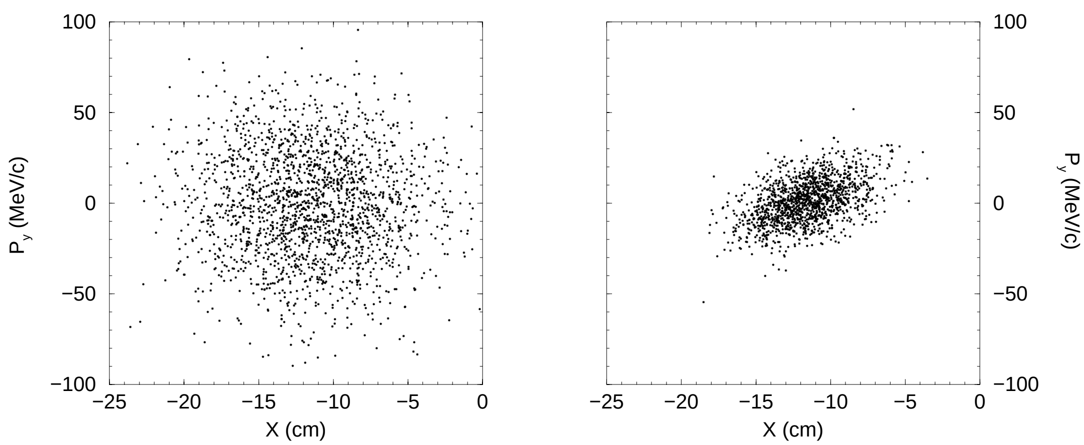

Figure 12: Projection of phase ellipsoid on $x - p_y$ plane before (left) and after the cooling (right)

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considerable correlations. Several conclusions follow by this:

- Rather strong $x - y$ coupling exists in reality;
- Initial upright phase ellipsoid is mismatched with the cooler;
- Finale transverse emittance is probably less then given above because it is obtained as a simple production of corresponding dispersions i.e. the correlations are not taken into account. Both problems of matching the beam at injection and suppression of the correlation at the end should be investigated additionally.

5 Conclusion

High performances of RFOFO cooling ring with tilted solenoids confirmed by simulation with realistic magnetic field. The validity of approximate methodic used in [1] is confirmed also. The nearest goal should be some more correct consideration of $x - y$ coupling producing by the bend, and its suppression if needed. Important problem is also an investigation of dispersion in dependence on another parameters of the ring. Its purposeful change can extend the capabilities of the ring, e.g. usage for longitudinal compression of muon bunch after phase rotation by low frequency accelerating field.

References

[1] J.S.Berg, R.C.Fernow, R.B.Palmer, "An Alternating Solenoid Focused Ionization Cooling Ring". McNote 239, March 2002.

[2] V.Balbekov, "Investigation of RFOFO Like Cooling Rings". McNote 263, November 2002.