Muon Collaboration

Ring Coolers and the Summary of the Emittance Exchange Workshop

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Fermilab

MUTAC meeting
Berkeley, 18-19 October 2001

Format of talk

- Emittance Exchange- Why do we need it? Some elementary theory.
- Ring Coolers-
  - » Balbekov Rings (solenoidal focusing)
    - 0.225 GeV/c momentum
  - » Garren Ring (Quad focusing)
    - Initially 1 GeV/c – Now at 0.5GeV/c
  - » Tollestrup Ring (Quad focusing)
    - At 1 GeV/c- good to explore behavior
  - » Palmer Ring (Solenoid focusing)
    - 0.225GeV/c RFOFO linear channel turned into rings
- Summarize rest of Workshop
  - » Helical coolers (Penn)
  - » Rf manipulations(Koscielniak)
  - » Quad cooling channels (Johnstone)
  - » COSY as a tool for cooling simulations?(Makino,Berz)
  - » Merit factors
- Conclusions

Why We Need Emittance Exchange

- We need emittance exchange for two reasons-
- Linear Cooling channels cool transversely. Longitudinal emittance grows due to straggling.
- Muon Colliders- After collection and phase rotation, the longitudinal and transverse emittance of single bunch of muons are L/T)=(42cm/1.2cm) respy. Collider needs (L/T)=(2cm/0.006cm). So clearly transverse and longitudinal emittances have to be reduced.
- Neutrino Factories- During the cooling produced by Study II linear cooler, the longitudinal emittance grows and results in a factor of 2 loss in muons. Longitudinal cooling could improve performance considerably.

Ionization Cooling theory for pedestrians

- In a Hamiltonian system, a particle's motion along the beam direction may be specified by a set of 6 canonical variables $(x,p_x),(y,p_y),(z,p_z)$ or $(x,p_x),(y,p_y),(E,t)$. Let us define a 6-vector X_i ,(i=1,6), which refers to the above set. Over an ensemble of particles, let us define a 6-vector Y such that $Y_i=X_i-\langle X\rangle_i$
- Then the error matrix

$$E_{ij} = \langle Y_i Y_j \rangle$$

- Then the 6-Dimensional emittance ε_6 is defined as

$$(\varepsilon_6)^2 = \mathrm{determinant}(E)/(m_\mu c)^6$$

- In a Hamiltonian system, the 6-vector X' at a later time is given by a linear transformation U such that

$$X' = U\,X, \quad \text{leading to } E' = UEU^T$$

- I.e. Det(E') = Det(E), emittance is preserved if Det(U)=1. Such transformations are known as Symplectic transformations. Liouville's theorem.
- Cooling is a non-Hamiltonian transformation with Det(U)<1, leading to emitance reduction.

Ionization cooling theory for pedestrians

- In the special case where correlations between x,y and z sets of variables can be neglected, the 6-dimensional emittance can be written as

$$(\varepsilon_6) = (\varepsilon^x_n)(\varepsilon^y_n)(\varepsilon^z_n)$$

where ε^x_n is the normalized emittance in the x direction etc. The x and y emittances are referred to as the transverse emittance and the z emittance is known as the normalized emitance.

When angles wrt beam direction are small, it can be trivially shown that

$$(\varepsilon^x_n)^2 = \{<x^2><\theta^2> - <x\theta>^2\}\gamma^2\beta^2$$

- The term in the {} is known as the (unnormalized) emittance.
- Defining E as the particle energy, $\beta_\perp$ as the beta function, $\beta\gamma$ as the usual Lorentz factors and L_R as the radiation length of cooling material, and m_μ as the mass of the muon, leads to the following expressions.

Ionization cooling theory for pedestrians

- We can show
  - rate of cooling decreases as emittance decreases
  - Effects due to multiple scattering are ameliorated by placing absorbers at points where angular divergence of beams is large so that the additional angular spread due to MS is not a large increse in emittance. This translates to areas of small beta function $\beta_\perp$
-

Differentiating,

$$\frac{d\varepsilon_N^x}{dz} = \varepsilon^x \frac{d(\beta\gamma)}{dz} + \beta\gamma \frac{d\varepsilon^x}{dz}$$

cooling heating

$$\frac{d\varepsilon_N^x}{dz}(cool) = -\frac{1}{\beta^2}\frac{\varepsilon_N^x}{E}\left|\frac{dE}{dz}\right|$$

$$\frac{d\varepsilon_N^x}{dz}(heat) = \frac{\beta\gamma}{2\varepsilon_x}\left\{<x^2>\frac{d<\theta^2>}{dz} + <\theta^2>\frac{d<x^2>}{dz} - 2<x\theta>\frac{d<x\theta>}{dz}\right\}$$

neglecting correlations

$$\frac{d\varepsilon_N^x}{dz}(heat) = \beta\gamma\frac{\beta_\perp}{2}\frac{d<\theta^2>}{dz} \rightarrow \text{multiple scattering}$$

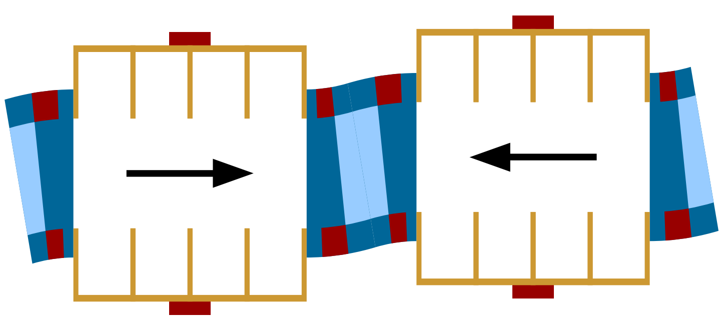
leading to

$$\frac{d\varepsilon_N^x}{dz} = -\frac{1}{\beta^2}\frac{\varepsilon_N^x}{E}\left|\frac{dE}{dz}\right| + \frac{\beta_\perp}{}\frac{(0.014)^2}{2\beta^3 E m_\mu L_R}$$

Linear Cooling Theory

Berg,Berz,Fernow,Kim,Lebedev,Makino,Neuffer, Palmer,Penn,Wang,Wurtele

If you the transfer matrix U, the 6 eigenvalues come in 3 complex conjugate pairs. If the mixings are not too large, then the eigenfunctions are 2 transverse like and one longitudinal like. To illustrate, an sFOFO lattice with a bend dipole in between, can be analyzed using linear theory (Berg).

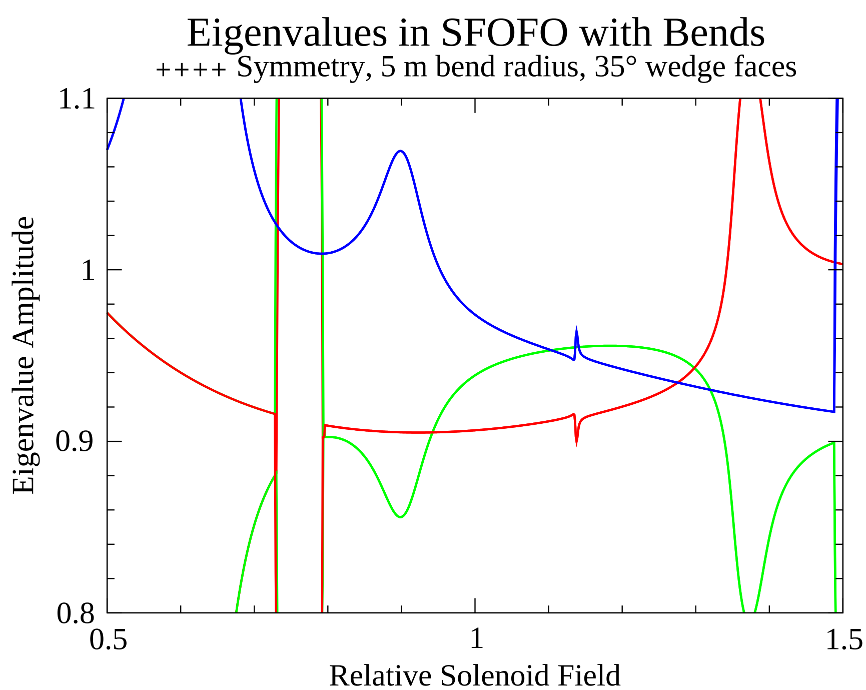


Linear theory-Eigenvalues of the transfer matrix

- In the presence of cooling, the product of eigenvalues of the transfer matrix has the form-

