Emittance Exchange in Ionization Cooling

David Neuffer
Fermilab
Emittance Exchange Workshop

Outline

- Ionization Cooling - Longitudinal “Cooling”
- Heating - Energy Straggling
- “Wedge Absorbers” - Emittance Exchange
- Partition Numbers
- Differential equations
- Matrix formulation - Thick wedges

References

- A. N. Skrinsky and VV Parchomchuk, Sov. J. Nucl. Phys. 12, 3 (1981)
- D. Neuffer, Part. Accel. 14, 75 (1983)
- D. Neuffer, Nucl. Inst. And Meth. A350, 27 (1994)
- D. Neuffer, AIP Conf. Proc. 441, 270 (1998)
- M. Sands, SLAC-121 (1970)
- R. Palmer, lecture notes (Vanderbilt PAS)
- D. Neuffer, CERN 99-12 (1999)

Ionization Cooling Principle

Loss of transverse momentum in absorber:

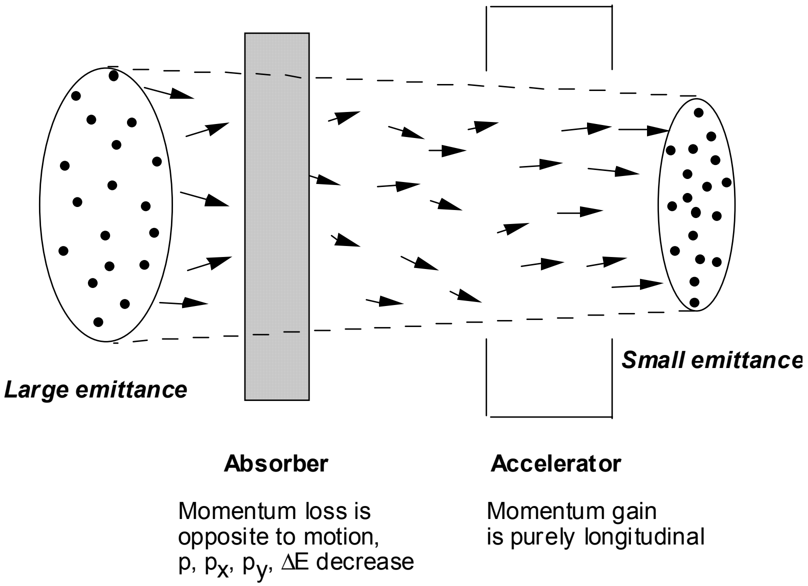
$$p_x \rightarrow p_x \left[1 - \frac{dp}{ds} \frac{\Delta s}{p} \right]$$

Changes transverse emittance:

$$\varepsilon_{\perp,N} = \frac{1}{m}\left[\langle x^2\rangle\langle p_x{}^2\rangle - \langle x p_x\rangle^2\right]^{1/2} = \beta\gamma\left[\langle x^2\rangle\langle \theta_x{}^2\rangle - \langle x\theta_x\rangle^2\right]^{1/2}$$

$$\varepsilon_{\perp,N} \rightarrow \varepsilon_{\perp,N} \left[1 - \frac{dp}{ds} \frac{\Delta s}{p} \right]$$

(**Reacceleration does not change normalized emittance)**



Heating by multiple scattering

Multiple Scattering in material increases rms emittance

$$\left\langle \theta_x^{\ 2} \right\rangle \rightarrow \left\langle \theta_x^{\ 2} \right\rangle + \frac{E_s^{\ 2} \Delta s}{(\beta c p)^2 L_R}$$

$$\varepsilon_{\perp,N} = \beta\gamma \left[\left\langle x^2 \right\rangle \left(\left\langle \theta_x^{\ 2} \right\rangle + \frac{E_s^{\ 2} \Delta s}{(\beta c p)^2 L_R} \right) - \left\langle x\theta_x \right\rangle \right]^{1/2}$$

$$\Rightarrow \quad \Delta\varepsilon_{\perp,N} = \beta\gamma \frac{\left\langle x^2 \right\rangle}{\varepsilon_\perp} \left(\frac{E_s^{\ 2} \Delta s}{2(\beta c p)^2 L_R} \right) = \beta\gamma \left(\frac{\beta_\perp E_s^{\ 2} \Delta s}{2(\beta c p)^2 L_R} \right)$$

Combining cooling and heating:

$$\frac{d\varepsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\varepsilon_N + \frac{\beta\gamma\ \beta_\perp}{2}\frac{d\left\langle \theta_{rms}^{\ \ 2} \right\rangle}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\varepsilon_N + \frac{\beta_\perp E_s^2}{2\beta^3 m_\mu c^2 L_R E}$$

Ionization Cooling Considerations

Material Properties for Ionization Cooling

Material	Symbol	Z	A	dE/ds (min.) MeV/cm	L_R Cm	L_R dE/ds MeV	Density Gm/cm^3	$G_x\beta\varepsilon_N/\beta_\perp$ mm-mrad/cm
Hydrogen	H_2	1	1.01	0.292	865	252.6	0.071	**37**
Lithium	Li	3	6.94	0.848	155	130.8	0.534	**71**
Lith. Hydride	LiH	3+1	7+1	1.34	102	137	0.9	**68**
Beryllium	Be	4	9.01	2.98	35.3	105.2	1.848	**88**
Carbon	C	6	12.01	4.032	18.8	75.8	2.265	122
Aluminum	Al	13	26.98	4.37	8.9	38.9	2.70	238
Copper	Cu	29	63.55	12.90	1.43	18.45	8.96	503
Tungsten	W	74	183.8	22.1	0.35	7.73	19.3	1200

- **Want materials with small multiple scattering (large L_R), but relatively large dE/ds, density (ρ)**
- **Want small β^* at absorbers => strong focusing**
- **- equilibrium emittances (/ β^*) smallest for low-Z materials**

Longitudinal (energy) cooling

- **Longitudinal "cooling" follows energy dependence of dE/ds:**

$$\frac{d\,\sigma_E^2}{ds} = -2\frac{\partial\left(\frac{dE}{ds}\right)}{\partial E}\sigma_E^2 + \frac{d\left\langle \Delta E_{rms}{}^2 \right\rangle}{ds}$$

In radiation damping, dE/ds $\propto$ E^2, $\Rightarrow$ strong energy cooling effect

But in ionization cooling dE/ds has a less favorable energy dependence.

$$\frac{dE}{ds} = 4\pi N_A r_e{}^2 m_e c^2 \rho \;\; \frac{Z}{A}\left[\frac{1}{\beta^2}\ln\left(\frac{2m_e c^2\gamma^2\beta^2}{I}\right) - 1 - \frac{\delta}{2\beta^2}\right]$$

$$\frac{dE}{ds} \cong 0.3071\rho\frac{Z}{A}\left[\frac{1}{\beta^2}\ln\left(\frac{2m_e c^2\gamma^2\beta^2}{I}\right) - 1\right] \quad \frac{MeV}{cm}$$

Energy "Cooling"

- **Energy cooling occurs if**

the derivative $\partial(dE/ds)/\partial E =$

$g_L (dp/ds)/p \;\; > \;\; \mathbf{0}$

$$\left(g_L \equiv \frac{\frac{d\varepsilon_L / ds}{\varepsilon_L}}{\frac{dp/ds}{p}} = \frac{\beta^2 \frac{\partial(dE/ds)}{\partial E}}{\frac{dE/ds}{E}} \right)$$

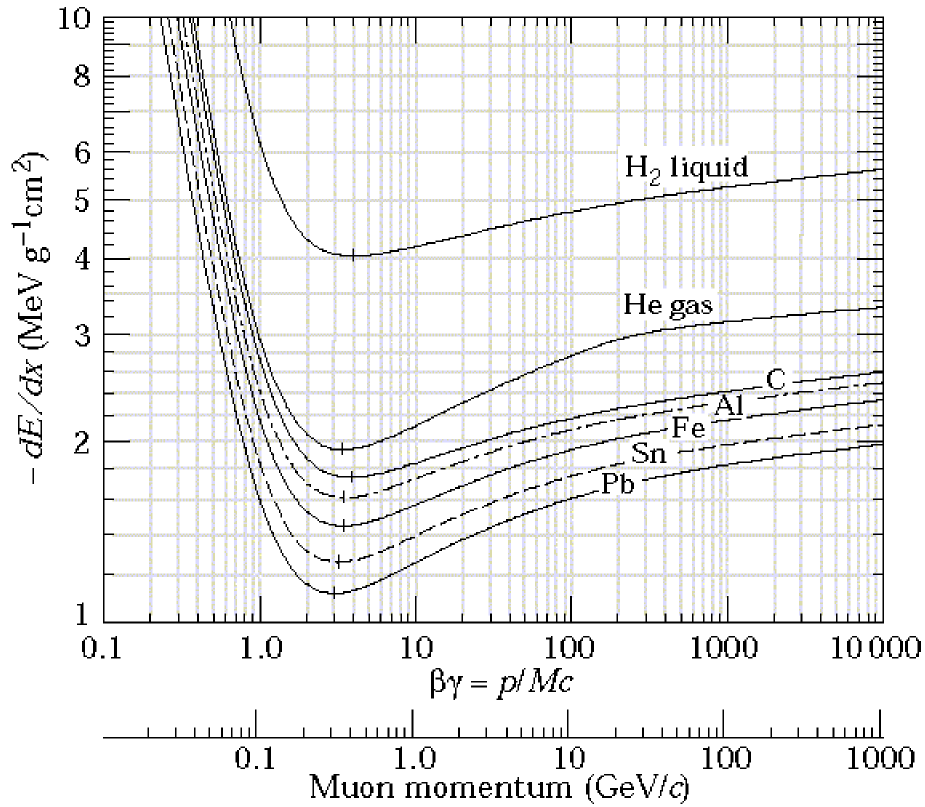
- $\partial(dE/ds)/\partial E$ is negative for

