

# Multiple Scattering in SF0F01 Type Absorber

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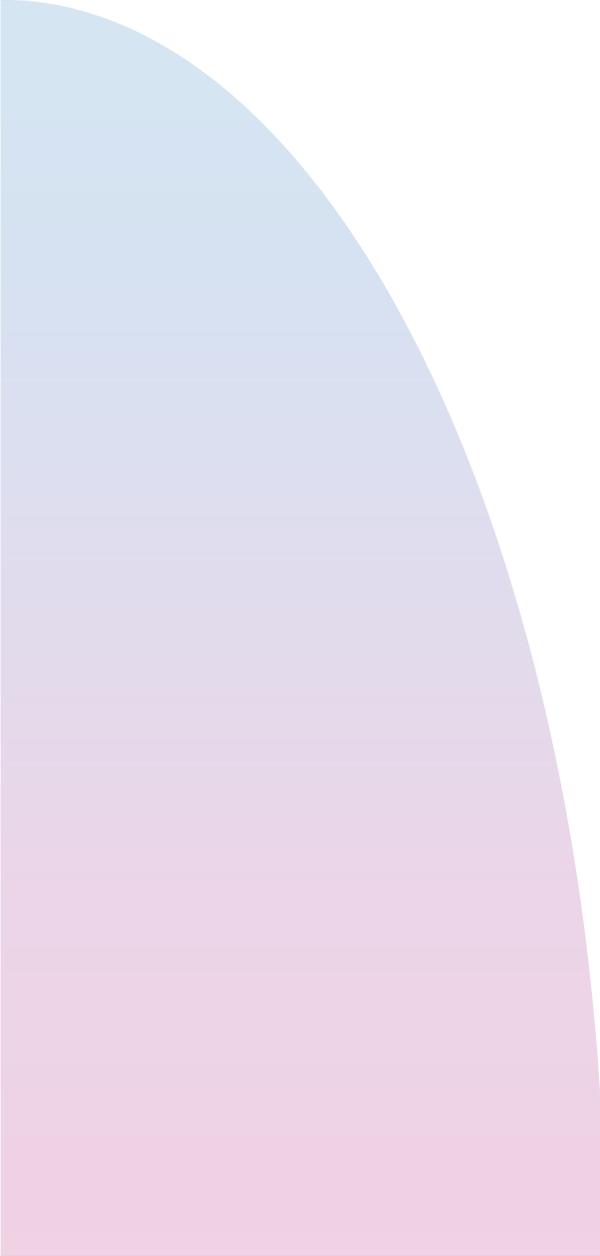
University of Illinois