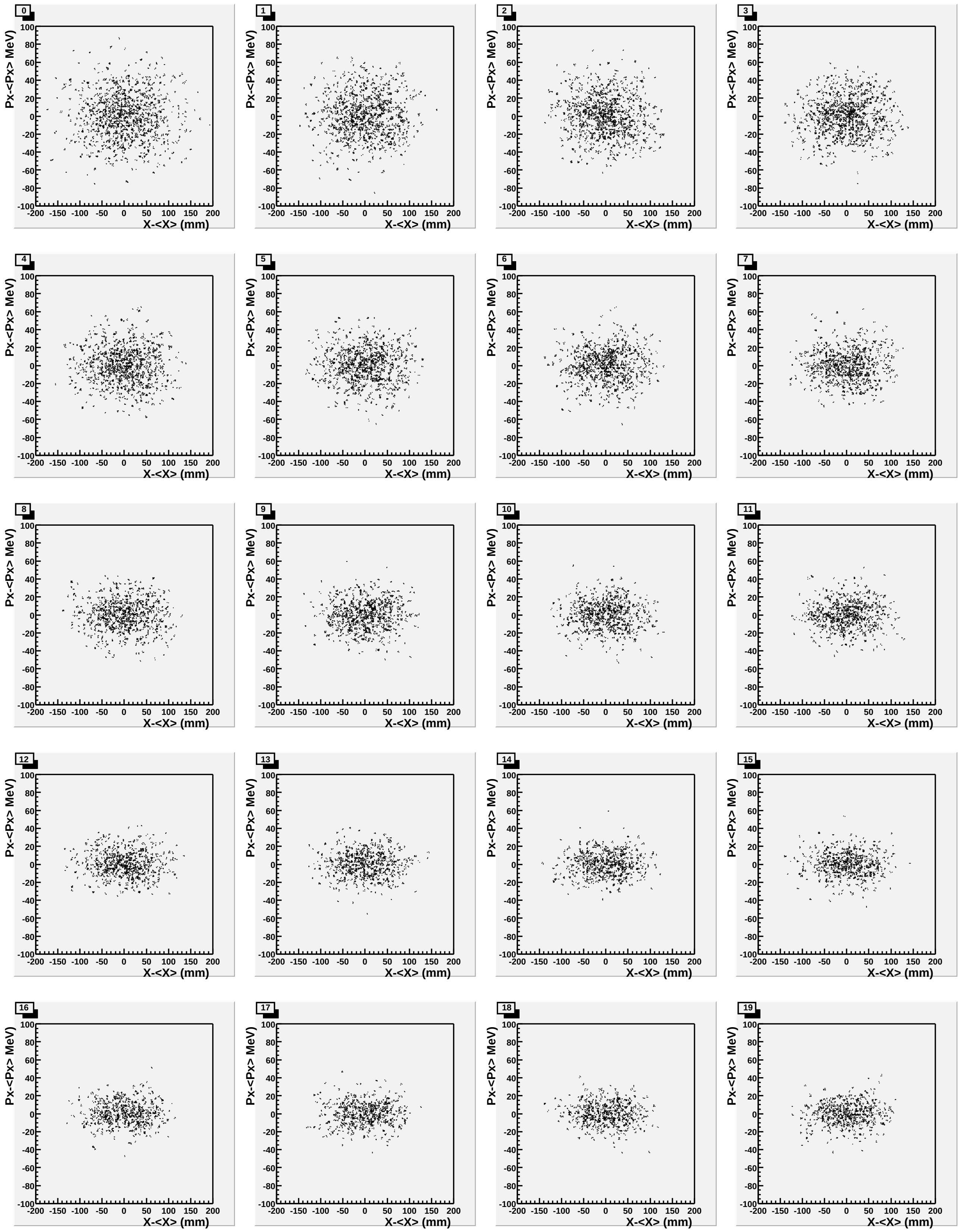
$$\prod_{i=1}^{i=6} \lambda_i = 1 - \delta$$

- Where δ is the cooling factor. Notice that if one has more transverse cooling (λ_1 and λ_2 being less than 1), then longitudinally we may get heating. The following picture shows the magnitude of the 3 eigenvalues

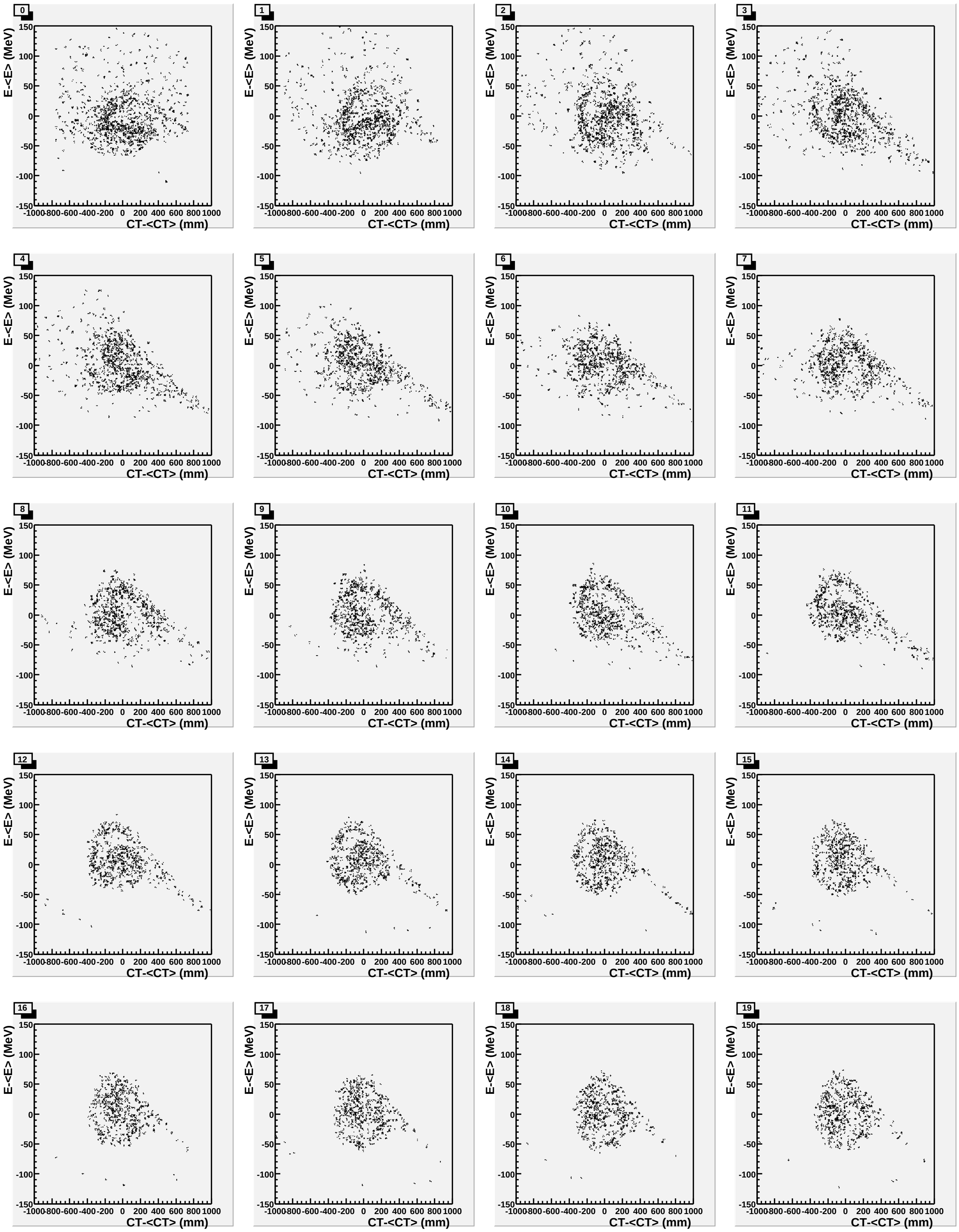


- V.Lebedev and A. Tollestrup have analyzed a 1 GeV cooler using linear theory.

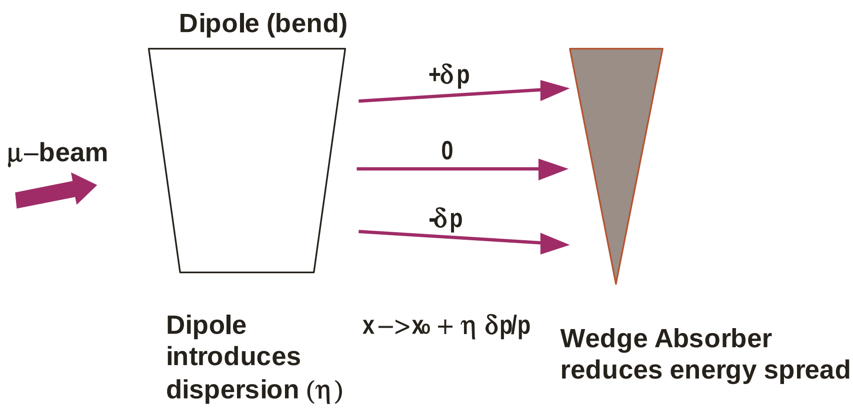
Conventional cooling- Double Flip linear channel-results Every 2.5 meters xpx plots



Conventional cooling- Double Flip linear channel-results Every 2.5 meters En-ct plots