$$P < \sim 0.3 \text{ GeV/c}$$

and only weakly positive for

$$p > \sim 0.3 \text{ GeV/c}$$

$\Rightarrow$ **Ionization cooling does not effectively cool longitudinally**

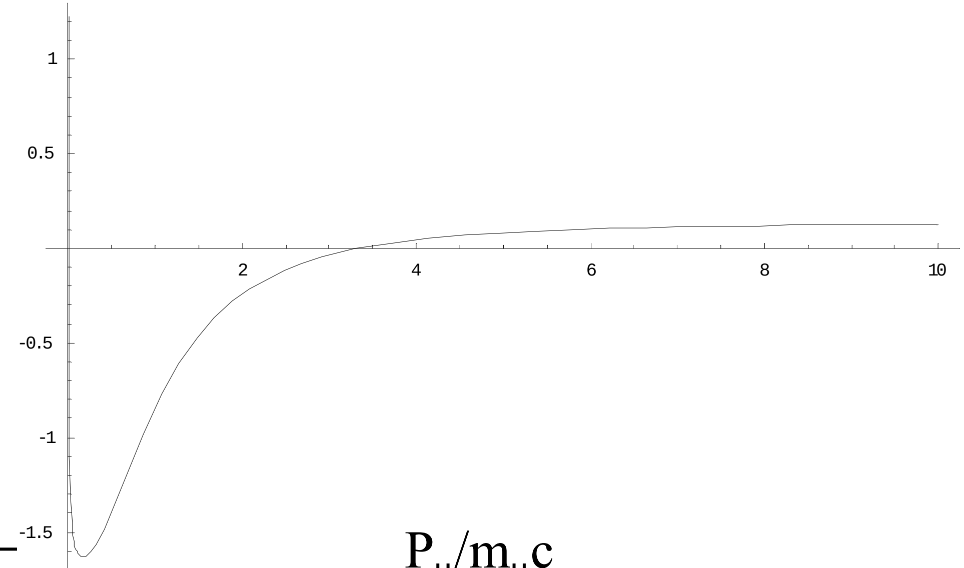


But

Energy cooling can be enhanced by "emittance exchange"

Partition Numbers

$$\left(g_L \equiv \frac{\frac{d\varepsilon_L / ds}{\varepsilon_L}}{\frac{dp/ds}{p}} = \frac{\beta^2 \frac{\partial(dE/ds)}{\partial E}}{\frac{dE/ds}{E}} \right)$$



$P_\mu/m_\mu c$

can be evaluated as:

$$g_{L,0} = \frac{-2\,\mathrm{Ln}[K(\gamma^2-1)] + 2\gamma^2}{\gamma^2 \mathrm{Ln}[K(\gamma^2-1)] - (\gamma^2-1)}$$

where:

$$K = \frac{2 m_e c^2}{I} \cong \frac{2 \cdot 511000.}{12Z+7}$$

g_L is negative (ΔE - heating) for $P_\mu/m_\mu c < 3$

Energy straggling ...

Energy loss in material is accompanied by energy straggling

$$\frac{d\left\langle \Delta E_{rms}{}^{2}\right\rangle}{ds} = 4\pi\left(r_e m_e c^2\right)^2 n_e \gamma^2\left(1-\frac{\beta^2}{2}\right) \cong 0.157\rho\frac{Z}{A}\gamma^2\left(1-\frac{\beta^2}{2}\right) \quad (\mathrm{MeV})^2\,\mathrm{cm}^2/\mathrm{gm}$$

"Cooling" term + energy straggling

obtains energy spread (σ_E) cooling equation:

$$\frac{d\sigma_E^2}{ds} = -2\frac{g_L \frac{dE}{ds}}{\beta^2 E}\sigma_E^2 + 4\pi\left(r_e m_e c^2\right)^2 n_e \gamma^2\left(1-\frac{\beta^2}{2}\right)$$

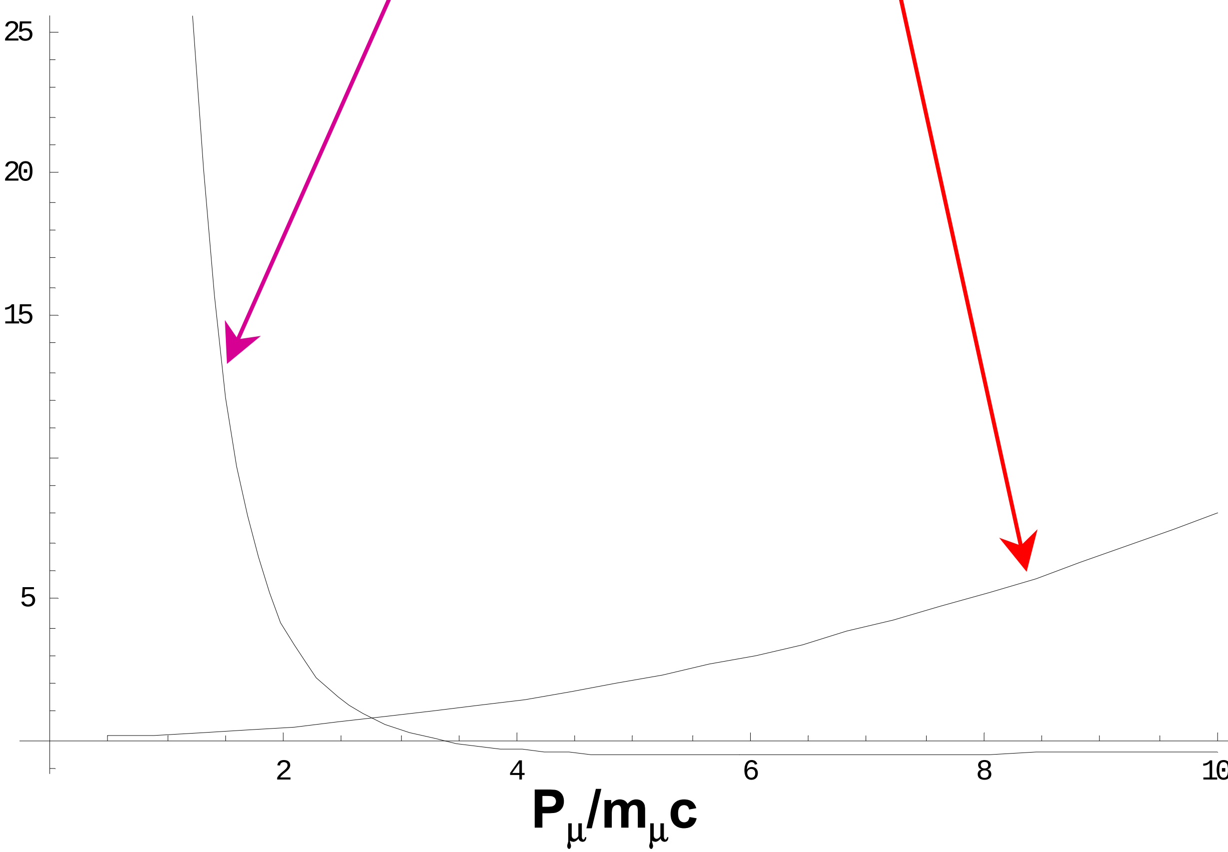
(At "typical" parameters, $\sigma_p/p_{equilibrium}$ ~ 2%)

Optimal Emittance Exchange Energy ?

- Equation for increase in energy spread:

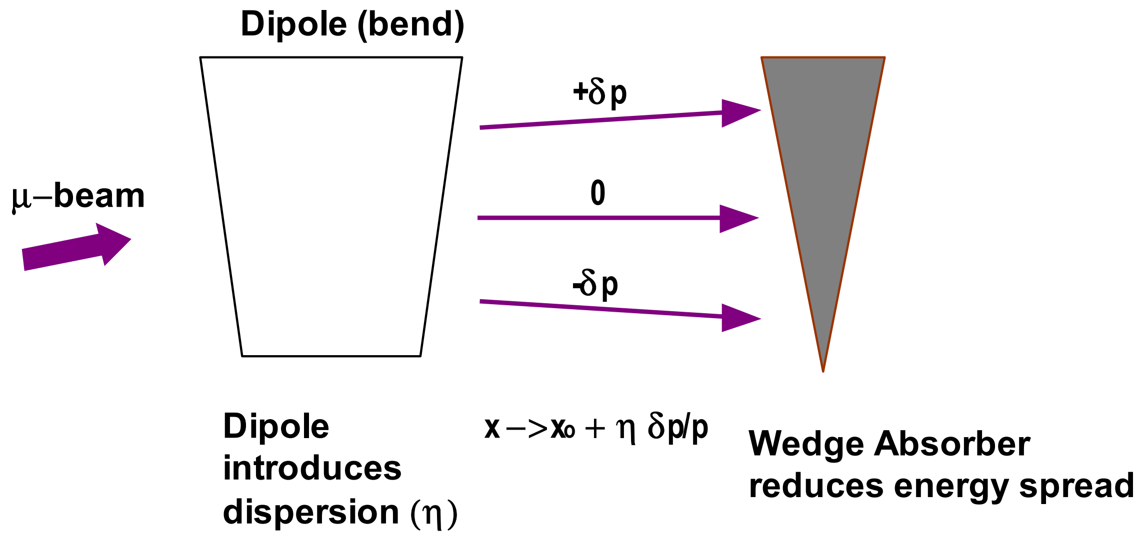
$$\frac{d\sigma_E^2}{ds} = -2\frac{g_L \frac{dE}{ds}}{\beta^2 E}\sigma_E^2 + 4\pi\left(r_e m_e c^2\right)^2 n_e \gamma^2\left(1-\frac{\beta^2}{2}\right)$$

- For $E_\mu < 400$ MeV, g_L is negative (antidamping)
- Sample parameters: $\sigma_E = 20$ MeV
- **Sum of heating terms is minimum for $E_\mu = \sim 400$ MeV**



Shouldn't we cool where longitudinal heating is minimized ?