Urbana-Champaign

**MuCool Experiment**

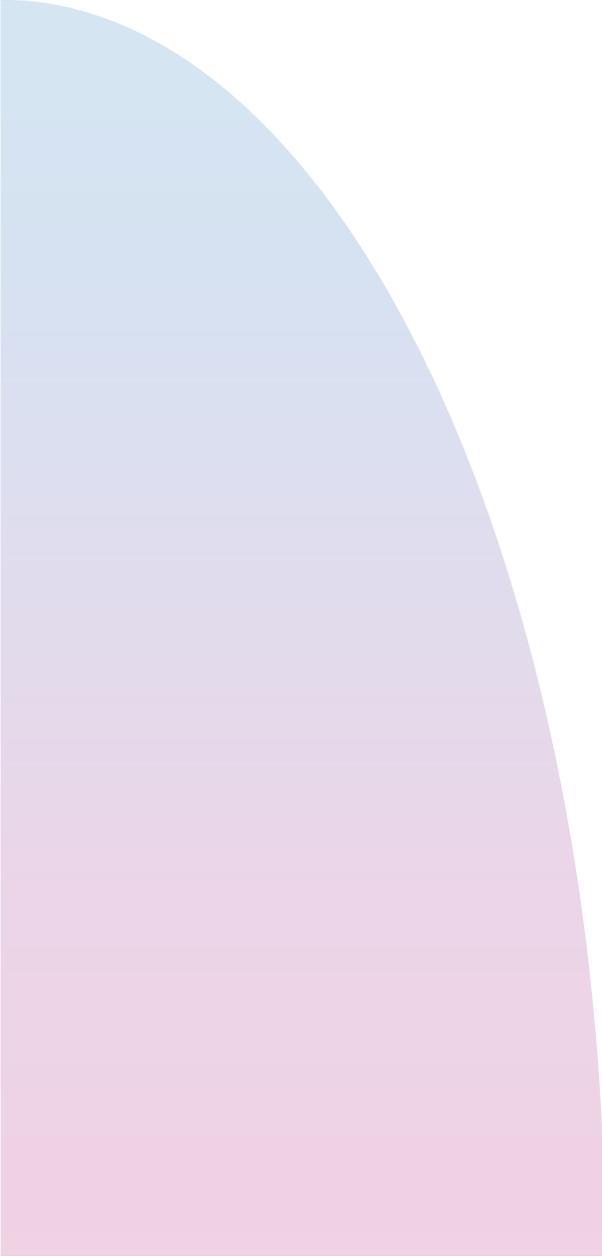
Emittance Exchange Workshop at LBL

Oct 8, 2001



# Considerations...

- $dE/dx$  energy loss affects momentum hence multiple scattering.  
Initially ignore this effect.
- For small angle scattering the pathlength is the same as the unscattered pathlength  
 $z' = z$



# Considerations...

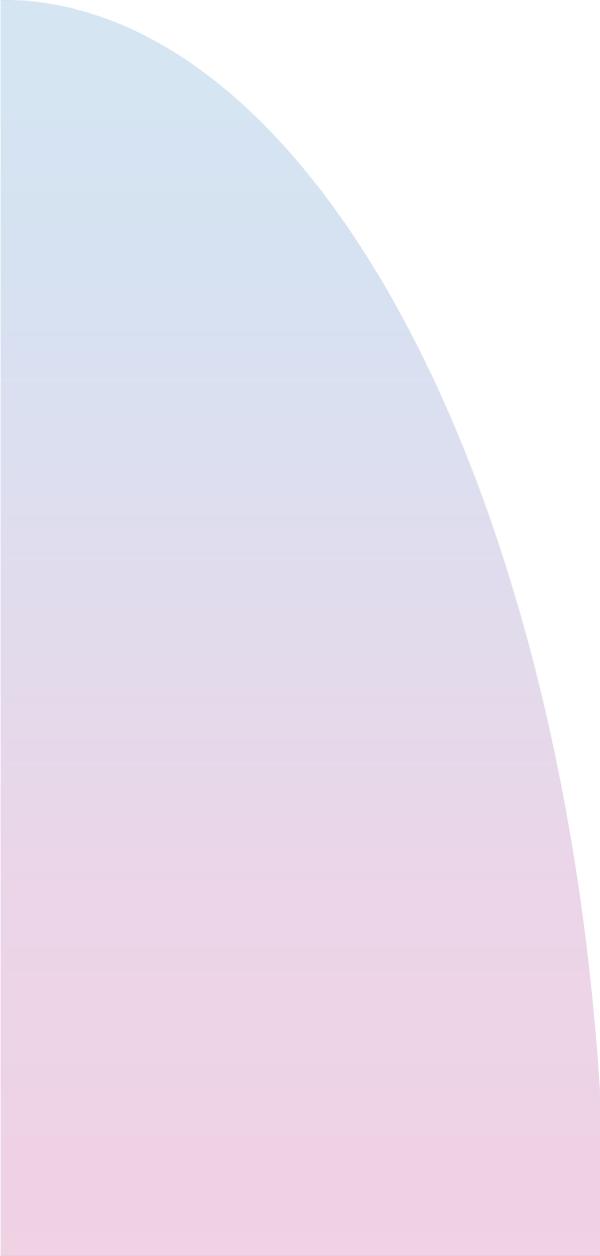
- Since  $\beta$  and  $p$  are dependent for a set of particles of all the same mass...

introduce  $s = (p/m_\mu)^2$

$$= \gamma^2 \beta^2$$

$$= \beta^2 / (1 - \beta^2)$$

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# Considerations

- Magnetic Field  
(Consider  $B = 0$  initially)  
May have strong effect on  $z$ , the pathlength.
- Absorber Geometry  
 $R = 18\text{cm}$   
 $L = 35\text{cm}$   
 $t \sim 300$  microns....sfofo lattice 1  
Initially detailed geometry ignored

# Rossi Formula

$$\theta_x : P(\theta_x) = \frac{1}{\sqrt{2\pi\sigma_o^2}} e^{-\frac{\theta_x^2}{2\sigma_o^2}}$$