Emittance exchange overview

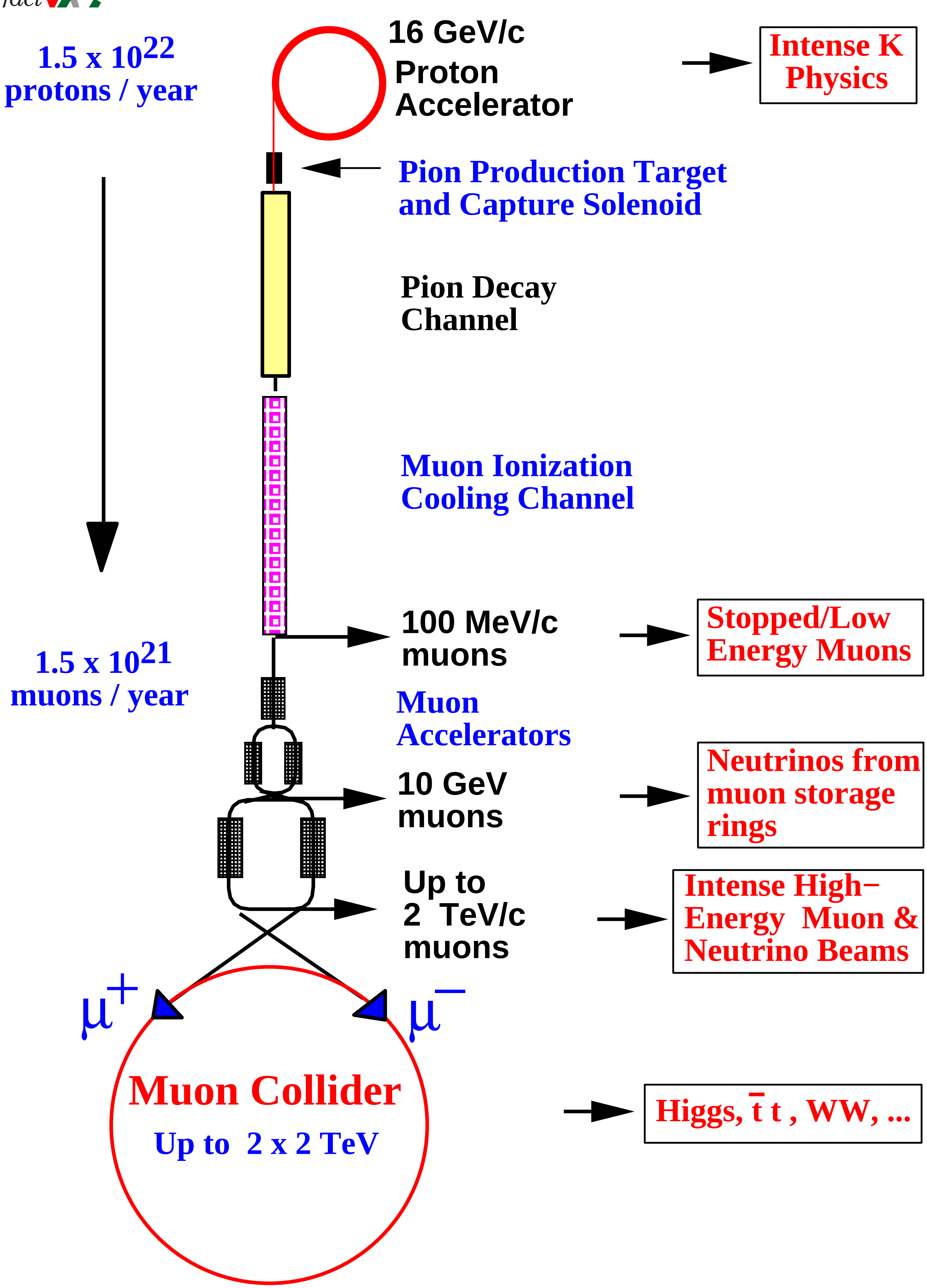


- Cooling derivative is changed by use of dispersion + wedge

  (Dependence of energy loss on energy can be increased)

$$\frac{\partial \frac{dE}{ds}}{\partial E} \Rightarrow \left.\frac{\partial \frac{dE}{ds}}{\partial E}\right|_0 + \frac{dE}{ds}\frac{\eta\rho'}{\beta c p \rho_0} = g_L \frac{dp/ds}{p}$$

Muon Collider Schematic



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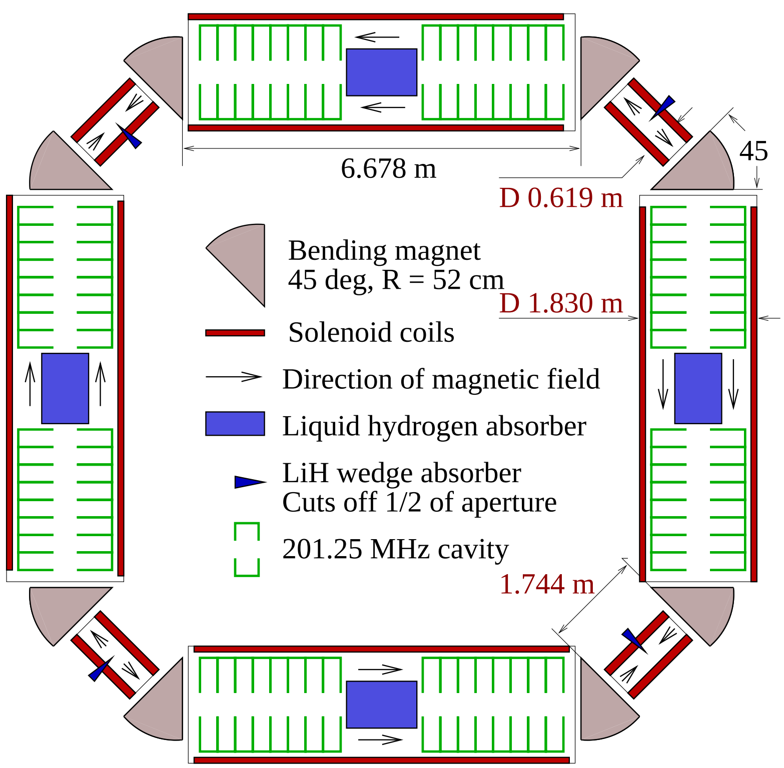
Balbekov Ring Cooler

- Working group
  - » V.Balbekov,G.Hanson,S.Kahn,D.Krop, R.Raja,P.Schwandt,Z.Usubov
- Scheme is to have 3 rings in tandem for a collider. Produces bunch compression and cooling.

Ring Cooler Update

V.Balbekov 04/24/01

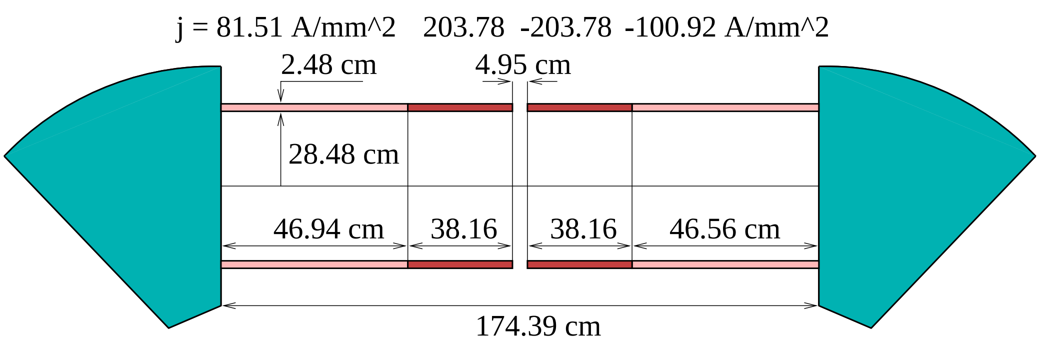


The ring cooler layout (new version, $R_{bend} = 52$ cm).

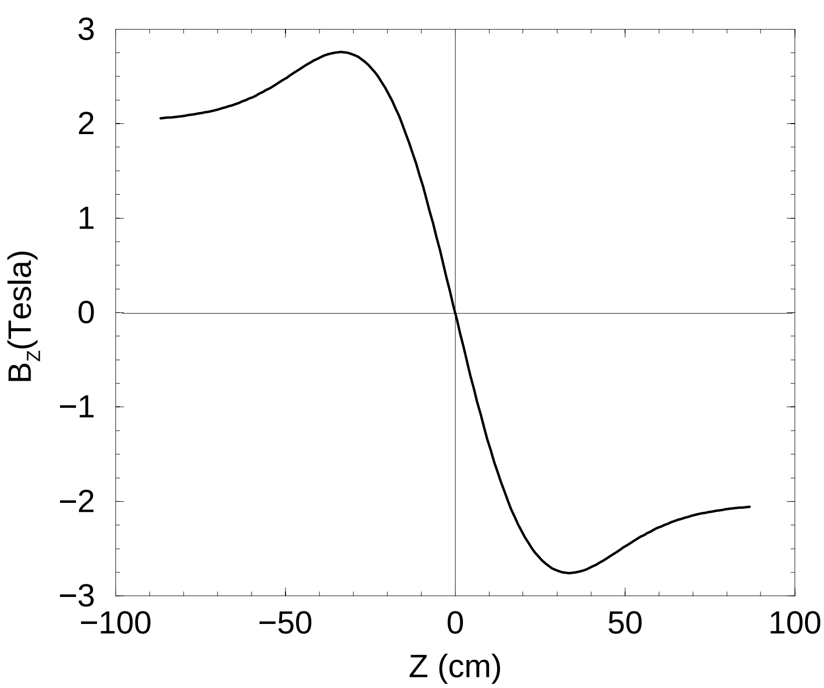
New design with bend magnet radius = 53cm

Short Straight Section

Function of the section: to create dispersion only in SSS



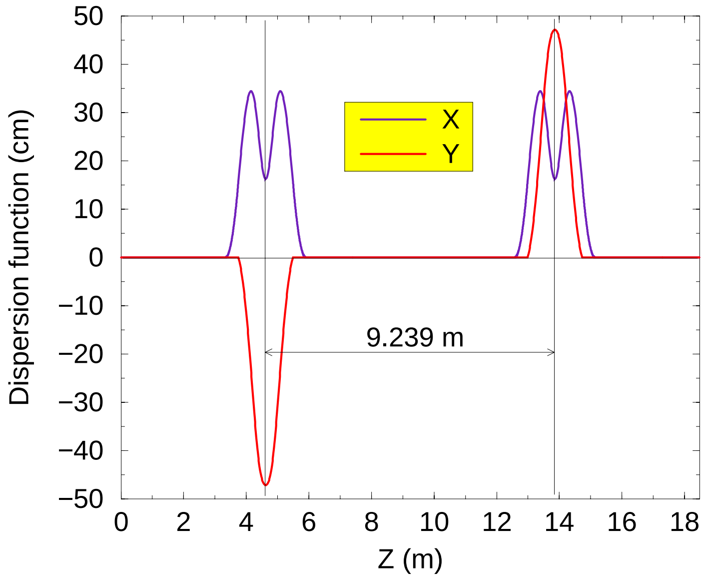
Layout of the Short Straight Section.



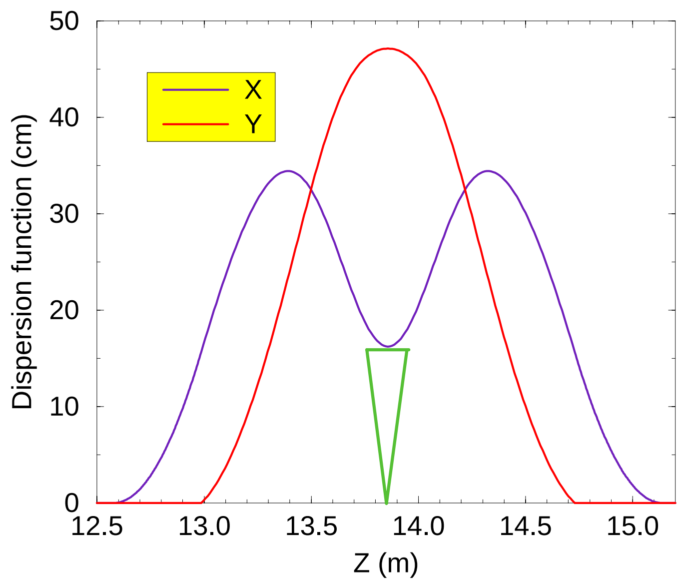
Axial field of the Short Straight Section.