Emittance exchange overview



- **Cooling derivative is changed by use of dispersion + wedge**
  (Dependence of energy loss on energy can be increased)

$$\frac{\partial \frac{dE}{ds}}{\partial E} \Rightarrow \left.\frac{\partial \frac{dE}{ds}}{\partial E}\right|_0 + \frac{dE}{ds}\frac{\eta\rho'}{\beta c p \rho_0} = g_L \frac{dp/ds}{p}$$

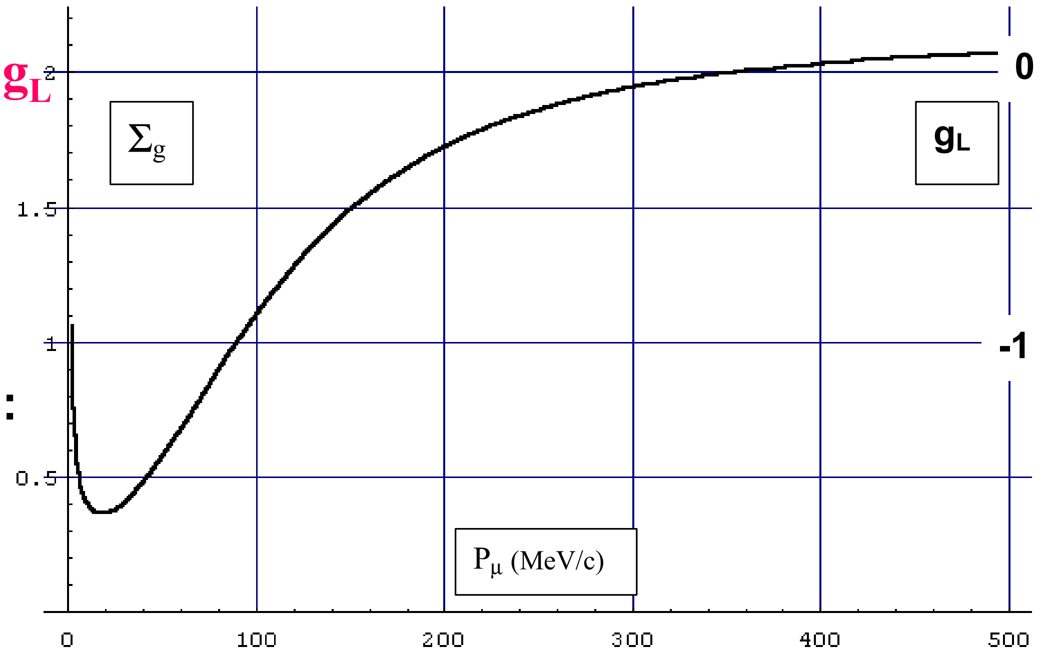
Partition Numbers

With emittance exchange the longitudinal partition number g_L changes:

$$g_L \Rightarrow g_{L,0} + \frac{\eta\rho'}{\rho_0}$$

But the transverse cooling partition number decreases:

$$g_x \Rightarrow 1 - \frac{\eta\rho'}{\rho_0}$$



The sum of the cooling partition numbers (at P = P_μ) remains constant:

$$\Sigma_g(P_\mu) = g_x + g_y + g_L = 2 + g_{L,0}$$

Longitudinal motion (ΔE-ϕ space)

$$\frac{d\Delta E}{ds} = eV'(\cos(\phi+\phi_s) - \cos\phi_s) \cong -eV'\sin\phi_s\,\phi$$

$$\frac{d\phi}{ds} \cong \frac{1}{\beta^3\gamma}\left(\frac{1}{\gamma^2} - \frac{1}{\gamma_t^2}\right)\frac{2\pi}{\lambda_0}\frac{\Delta E}{mc^2} = \frac{1}{\beta^3\gamma}\alpha_p\frac{2\pi}{\lambda_0}\frac{\Delta E}{mc^2}$$

Where $M_{56}' = 1/\gamma_t^2 = \eta/R$ **indicates a (possibly) nonisochronous transport**

Equilibrium trajectories follow:

$$eV'\sin\phi_s\;\phi^2 \;+\; \frac{1}{\beta^3\gamma}\frac{2\pi}{\lambda_0}\frac{\alpha_p}{mc^2}\,\Delta E^2 = \text{constant}$$

which implies:

$$\frac{\langle\phi^2\rangle}{\langle\Delta E^2\rangle} = \frac{1}{\beta^3\gamma\; eV'\sin\phi_s}\frac{2\pi}{\lambda_0}\frac{\alpha_p}{mc^2} \equiv \beta_\phi{}^2$$

, where β_ϕ is a "longitudinal betatron function"

Note: longitudinal cooling requires reducing $\lambda_0\,\beta_\phi$; (by smaller α_p, λ_0; larger V')

Energy Cooling → Longitudinal Emittance Cooling

Muon Collaboration

Use ΔE-ϕ coordinates: $\varepsilon_L = \sqrt{\langle \phi^2 \rangle \langle \Delta E^2 \rangle - \langle \phi \Delta E \rangle^2}$

Cooling term: $\Delta E \rightarrow \left(1 - \frac{g_L}{\beta^2 E} \frac{dE}{ds} ds\right) \Delta E \quad \Rightarrow \quad \varepsilon_L \rightarrow \left(1 - \frac{g_L}{\beta^2 E} \frac{dE}{ds} ds\right) \varepsilon_L$

Heating term: $\frac{d\varepsilon_L}{ds} = \frac{\langle \phi^2 \rangle}{2\varepsilon_L} \frac{d\langle \Delta E_{rms}{}^2 \rangle}{ds} = \frac{\beta_\phi}{2} \frac{d\langle \Delta E_{rms}{}^2 \rangle}{ds}$

Combining these obtains:

$$\frac{d\varepsilon_L}{ds} = -\frac{g_L}{\beta^2 E} \frac{dE}{ds} \varepsilon_L + \frac{\beta_\phi}{2} \frac{d\langle \Delta E_{rms}{}^2 \rangle}{ds}$$

Emittance exchange

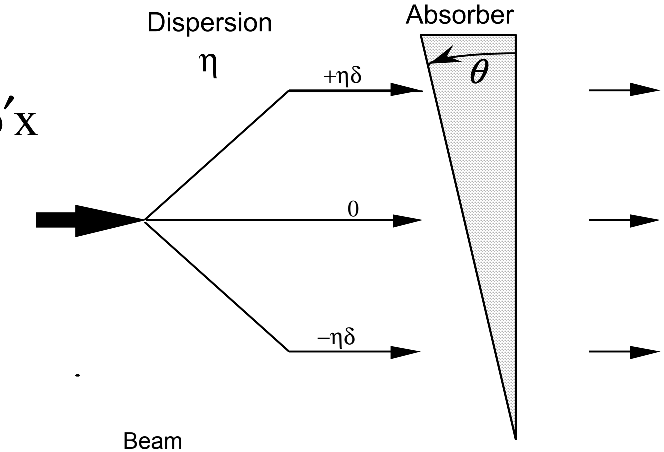
Dispersion:

$$x \rightarrow x + \eta\delta$$

Wedge:

$$\delta \rightarrow \delta - \frac{\frac{dp}{ds}\tan\theta}{p}x = \delta - \delta' x$$

(η, δ_p are in $\delta p/p$ units)

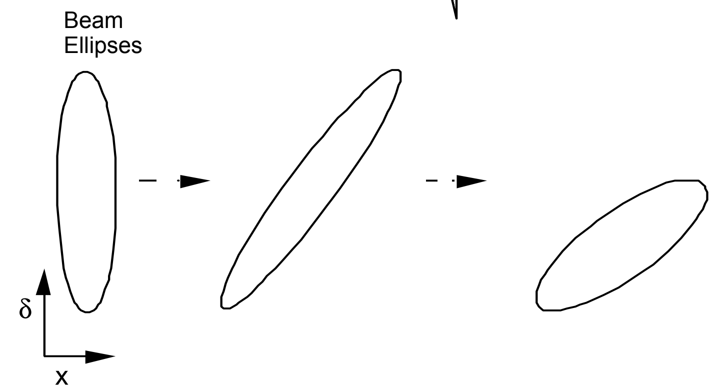


Emittance exchange:

$$\delta \rightarrow \delta_0 \sqrt{(1-\eta\delta')^2 + \delta'^2 \frac{\sigma_x{}^2}{\sigma_\delta{}^2}}$$