$\theta_y : P(\theta_y)$  is distributed identically.

where 
$$\sigma_o^2 = \left( \frac{14\text{MeV}}{p\beta} \right)^2 \frac{z}{X_0}$$

or 
$$\sigma_o^2 = \left( \frac{14\text{MeV}}{m_\mu} \right)^2 \frac{s+1}{s} \frac{z}{X_0}$$

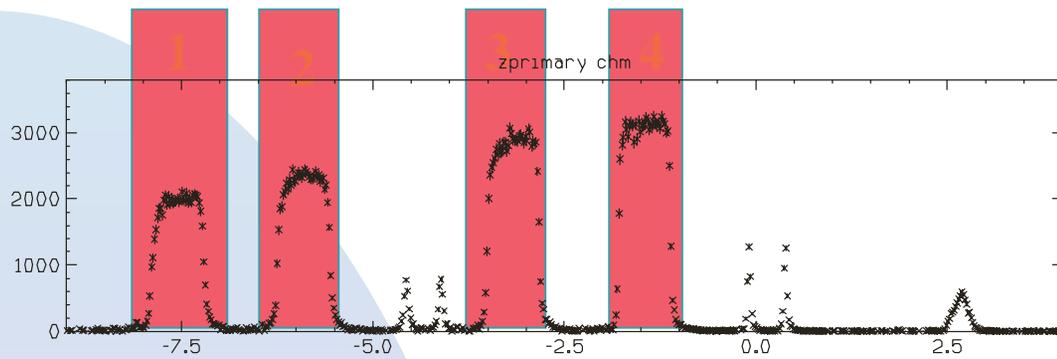
where  $z$  is pathlength,  $X_0$  radiation length,  $s=p^2/m^2$

The following two transparencies are provided by Prof. Jim Wiss from UIUC and the FOCUS experiment at Fermilab.

These demonstrate that the Rossi formulation describes multiple scattering as well or better than the formulation which includes the logarithmic term.

This allows us to use 3 gaussian distributions folded together to describe the behavior of scattering in the three regions of the absorber.

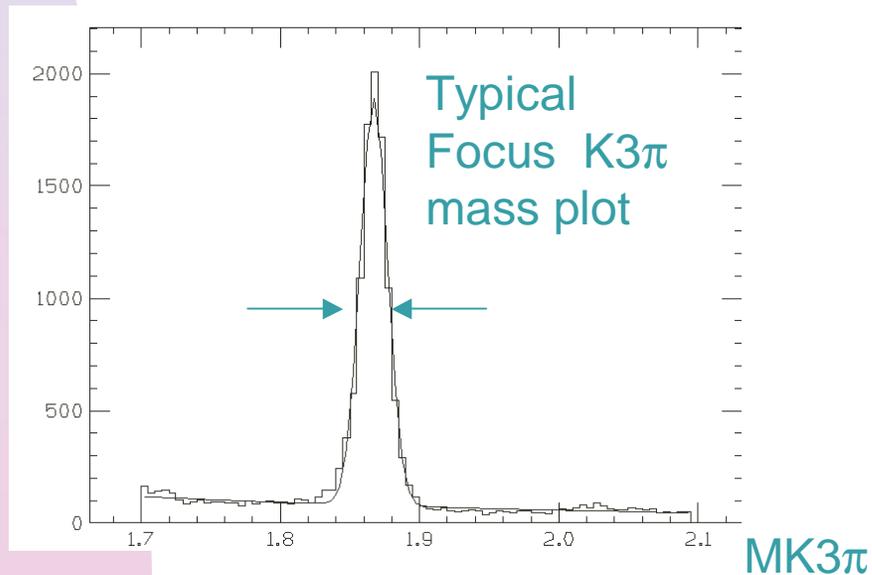
# MCS studies from FOCUS



FOCUS uses a 4 segment BeO target interleaved with 4 target microstrip planes. Each target segment is about 3% of a radiation length.