New design with bend magnet radius = 53cm

Dispersion Function



Dispersion function at 1/2 of the ring.



Dispersion function and wedge absorber.

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Ring Cooler

Phase space of the beam

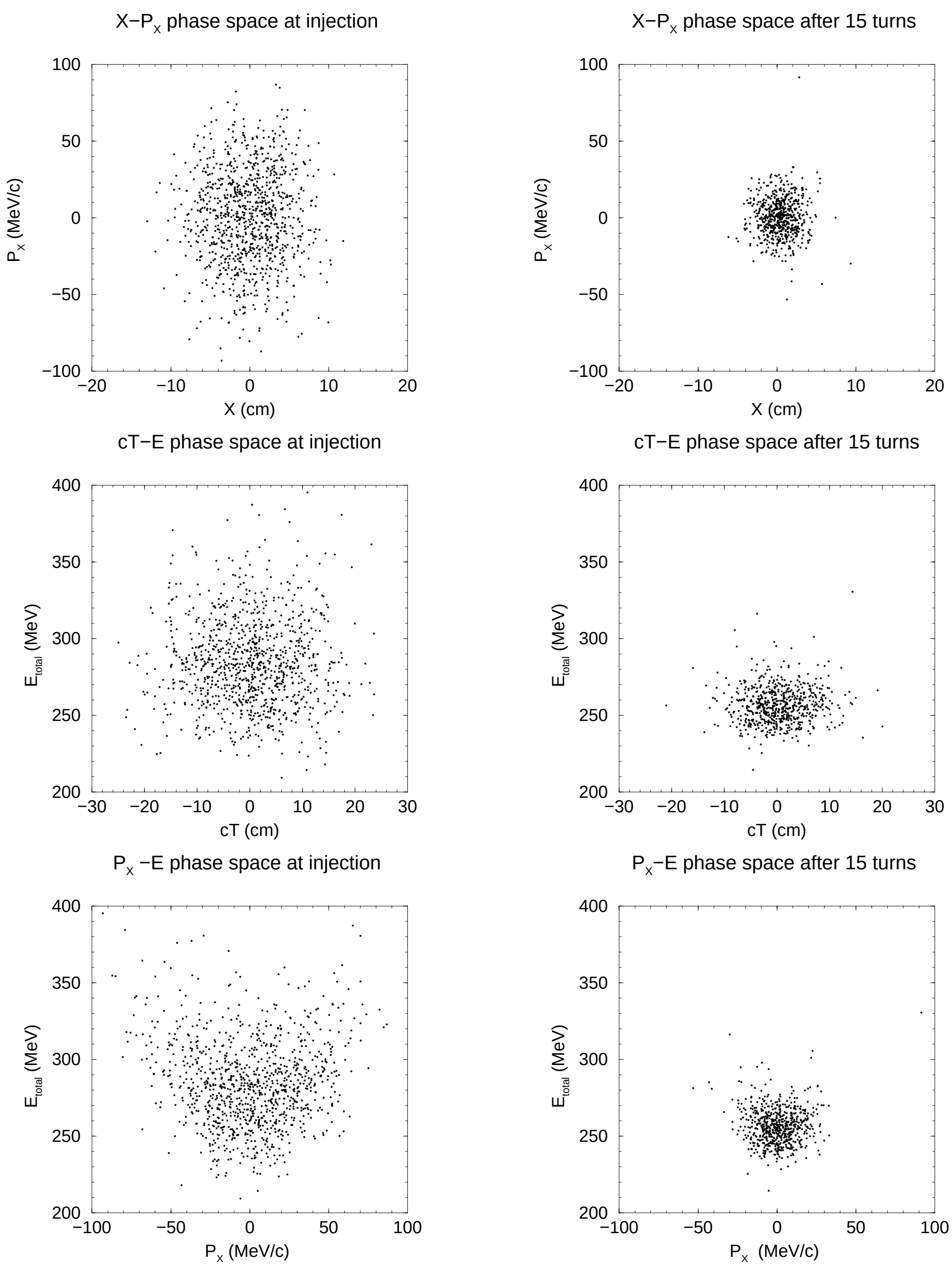

Figure 6:

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Ring Cooler -Performance

Bunching in the ring

- Current carrying target 15 cm Cu
- Primary focusing: Li lense
- Decay channel: solenoid $B = 3.56$ T, $R = 15$ cm, $L = 16$ m

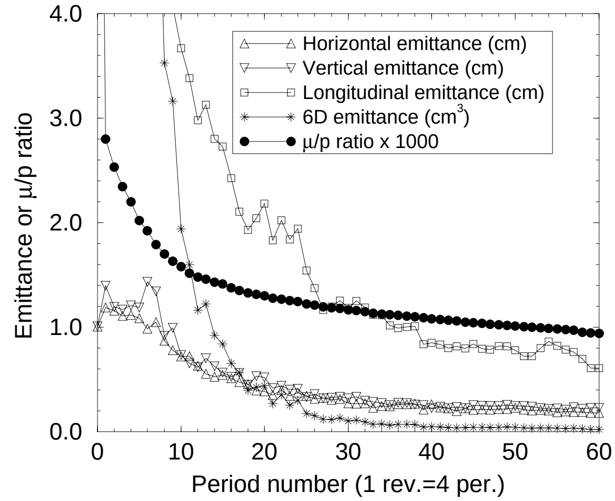

Figure 7: Beam emittance and μ/p ratio in the ring cooler

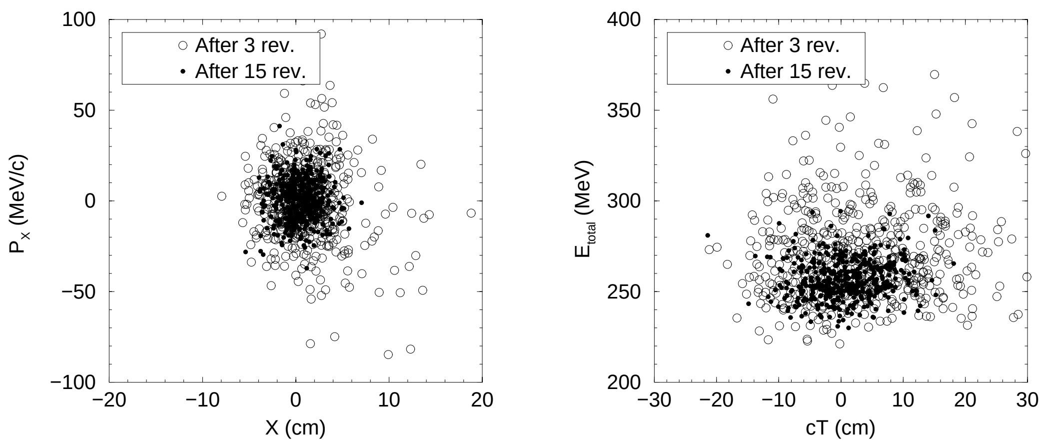

Figure 8: Phase space of the beam after 3 and 15 turns

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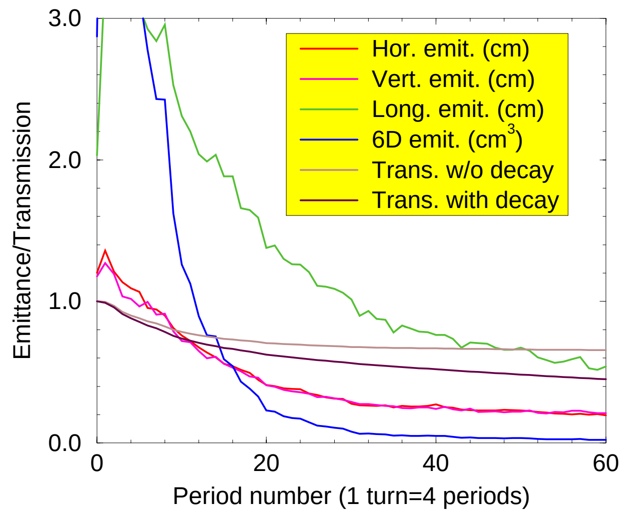
Balbekov cooler results

Conclusion

Low RF ring coolers with strong emittance exchange can provide longitudinal bunch compression required for the Higgs factory. Evolution of the beam parameters in 2 ring coolers, 40 turns in each, is presented in the Table:

Parameter	Begin	1st com.	2nd com.
Trans. emittance (cm)	1.2	1.0	0.8
Long. emittance (cm)	42	10	2.1
6D emittance (cm^3)	62	10	1.3
Transmission (%)	100	50	28