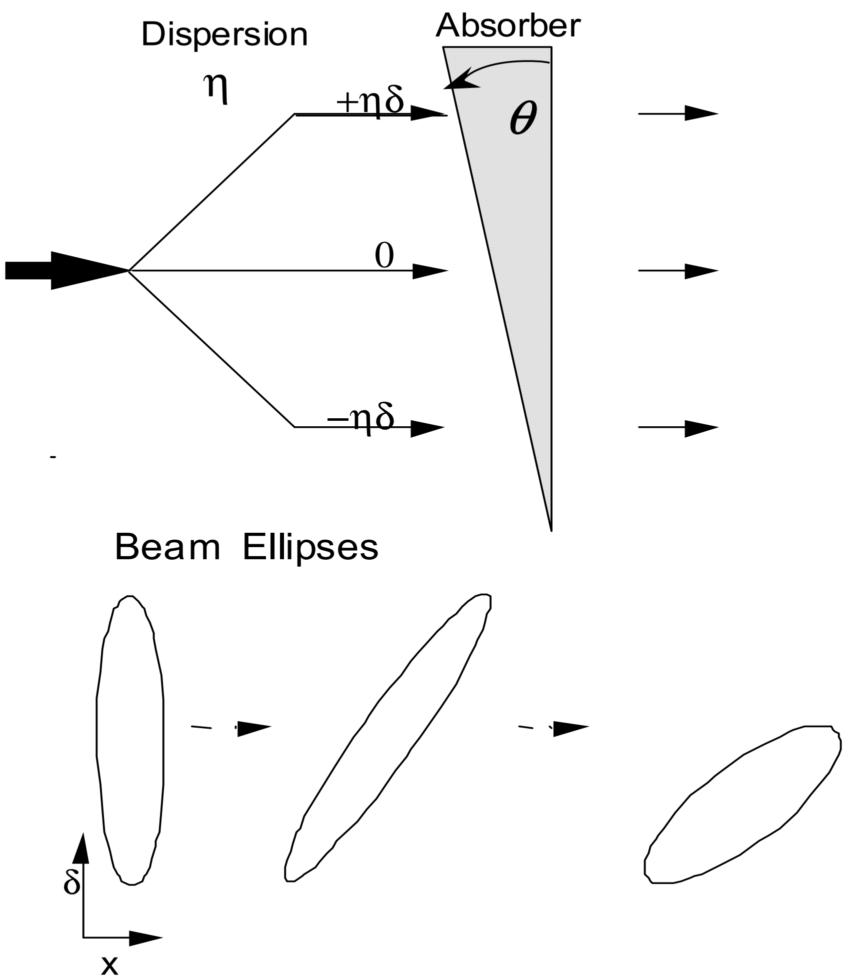
- **transverse emittance change is opposite:**

$$\varepsilon_x \rightarrow \varepsilon_{x,0} \Big/ \sqrt{(1-\eta\delta')^2 + \delta'^2 \frac{\sigma_x{}^2}{\sigma_\delta{}^2}}$$



Dispersion $\rightarrow$ Wedge

Emittance Exchange



Dispersion and wedge are equivalent to transport elements (in x - δ space) ($\delta_p = \delta p/p$)

Dispersion:

$$x \rightarrow x + \eta\delta_p$$

Wedge:

$$\delta_p \rightarrow \delta_p - \frac{\frac{dp}{ds}\tan\theta}{p}x = \delta_p - \delta' x$$

Emittance Exchange - Matrix formalism

Dispersion + wedge - equivalent to transport matrix:

$$\begin{matrix} x_1 \\ \delta_{p,1} \end{matrix} = \begin{bmatrix} 1 & \eta \\ -\delta' & 1-\eta\delta' \end{bmatrix} \begin{matrix} x_0 \\ \delta_{p,0} \end{matrix}$$

From linear transport matrix formalism:

$$\sigma_\delta{}^2 \rightarrow (1-\eta\delta')^2\sigma_\delta{}^2 + \delta'^2\sigma_x{}^2 \;;\; \sigma_x{}^2 \rightarrow \sigma_x{}^2 + \eta^2\sigma_\delta{}^2$$

Changes in emittances:

$$\varepsilon_L \rightarrow \varepsilon_L\sqrt{(1-\eta\delta')^2 + \delta'^2\frac{\sigma_x{}^2}{\sigma_\delta{}^2}} \;;\; \varepsilon_x \rightarrow \varepsilon_x \Big/ \sqrt{(1-\eta\delta')^2 + \delta'^2\frac{\sigma_x{}^2}{\sigma_\delta{}^2}}$$

Changes in betatron functions:

$$\eta \rightarrow \frac{\eta(1-\eta\delta') - \delta'\frac{\sigma_x{}^2}{\sigma_\delta{}^2}}{(1-\eta\delta')^2 + \delta'^2\frac{\sigma_x{}^2}{\sigma_\delta{}^2}} \;;\; \beta_x \rightarrow \beta_x \Big/ \sqrt{(1-\eta\delta')^2 + \delta'^2\frac{\sigma_x{}^2}{\sigma_\delta{}^2}}$$

Rewrite exchange equations

Dispersion + wedge - equivalent to transport matrix:

$$\begin{matrix} x_1 \\ \delta_{p,1} \end{matrix} = \begin{bmatrix} 1 & \eta \\ -\delta' & 1-\eta\delta' \end{bmatrix} \begin{matrix} x_0 \\ \delta_{p,0} \end{matrix}$$

From linear transport matrix formalism:

$$\sigma_\delta{}^2 \rightarrow (1-\eta\delta')^2 \sigma_\delta{}^2 + \delta'^2 \sigma_x{}^2 \;\; ; \;\; \sigma_x{}^2 \rightarrow \sigma_x{}^2 + \eta^2 \sigma_\delta{}^2$$

Changes in emittances:

$$\varepsilon_L \rightarrow \varepsilon_L \sqrt{(1-\eta\delta')^2 + (\eta\delta')^2 \frac{\sigma_x{}^2}{(\eta\sigma_\delta)^2}} \;\; ; \;\; \varepsilon_x \rightarrow \varepsilon_x \Big/ \sqrt{(1-\eta\delta')^2 + (\eta\delta')^2 \frac{\sigma_x{}^2}{(\eta\sigma_\delta)^2}}$$

Changes in betatron functions:

$$\eta \rightarrow \frac{\eta\left(1-\eta\delta'-\eta\delta'\frac{\sigma_x{}^2}{(\eta\sigma_\delta)^2}\right)}{(1-\eta\delta')^2 + (\eta\delta')^2 \frac{\sigma_x{}^2}{(\eta\sigma_\delta)^2}} \;\; ; \;\; \beta_x \rightarrow \beta_x \Big/ \sqrt{(1-\eta\delta')^2 + (\eta\delta')^2 \frac{\sigma_x{}^2}{(\eta\sigma_\delta)}}$$

(All equations are in units of $\eta\delta' = \eta \tan\theta \frac{\frac{dp}{ds}}{p}, \eta\sigma_\delta = \eta \frac{\delta p}{p}, \sigma_x$ **)**

Calculation of Dispersion, emittances

- Calculate dispersion (η, η') within simulations:

$$\eta = \frac{\left(\langle x P_\mu \rangle - \langle x \rangle \langle P_\mu \rangle\right)\langle P_\mu \rangle}{\langle P_\mu{}^2 \rangle - \langle P_\mu \rangle^2}$$

$$\eta' = \frac{\left(\langle x' P_\mu \rangle - \langle x' \rangle \langle P_\mu \rangle\right)\langle P_\mu \rangle}{\langle P_\mu{}^2 \rangle - \langle P_\mu \rangle^2}$$

$$\langle x^2 \rangle_{corr} = \langle x^2 \rangle - \frac{\eta^2\left[\langle P_\mu{}^2 \rangle - \langle P_\mu \rangle^2\right]}{\langle P_\mu \rangle^2}$$

$$\langle x\, x' \rangle_{corr} = \langle x\, x' \rangle - \frac{\eta\,\eta'\left[\langle P_\mu{}^2 \rangle - \langle P_\mu \rangle^2\right]}{\langle P_\mu \rangle^2}$$

$$\langle x'^2 \rangle_{corr} = \langle x'^2 \rangle - \frac{\eta'^2\left[\langle P_\mu{}^2 \rangle - \langle P_\mu \rangle^2\right]}{\langle P_\mu \rangle^2}$$

Emittance, betatron functions:

$$\varepsilon_x = \sqrt{\langle x^2 \rangle_{corr} \langle x'^2 \rangle_{corr} - \langle x\, x' \rangle_{corr}{}^2}$$

$$\beta_x = \frac{\langle x^2 \rangle_{corr}}{\varepsilon_x} \qquad \alpha_x = -\frac{\langle x\, x' \rangle_{corr}}{\varepsilon_x}$$

Other optics considerations

•Previous case assumed waist at absorber (α=0, η'=0)

If $\alpha \neq 0$ at (wedge) absorber, η' changes….

$$\left\langle x\,x' \right\rangle_{\text{before}} = \left\langle x\,x' \right\rangle_{\text{after}} + \eta\,\eta' \left\langle \left(\frac{\delta p}{p}\right)^2 \right\rangle$$

Matrix Formalism Examples - I

Thin wedge example:

With $\delta' \equiv \dfrac{\frac{dp}{ds}\tan\theta}{p}$ **small:**

$$\varepsilon_1 \to \varepsilon_0(1+\eta_0\delta')$$

$$\delta_1 \to \delta_0(1-\eta_0\delta')$$

$$\eta_1 \to \eta_0(1+\eta_0\delta') - \delta' \frac{\sigma_0{}^2}{\delta_0{}^2}$$

$$\beta_1 \to \beta_0(1+\eta_0\delta')$$

Adding intrinsic cooling, and using $\tan\theta \equiv \rho' / \rho_0$

$$\Rightarrow \frac{d\varepsilon_L}{ds} = -\frac{\partial \frac{dE}{ds}}{\partial E}\varepsilon_L - \eta\delta'\varepsilon_L = -\frac{g_L}{\beta^2 E}\frac{dE}{ds}\varepsilon_L$$

These results are the same obtainable from small increments in the continuous cooling case.