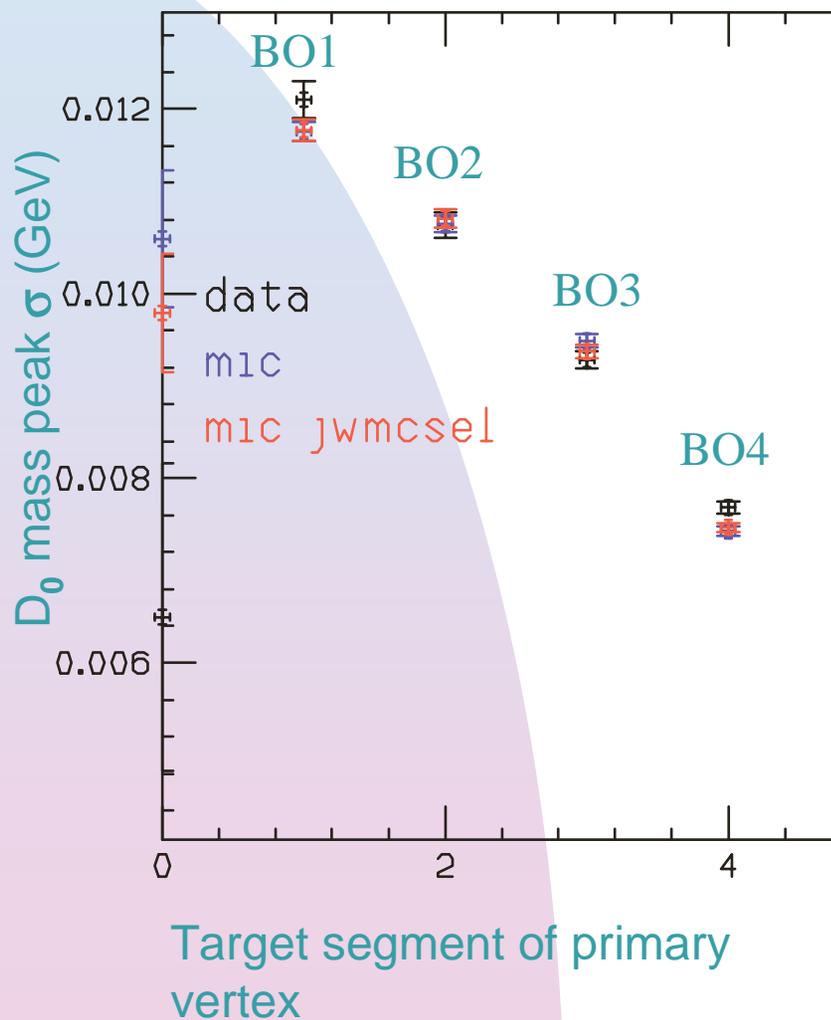
Z of charm primary vertices

We can study MCS by seeing how the mass resolution of D's and Ks's varies as a function of the vertex position. The Z of the vertex controls the amount of material the tracks scatter through prior to being measured by the downstream spectrometer. The lower the Z value -- the more multiple scattering.