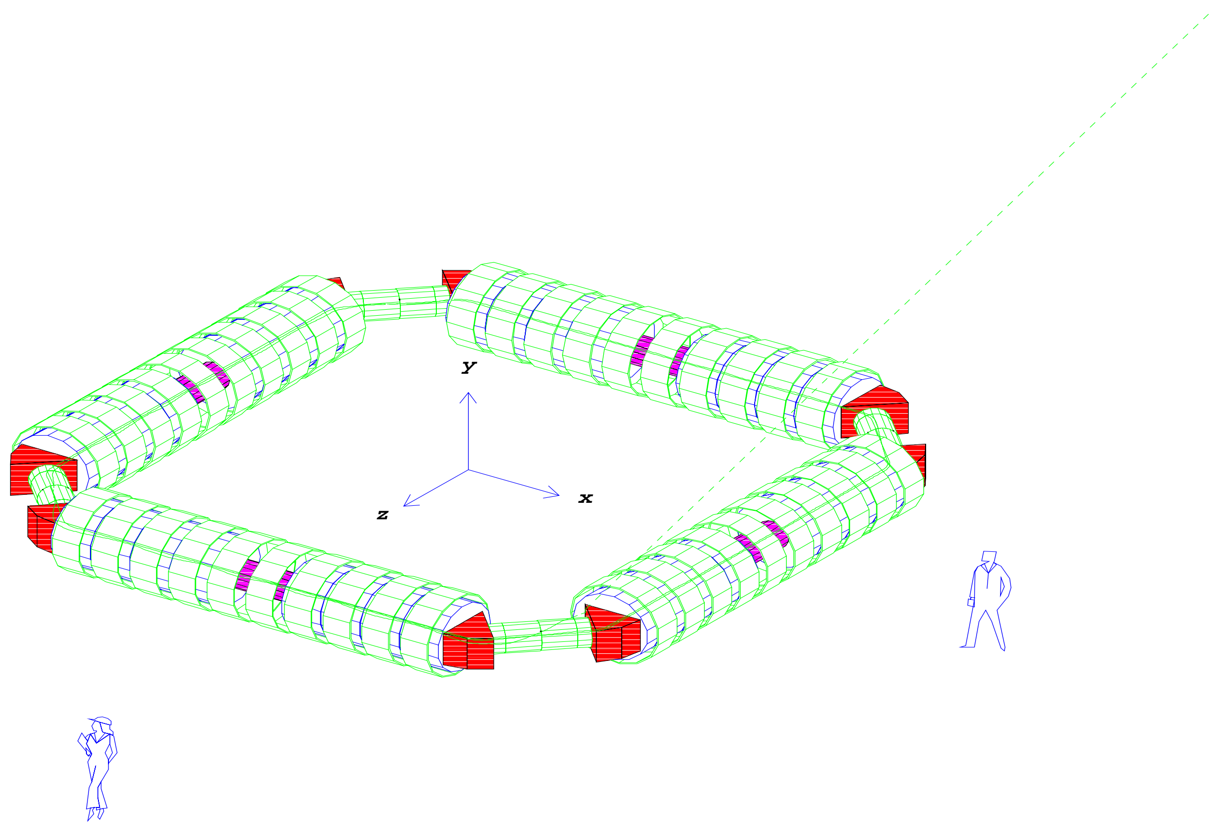
Cooling of this beam by 200 MHz ring cooler is plotted below:



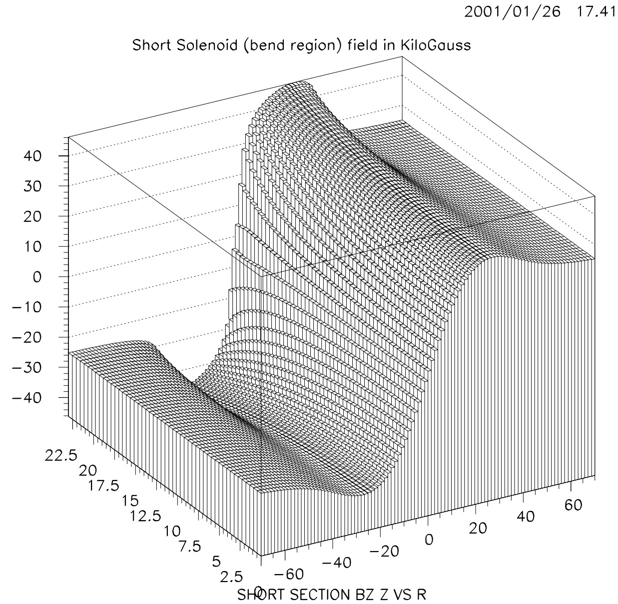
After 15 turns, trans./long. emit. are: **2/5 mm**.
Higgs factory requirements are: **0.06/20 mm**.

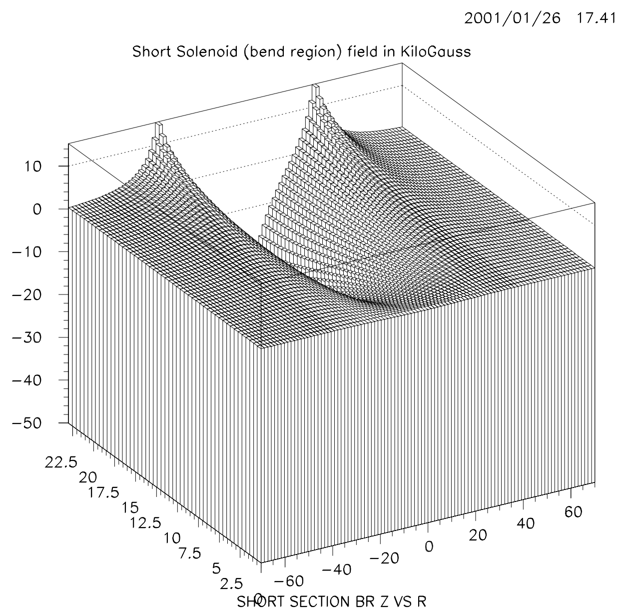
Geant Simulation of ring cooler

- Geant tracking modified to handle electric fields exactly.
- Utilities built to produce transfer maps with tracking.



Geant Simulation of ring cooler
Short Section Solenoid





Balbekov ring-Solenoid end field kick

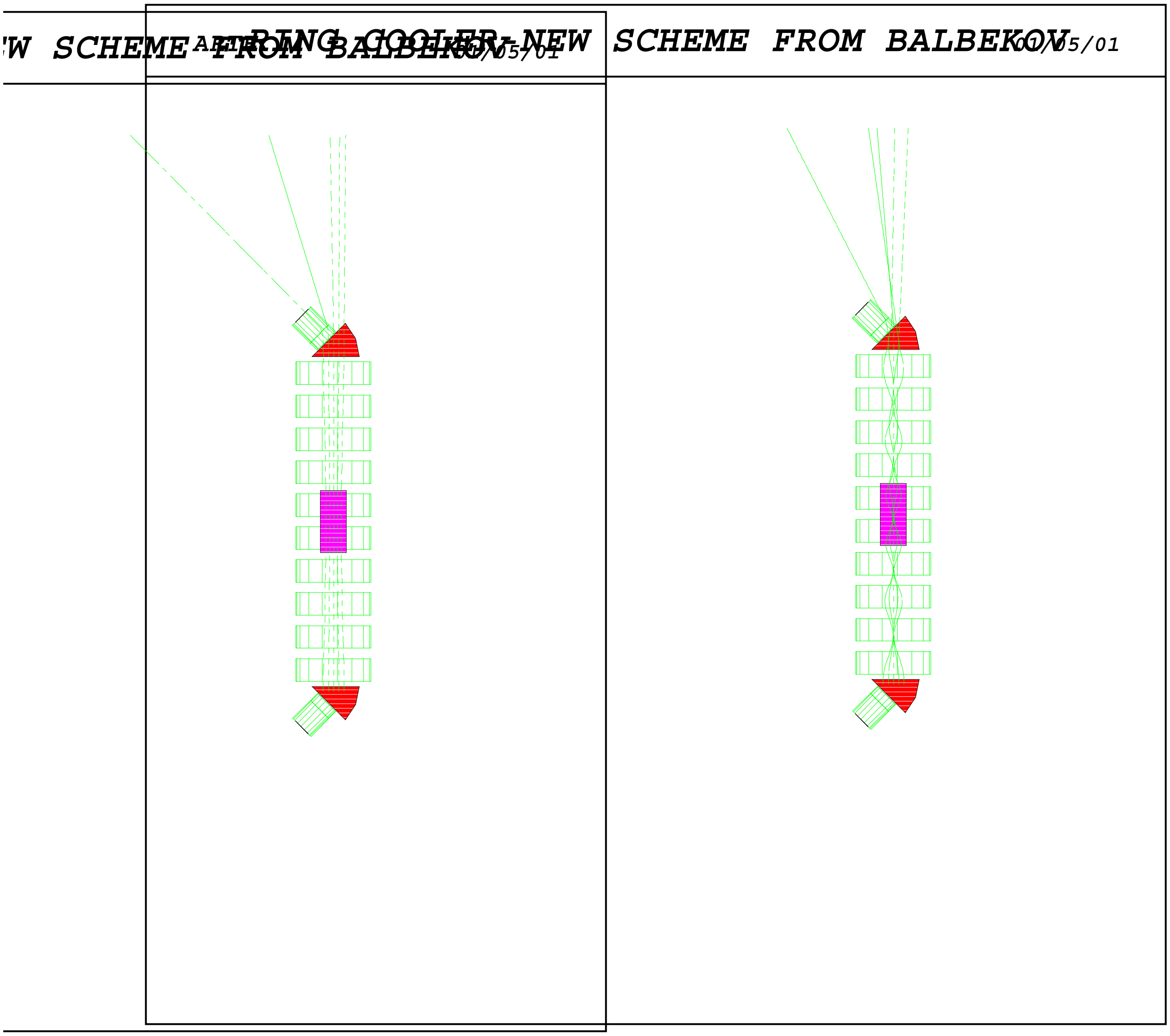
- Currently fields are non-Maxwellian in that there are no fringe fields. Fringe effects are approximated.
- One needs to simulate the end field of the solenoids by changing the transverse momentum of the particle by the following equations. These assume that the field of the solenoid B_z changes abruptly from its end value to zero while obeying Maxwell's equations. The kicks at entrance is given by

$$\Delta p_x = \frac{e}{2}(2.99\times10^{-4})yB_z$$

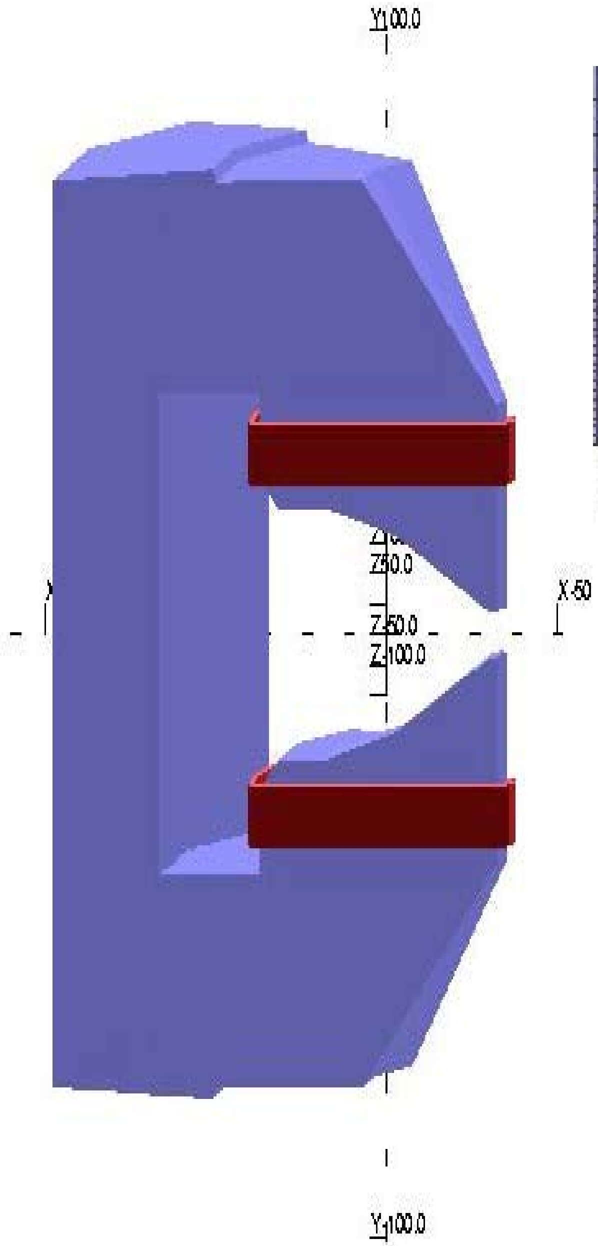
$$\Delta p_y = -\frac{e}{2}(2.99\times10^{-4})xB_z$$