(notation note: δ_1 and δ_0 indicate $\delta_p = \delta p/p$ rms widths (σ_δ) after and before wedge, respectively)

Matrix Formalism Examples - II

Maximum Exchange (occurs at $\delta' = 1/\eta_0$):

$$\delta_1 = \delta_0 \frac{\sigma_0}{\eta_0\delta_0}, \varepsilon_1 = \varepsilon_0 \frac{\eta_0\delta_0}{\sigma_0}, \beta_1 = \beta_0 \frac{\eta_0\delta_0}{\sigma_0}, \eta_1 = -\eta_0$$

- **Dispersion beam size ($\eta\delta_0$) and emittance beam size (σ_0) are simply exchanged.**

Exchange Matrix examples - III

Dispersion matched to zero after absorber:

Occurs when: $$\delta' = \frac{1}{\eta_0\left[1 + \frac{\sigma_0{}^2}{\eta_0{}^2\delta_0{}^2}\right]}$$

$$\delta_1 = \frac{\delta_0}{\sqrt{1 + \frac{\eta_0{}^2\delta_0{}^2}{\sigma_0{}^2}}}, \varepsilon_1 = \varepsilon_0\sqrt{1 + \frac{\eta_0{}^2\delta_0{}^2}{\sigma_0{}^2}}, \beta_1 = \beta_0\sqrt{1 + \frac{\eta_0{}^2\delta_0{}^2}{\sigma_0{}^2}}, \eta_1 = 0.$$

This solution greatly simplifies downstream optics; requires $\eta_0\delta_0 \cong \sigma_0$ for good solutions.

Conditions for “optimal” emittance exchange

- Wedge angle limited to tan $\theta \cong 1$ by geometry considerations
- Wedge material should have small multiple scattering (low-Z: H_2, Li, Be, …)
- Need strong focussing (small β^*) at wedge
- If no continuous focusing (solenoid, Li lens), need short wedge ($L < 2\beta^*$)

  from “hour-glass” effect
- For maximal exchange in single wedge, need high density (Be ?)

Emittance exchange example

Emittance exchange at parameters

at ~ beginning of cooling:

Be wedge; θ=45º

$\eta = 1\text{m} \rightarrow \sim 0$

$\eta\delta_0 = 7.3\text{cm}, \ \sigma_0 = 3.7\text{cm}$

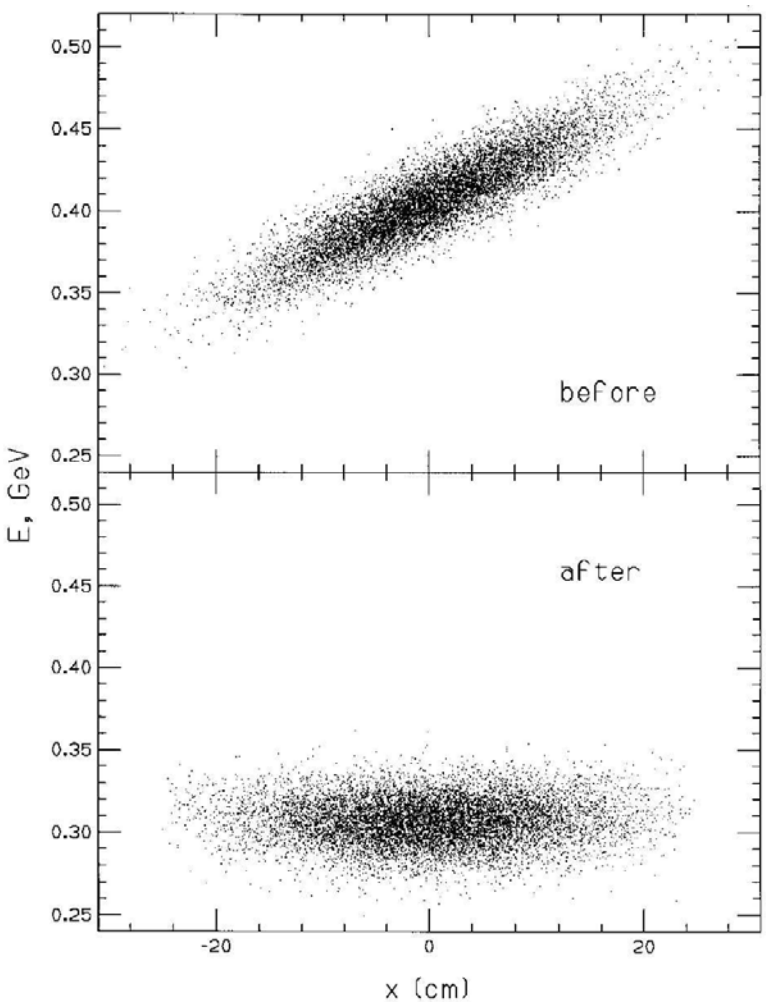
$P_{ave} = 391 \rightarrow 340$ MeV/c

$\delta P = 28.6 \rightarrow 13.9$ MeV/c

$\varepsilon_{\perp,N} = 1.5\text{cm} \rightarrow 2.76\ (x);\ 1.37\ (y)$

thick wedge: 6-D ε cools by 0.78

SIMUCOOL results; NIM A405, 1 (1998)



Exchange in ΔE-increase mode (anti-wedge)

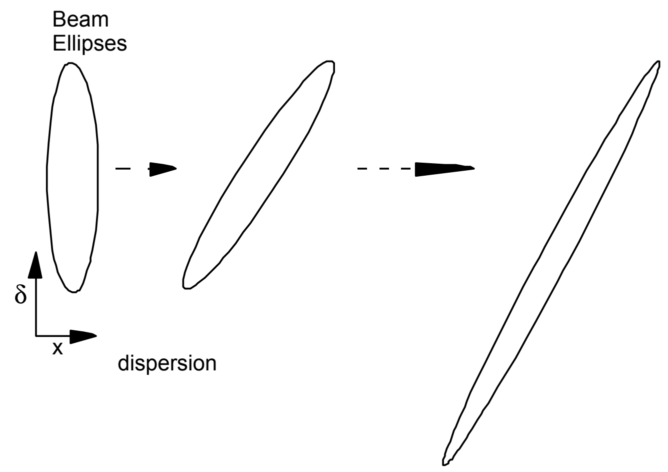
- **Emittance exchange can run in "reverse", with transverse emittance decreasing while longitudinal emittance increases**
- **Obtain with $\eta < 0$ (or $\theta < 0$)**

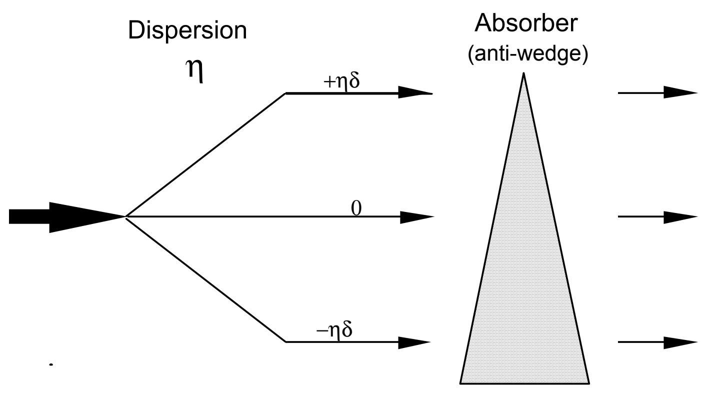
$$\delta_1 = \delta_0 \sqrt{(1+|\eta\delta'|)^2 + |\eta\delta'|^2 \frac{\sigma_0{}^2}{(\eta\delta_0)^2}}$$

$$\{\varepsilon_1, \beta_1, \sigma_1\} = \frac{\{\varepsilon_0, \beta_0, \sigma_0\}}{\sqrt{(1+|\eta\delta'|)^2 + |\eta\delta'|^2 \frac{\sigma_0{}^2}{(\eta\delta_0)^2}}}$$

$$\eta_1 = \frac{\eta_0 \left(1 + |\eta\delta'| + |\eta\delta'| \frac{\sigma_0{}^2}{(\eta\delta_0)^2}\right)}{\left((1+|\eta\delta'|)^2 + |\eta\delta'|^2 \frac{\sigma_0{}^2}{(\eta\delta_0)^2}\right)}$$

Beam ellipses in energy-spread increase mode (anti-wedge)





“Anti-wedge” example

- Example of low-energy emittance anti-exchange (near end of scenario)