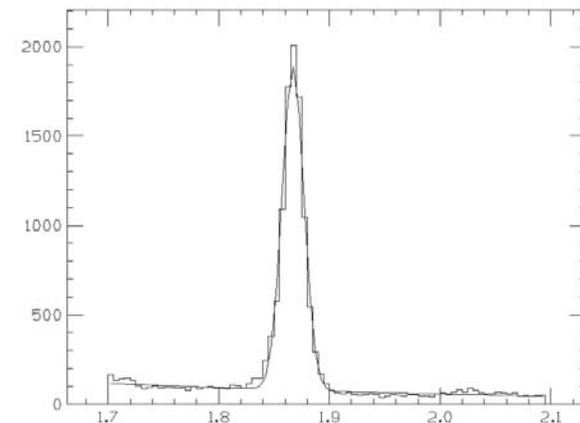
MK3π

# Width of $D^0 \rightarrow K3\pi$ : data vrs MC

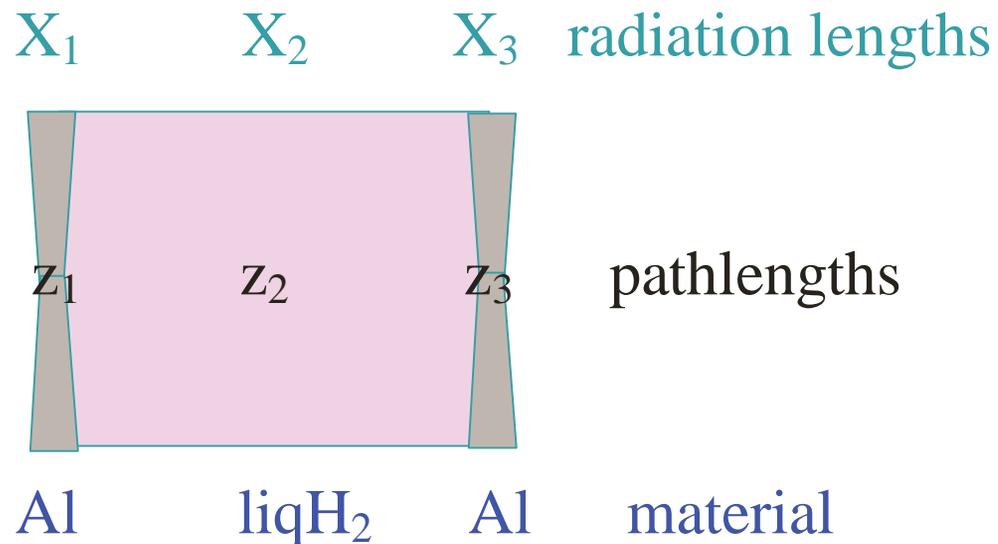


The **jwmcse1** Monte Carlo simulates just MCS and uses a simple Gaussian projected width of  $\sigma = (.014/P) \sqrt{x/x_0}$  with no log terms (**Rossi formula**)

The  $D^0 \rightarrow K3\pi$  mass resolution improves as the vertex moves downstream and the tracks pass through less scattering material



- For consecutive materials the variances add for the gaussian distributions.



Idealized geometry of absorber for simple calculation.

$$X_1 = 8.9 \text{ cm} = X_{\text{Al}} = X_3 \quad X_2 = 866 \text{ cm} = X_{\text{liqH}}$$

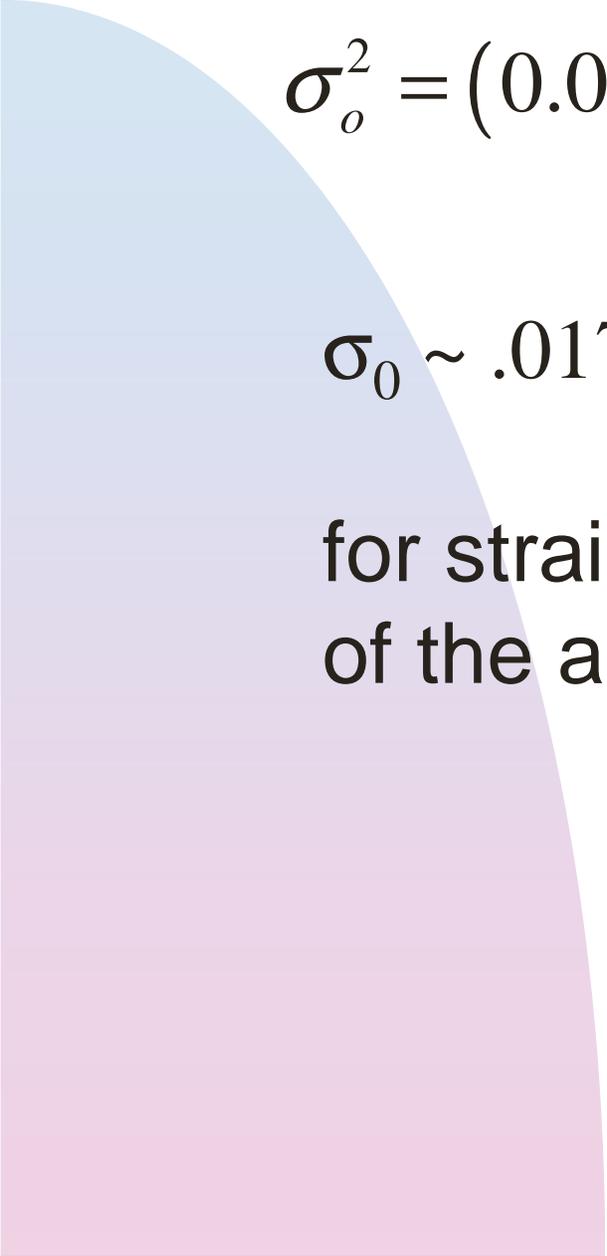
Note that the pathlength through each material depends on the position on the absorber window, initial angle, and eventually B-field.