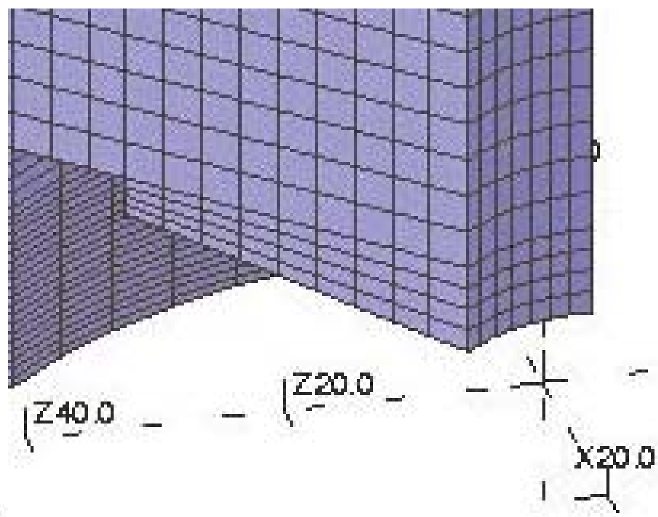
- At exit, the signs are reversed.

Solenoid end field kick



Proper design of magnetic fields
S.Kahn, P.Schwandt
Sketch of Dipole Magnet



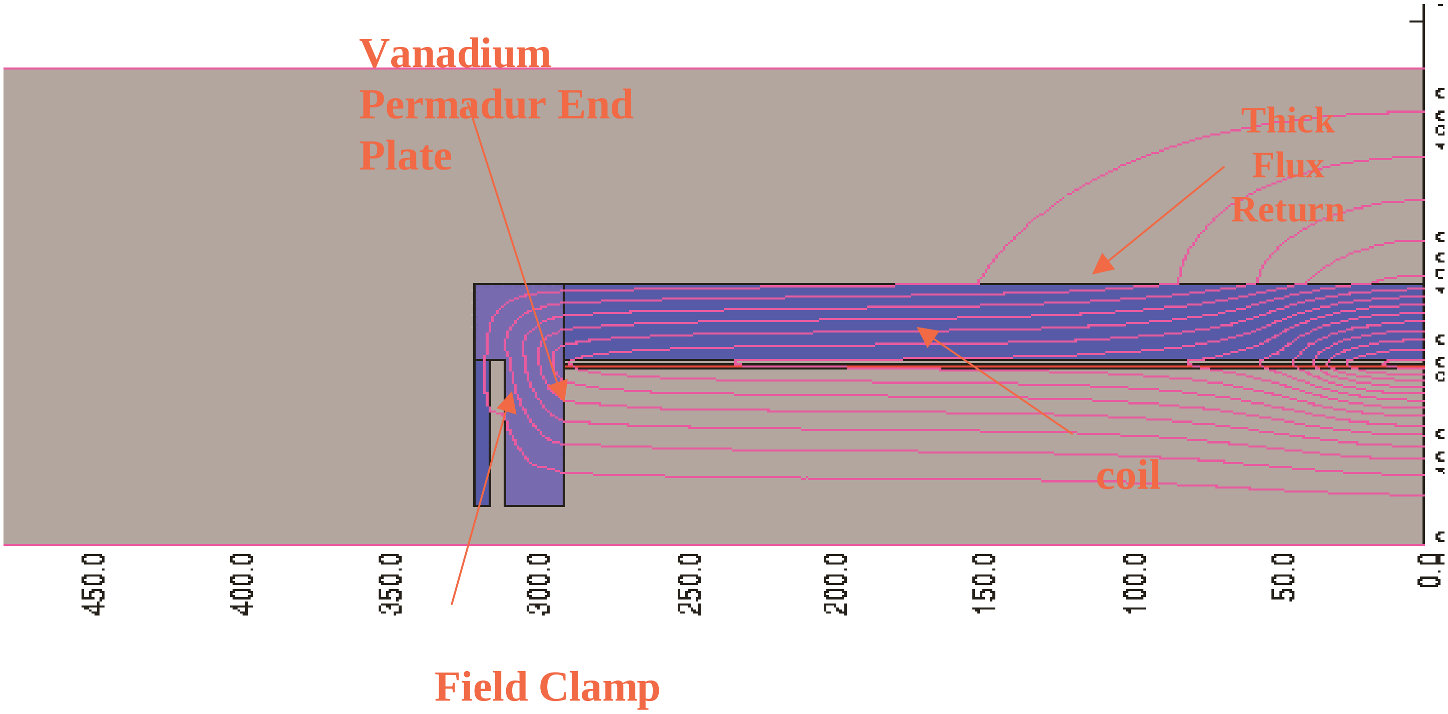


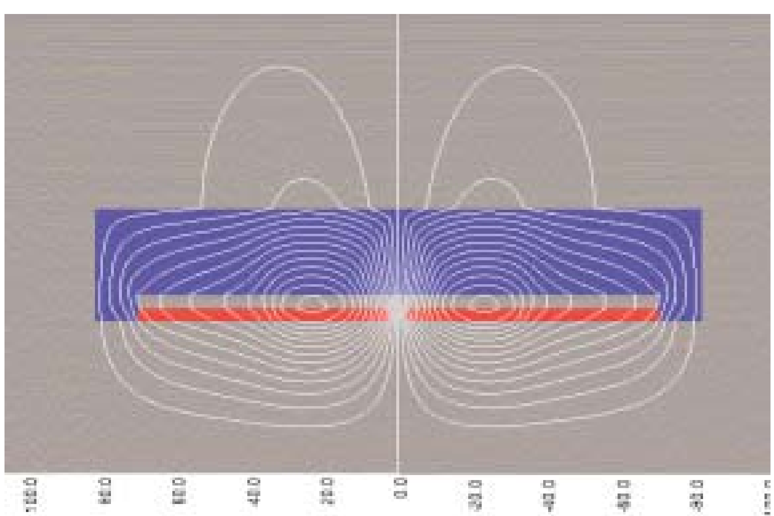
Pole face is shaped to achieve required gradient

The design of the wedge magnet is from P. Schwandt (dated 30 Jan 01).

It has been revised.