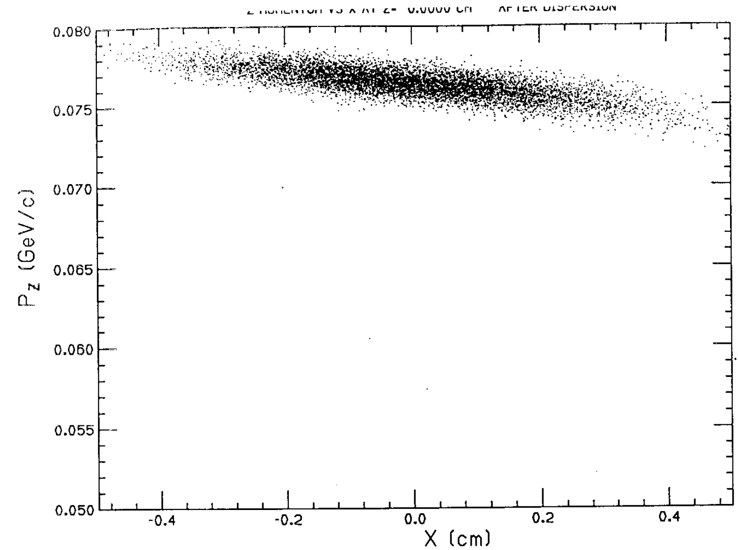
Initial beam:

T_μ = 25 MeV ; P_μ=77 MeV/c

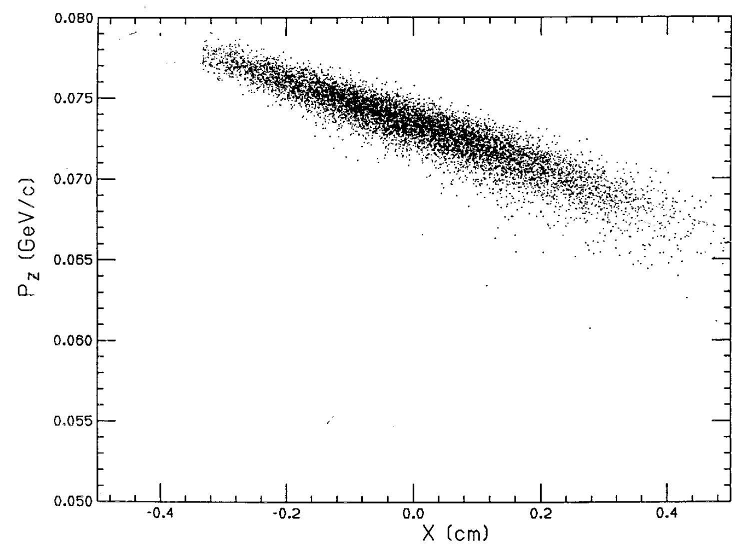
$\varepsilon_{t,N}$ = 0.0061 cm; δp = 0.77 MeV/c

LiH wedge: $\tan\theta$ = 1, 0.8cm long;

η= -0.105m; $\beta_\perp$=1.3cm



REVERSE WEDGE LiH ε = 0.0061 cm-rad η = 10.5 cm P=0.0769 GeV/c Δp/p = 0.00981 Z = 0.0 cm AFTER



Results(SIMUCOOL):

$\delta p \rightarrow$ 1.76 MeV/c, $P_\mu \rightarrow$ 73 MeV/c

$\varepsilon_{t,x} \rightarrow$ 0.0039; $\varepsilon_{t,x} \rightarrow$ 0.0061

$\varepsilon_{6D} \rightarrow 1.45 \times$

Low-Energy wedge heats beam

Alternative anti-wedge: low-energy Li lens

Similar anti-exchange can be obtained using low-energy Li lens

(Palmer 1996)

"End of cooling" Example

Lens:

14cm Li, 10000T/m, r=0.2cm

Beam:

P_μ=100 MeV/c, $\varepsilon_{t,N}$ = 0.01 cm;

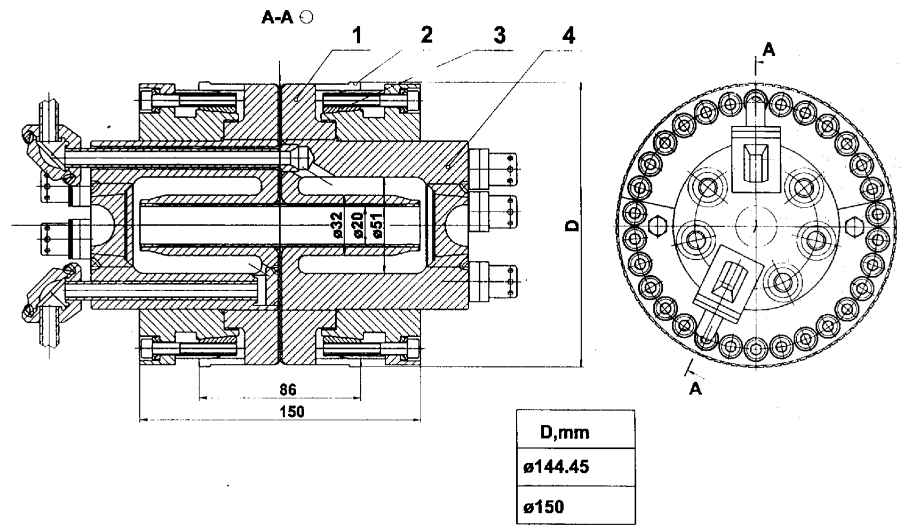
δp = 2 MeV/c

Results(SIMUCOOL):

$\delta p \rightarrow$ 4.36 MeV/c, $P_\mu \rightarrow$ 68 MeV/c

$\varepsilon_{t,N} \rightarrow$ 0.0077; $\varepsilon_{6D} \rightarrow$ 1.29 $\times$

- still some 6-D heating



Bent Solenoid emittance exchange

- **Example of bent solenoid**

  + wedges for emittance exchange

(Muon Collider R&D Status Report, 1998)

Double bend/wedge to keep $\varepsilon_x, \varepsilon_y$ equal

B_z ~3.5T; B_y ~ 1T, $\eta \cong 0.6$m

P_μ = 180 $\rightarrow$ 150 MeV/c

Tranverse emittance increases:

0.087 $\rightarrow$0.17 π cm-rad (norm.)

Longitudinal δP decreases:

9.26 $\rightarrow$ 3.35 MeV/c

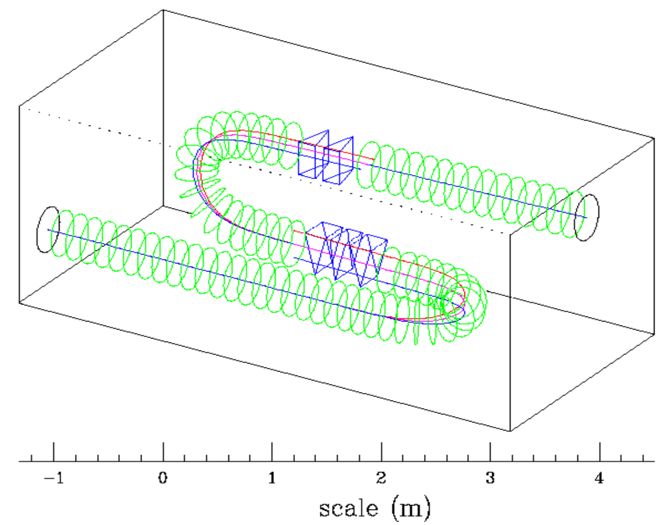
5-D emittance increases ($\times$1.37)

Longitudinal position not controlled

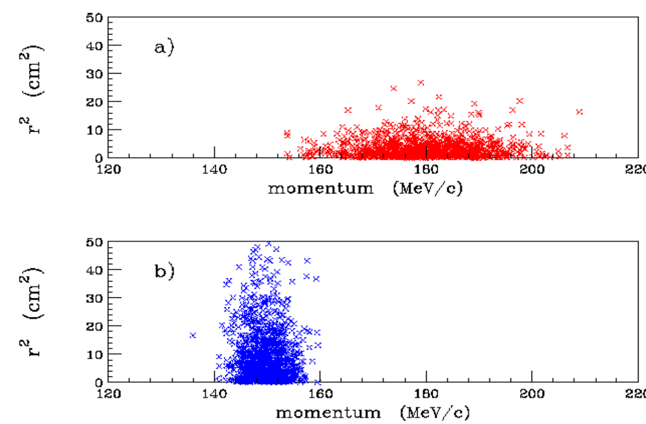
(6-D not solved yet …)

Bent Solenoid emittance exchange

(η_y = Bθ/Bρ, $\beta_\perp$ = 2 Bρ/B)



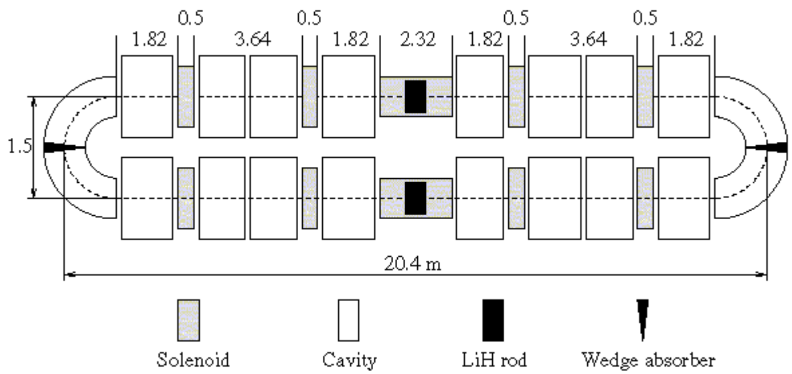
Simulation results:

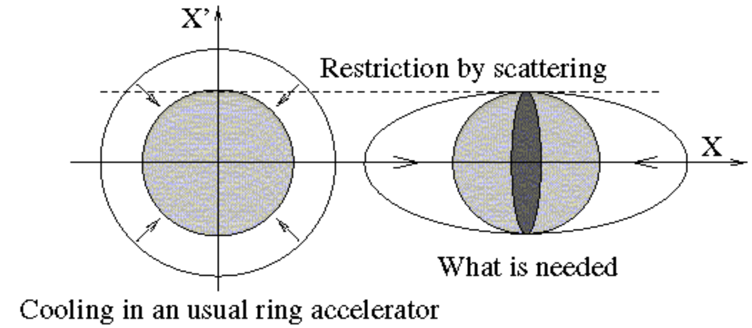


6-D Cooling in "Ring Cooler" (V. Balbekov)

Muon Collaboration

Need: decreasing $\beta_\perp$ to keep θ_{rms} ~constant (multiple scattering)





Solution: **set tune** (x, y, and z) **at integer** (or half integer); Cooling naturally damps betatron functions.