$$\sigma_o^2 = \left( \frac{14 \text{ MeV}}{m_\mu} \right)^2 \frac{s+1}{s} \left[ \frac{z_1(x, \alpha) + z_3(x, \alpha)}{X_{Al}} + \frac{z_2(x, \alpha)}{X_{liqH}} \right]$$

Assume  $s = (200 \text{ MeV} / 105 \text{ MeV})^2 = 3.58$

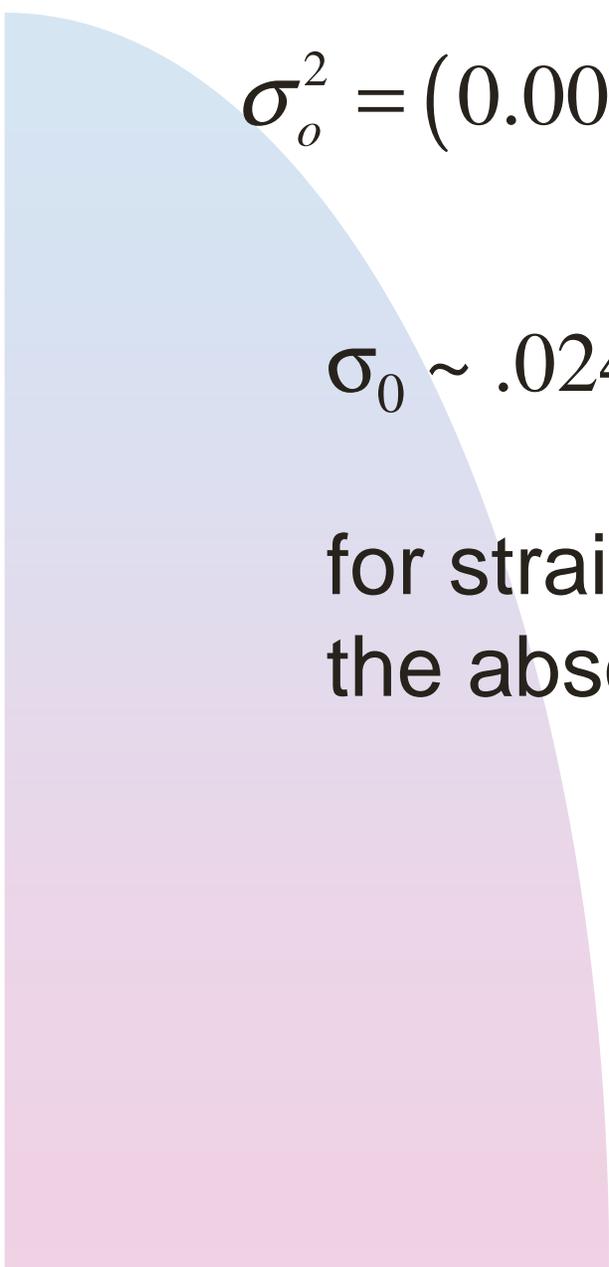
and  $z(\alpha) = t / \cos \alpha$ .

Since the angular spread for the quad channel is  $\leq 3^\circ$  and for the solenoid sfofo1 channel  $\leq 7^\circ$  it is valid to assume  $\cos \alpha \sim 1$  ( in the case  $B=0$  Tesla)


$$\sigma_o^2 = (0.0063587) \left[ \frac{2(0.03cm)}{8.9cm} + \frac{35cm}{866cm} \right]$$

$$\sigma_o \sim .0173 \text{ radians} \sim 1^\circ$$

for straight-throughs at the **center**  
of the absorber.


$$\sigma_o^2 = (0.0063587) \left[ \frac{2(0.2445cm)}{8.9cm} + \frac{35cm}{866cm} \right]$$

$$\sigma_0 \sim .0246 \text{ radians} \sim 1.4^\circ$$

for straight-throughs at the **edge** of the absorber.

## Conclusions:

The rms angular spread in both x and y is **1 degree** for straight-throughs at the **center** of the absorber.

The rms angular spread in both x and y is **1.4 degrees** for straight-throughs at the **outer edge** of the absorber.

These may be fine for the quad channel where  $B \sim 0$  Tesla but the trajectory through the magnetic field cusp inside the absorber in the solenoid channel will be much more complicated and possibly significantly longer (to be determined).