Long and Short Solenoid Magnets





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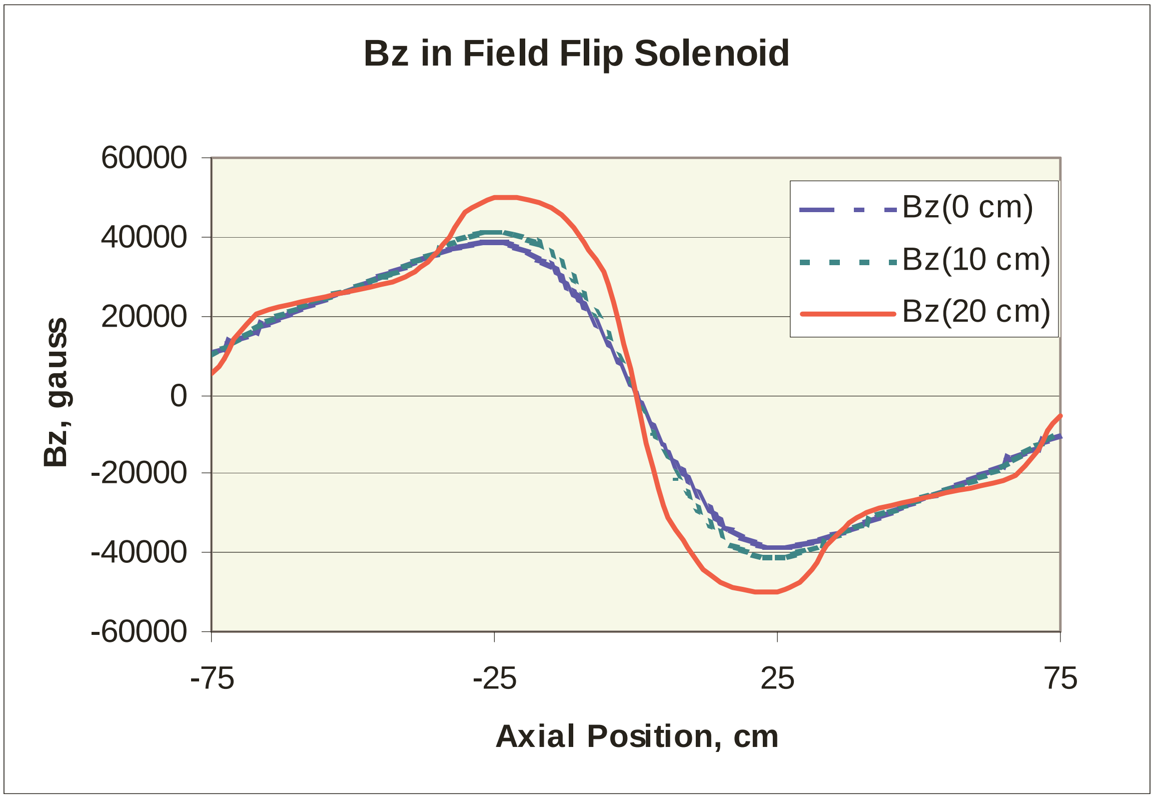
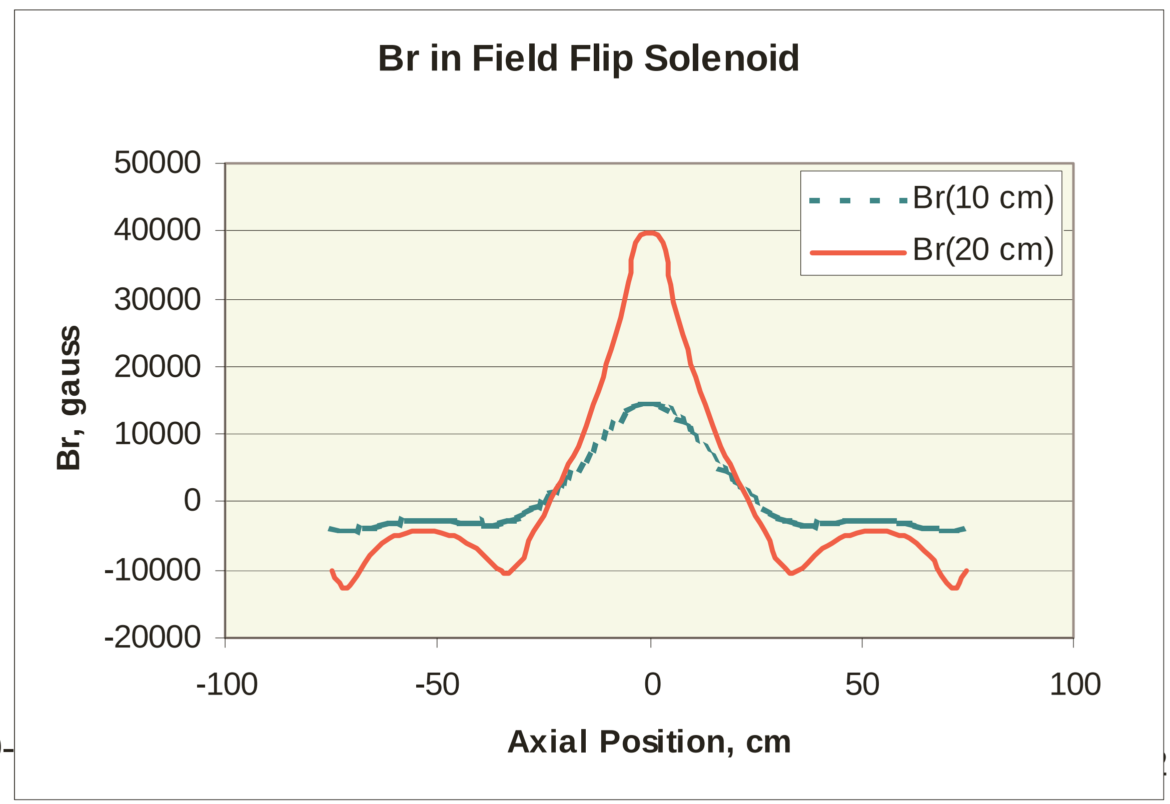
Short Solenoid Fields

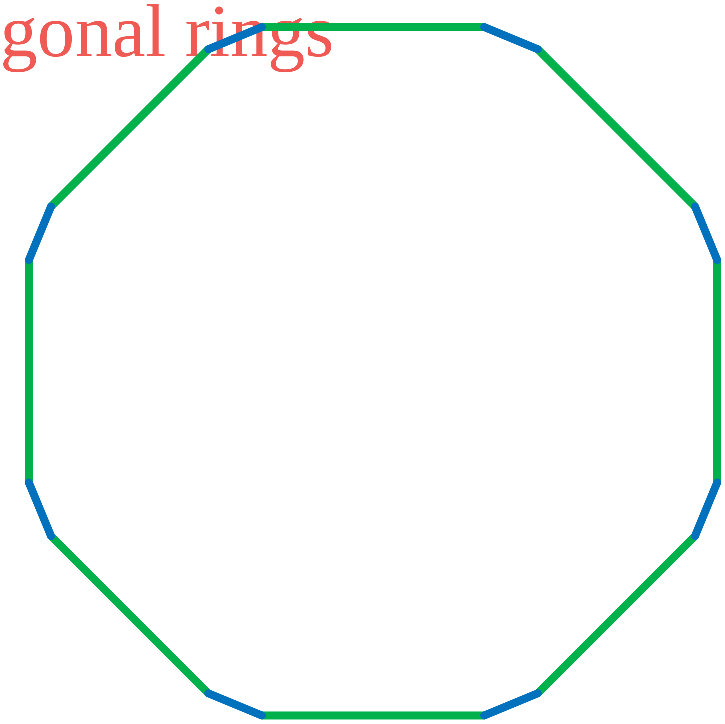




Injection/Extraction into Rings

Balbekov Octagonal Configuration

- Octagonal rings



Balbekov Snake Configuration

Octagon vs Rectangular coolers

- Comparison of beam parameters at cooling in the rings

- A - 2 tetragonal compressors (baseline design). The beam passes 40 cells (10 turns) in each compressor.

- B - Octagonal compressor. 160 cells = 20 turns. No Wedge

No	of cells	Tr.emit. (cm)	Ln.emit. (cm)	6D emit (cm^3)	Transm w/o decay	Transm. w/decay	Merit Factor
A	80	0.8	2.1	1.4	0.48	0.28	12
C	160	0.69	2.1	1	0.82	0.33	20.2

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Snake results.

Comparison of 'ring'(A) and 'snake'(B) coolers

A - 3 rings system:

40 cells (10 turns) in 1st ring, the same in 2nd,

60 cells (15 turns) in 3rd.

B - snake system:

40 cells from 1st ring cooler, 40 in 2nd,

60 cells in 3rd.

Angle of snake=18 degrees

	Stage	Tr.emit. (cm)	Ln.emit. (cm)	6D emit. (cm^3)	Transm. w/o dec.	Transm. w/decay	Merit factor
A	Inj	1.2	43	61.9	1	1	1
A	1st	1	10.1	10.1	0.64	0.49	3
A	2nd	0.8	2.11	1.35	0.48	0.28	12.8
A	3rd	0.21	0.55	0.024	0.37	0.15	386
B	Inj	1.22	43	64	1	1	1
B	1st	1.06	13.2	14.8	0.82	0.62	2.68
B	2nd	0.76	2.81	1.62	0.7	0.41	16.2
B	3rd	0.19	0.58	0.021	0.61	0.24	730

Neutrino factory

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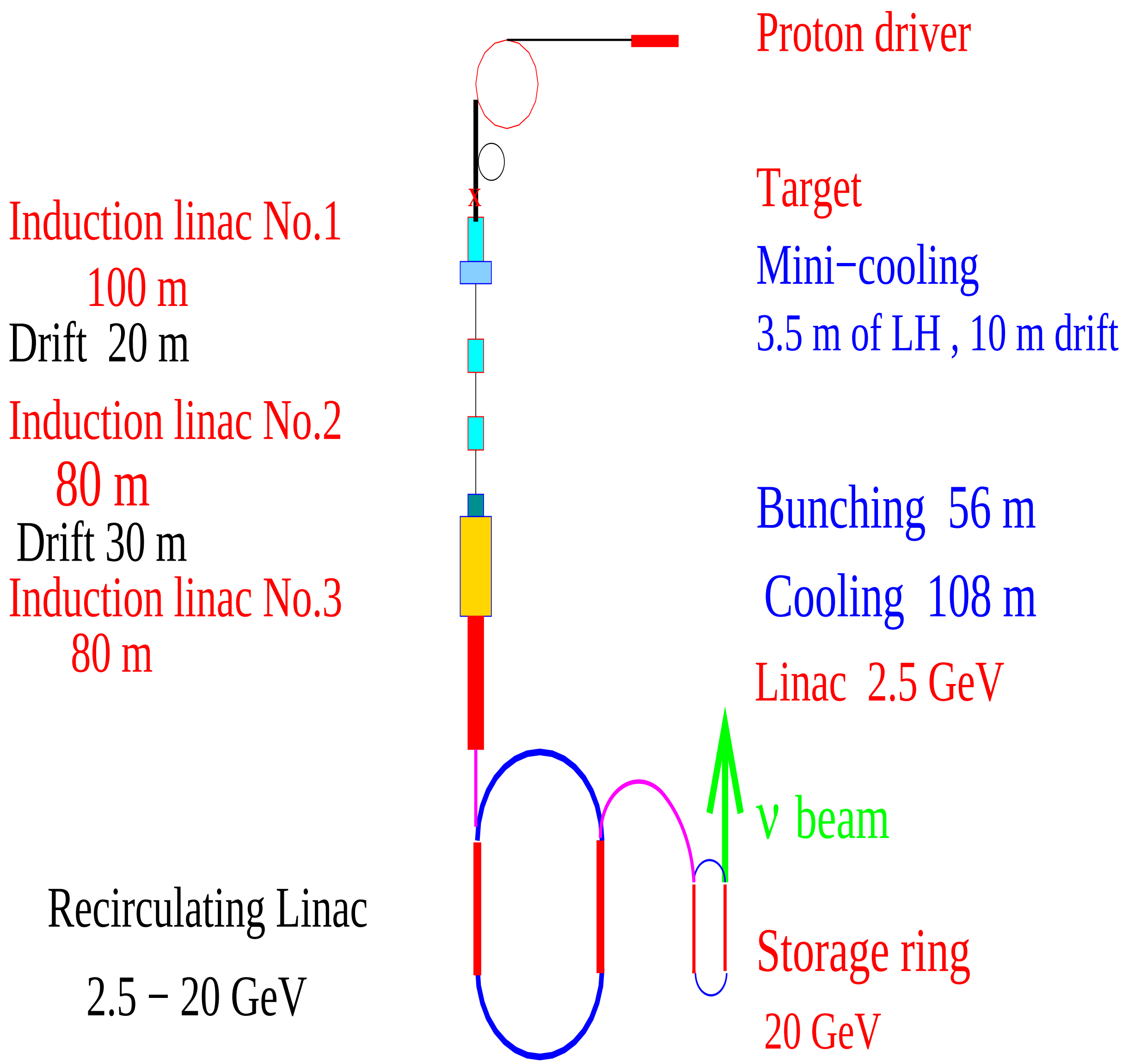
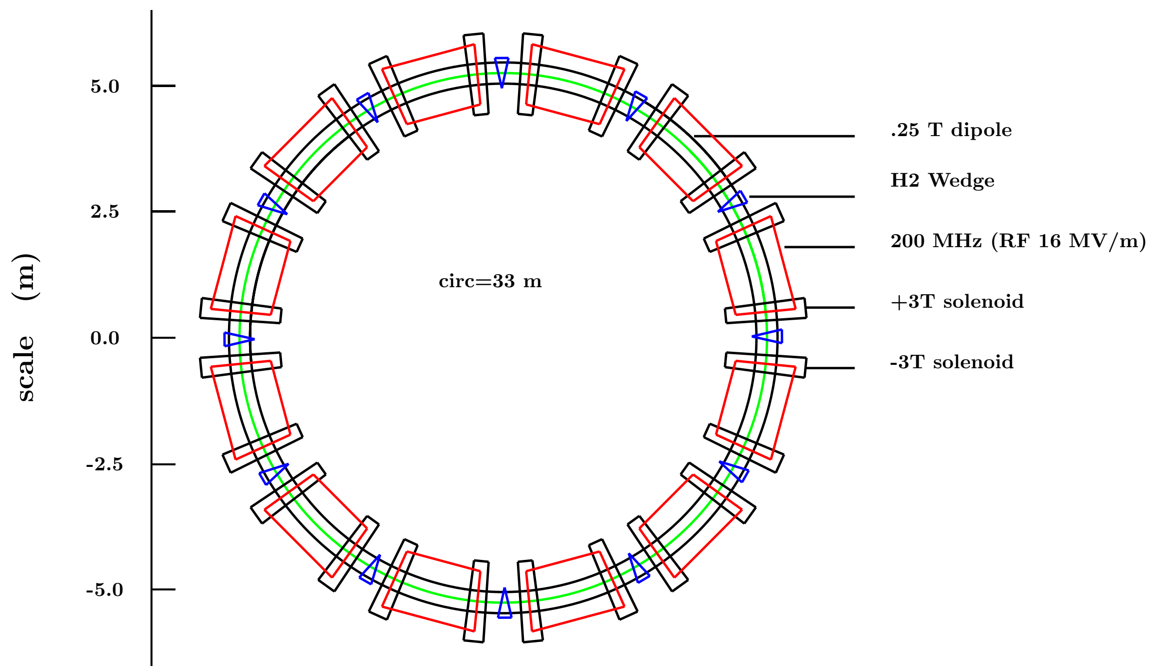