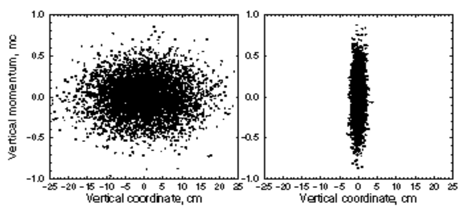


Figure 6: Initial and final transverse distributions.

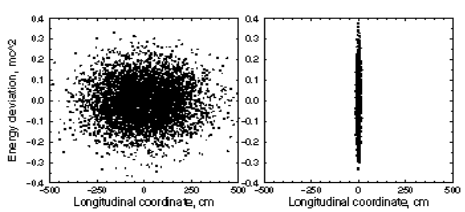
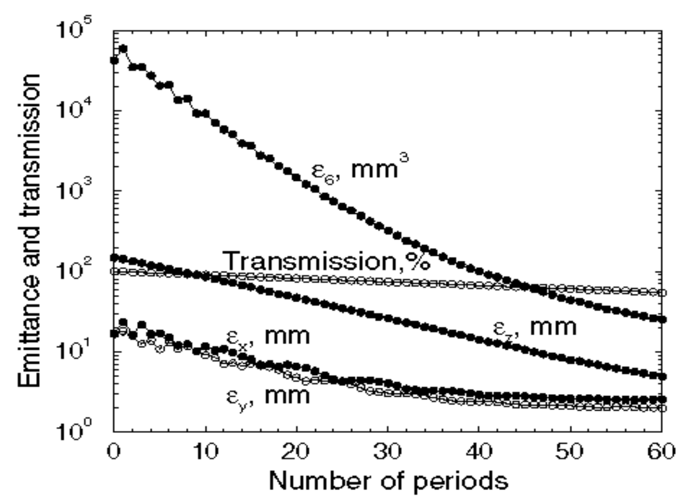
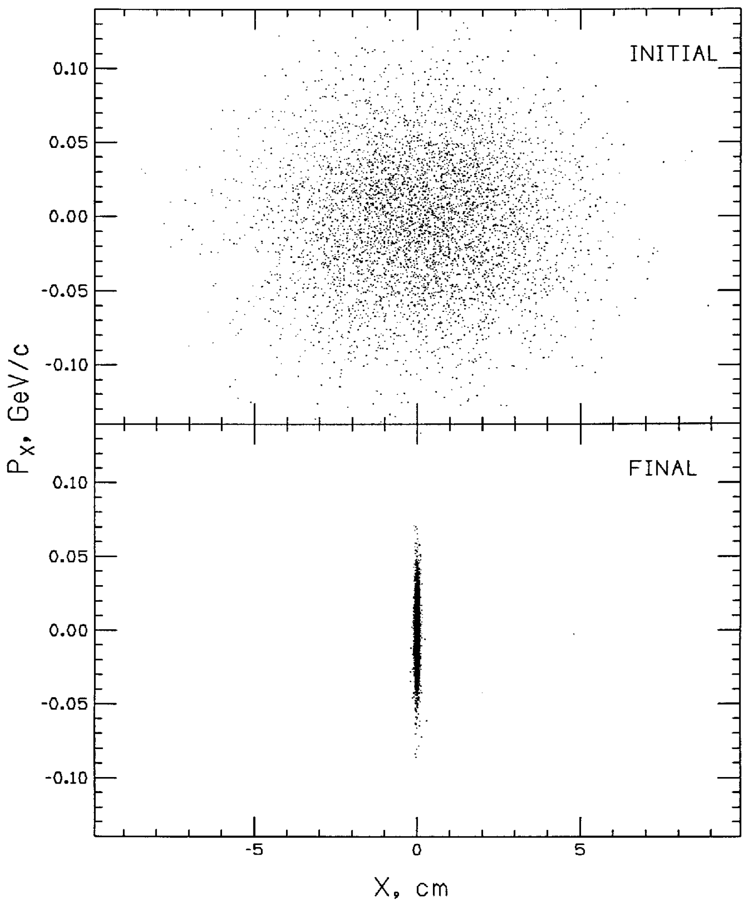
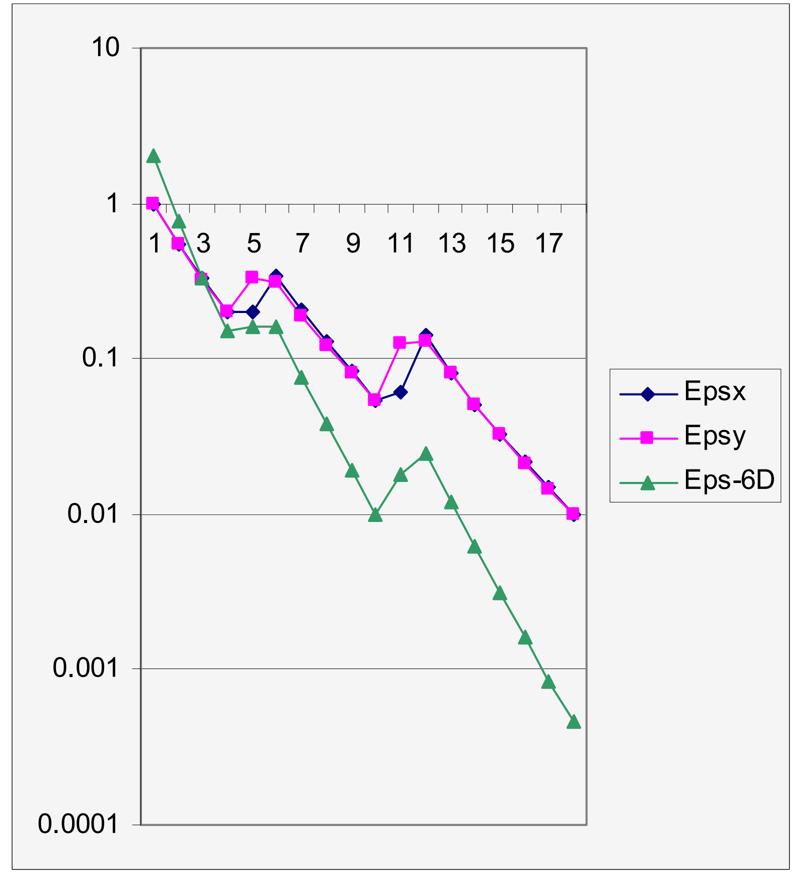


Figure 7: Initial and final transverse distributions.



5-D simulation of “complete cooling scenario” includes 2 emittance exchanges





Long - term Li-lens Cooling results:
($\varepsilon_\perp \times 1/100$; 6-D cooling by 10^{-4})

μ^+-μ^- Collider Cooling requirements

- μ^+-μ^- **Collider**

  requires energy cooling

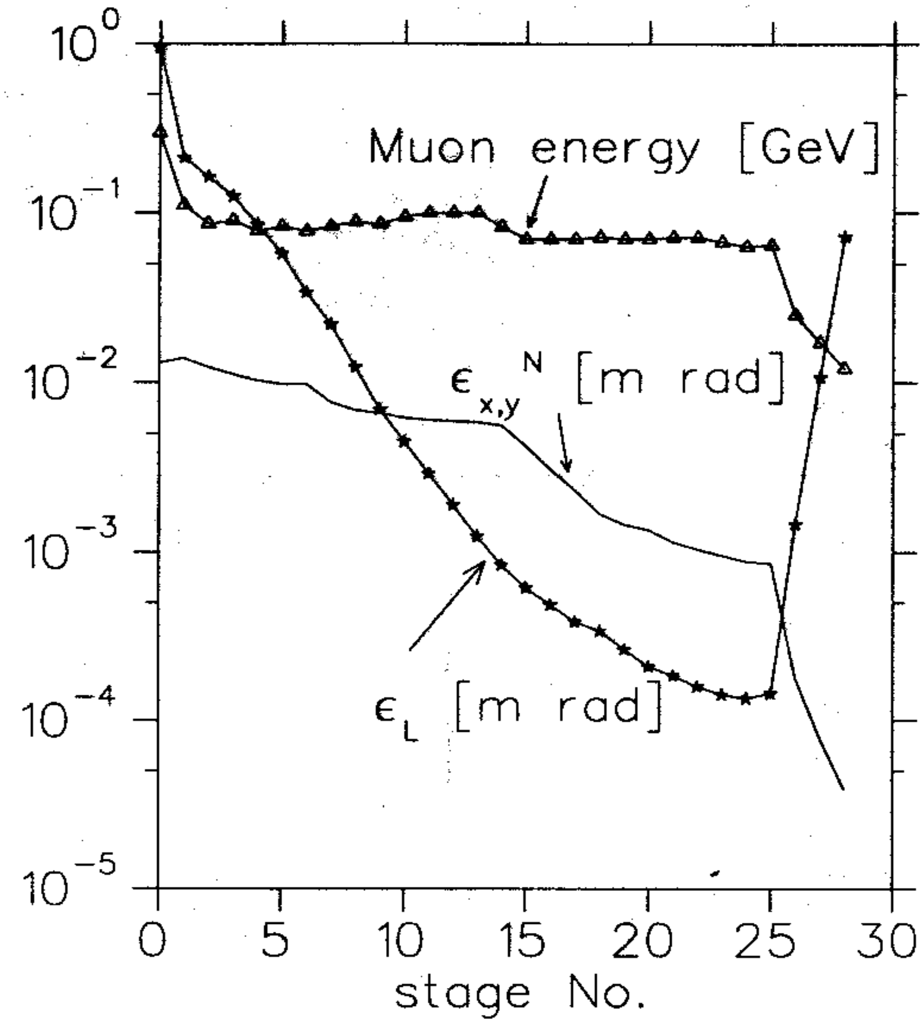
  and emittance exchange

  (and anti-exchange)

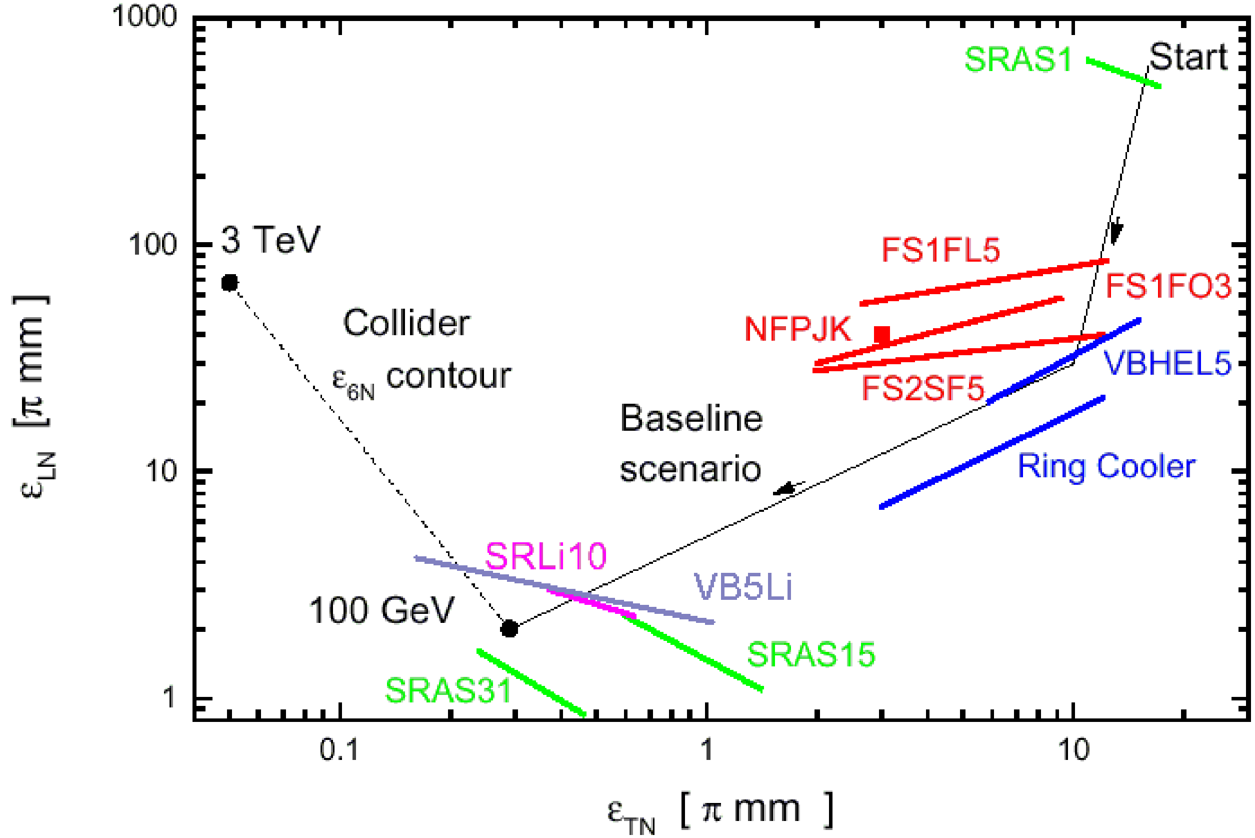
  to obtain small ε_L (ΔE–ϕ),

  and small x,y emittances

  required for high-luminosity



Cooling Status (Map)



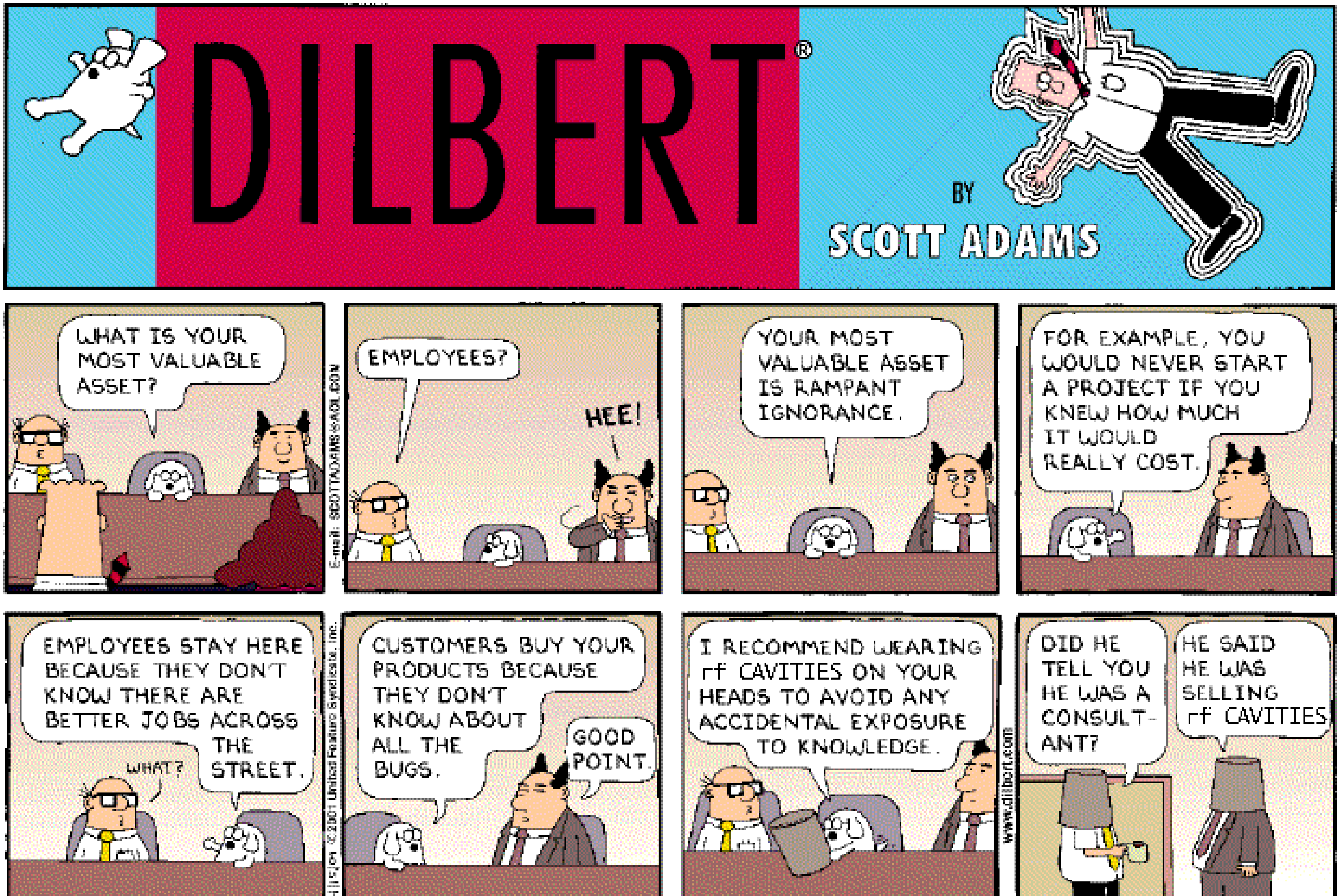
- **Cooling sections that have beem simulated in some detail**
  - ICOOL, Geant, VLB simulations
- From R. Fernow

Summary

- Emittance exchange is necessary to obtain longitudinal ionization cooling
- Emittance exchange requires:
  - Dispersion (η) at absorber
  - wedge-type absorber, with more material for higher-E beam
  - small-β^* optics at absorber to minimize multiple scattering
  - matched optics, with large momentum acceptance
  - longitudinal beam matching ….
- Round emittances require similar exchanges in both x and y

- Detailed solutions with all of these features have not yet been developed
- Integration into complete cooling scenario is also required

Snowmass Summary ?



Post-Snowmass Status