Bent Solenoids and rFoFo rings

Simulations of RFOFO Ring Emittance Exchange

R.B. Palmer R. Fernow S. Berg
(Oct 01 LBL)



An upward spiral would solve injection/extraction problems and allow tappering of beta function to give continuous cooling

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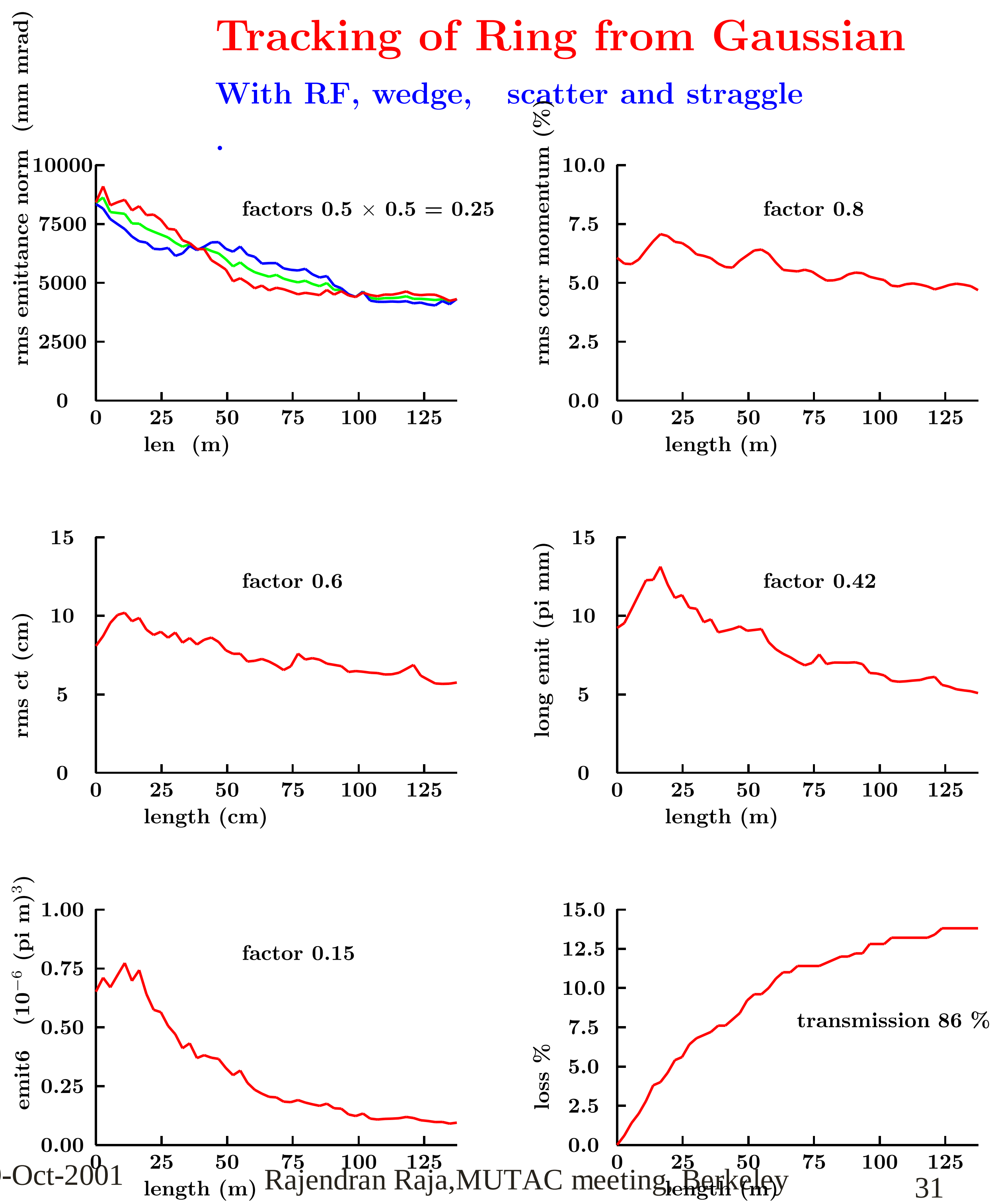
Bent Solenoids and Sfofo rings

Tracking of Ring from Gaussian

With RF, wedge, scatter and straggle



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Bent Solenoids and Sfofo rings

Summary

- **Longitudinal emittance** $\times$ 0.42
- **Transverse xy emittance** $\times (.5)^2 = 0.25$
- **6D emittance** $\times$ 0.15
- **Transmission 86 %**
- **Quality Factor** $(\epsilon_{in}/\epsilon_{out} \times$ Trans $=$ 5.7

Conclusions

- **There have been problems in tracking solenoid channels with bends in ICOOL, and several of them have been fixed;**
  - **Interpolation of tables of axial fields**
  - **Adding a missing term in the third order expansion**
  - **Allowing axial fields to be specified by Fourier components, thus elliminating discontinuities between cell fields.**
- **But the derivation of off axis fields is still limited to only 3rd order in radius (7th order is used in systems without bends), and this may still be causing difficulties.**
- **Using these third order calculations, 6D emittance cooling was achieved:**
- **Requires optimization, but should significantly improve study 2 performance.**
- **This, and the bend solenoid simulations, are the only emittance exchange demonstations with tracking in full Maxwellian focus and RF fields.**

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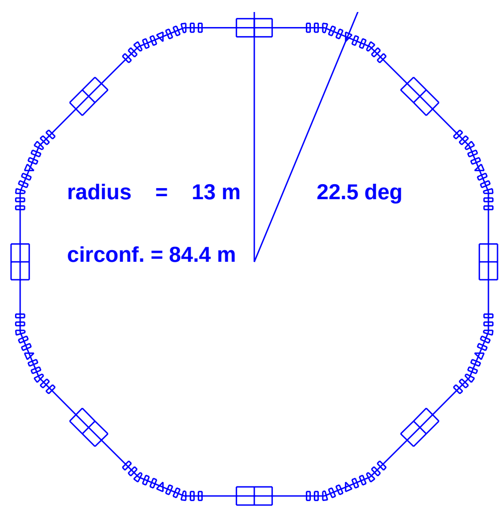
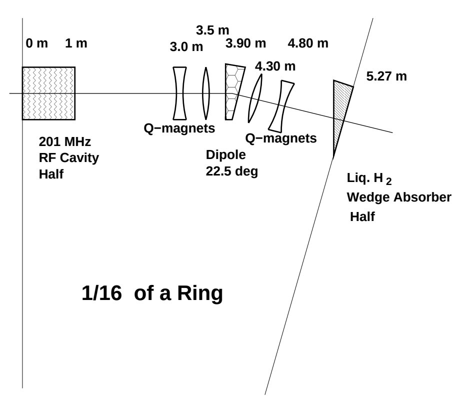
Quadrupole rings

- Fukui,Cline,Garren,He,Kirk,Mills
- Easier to inject in and out of.

Overall Layout of the Muon Ring Cooler



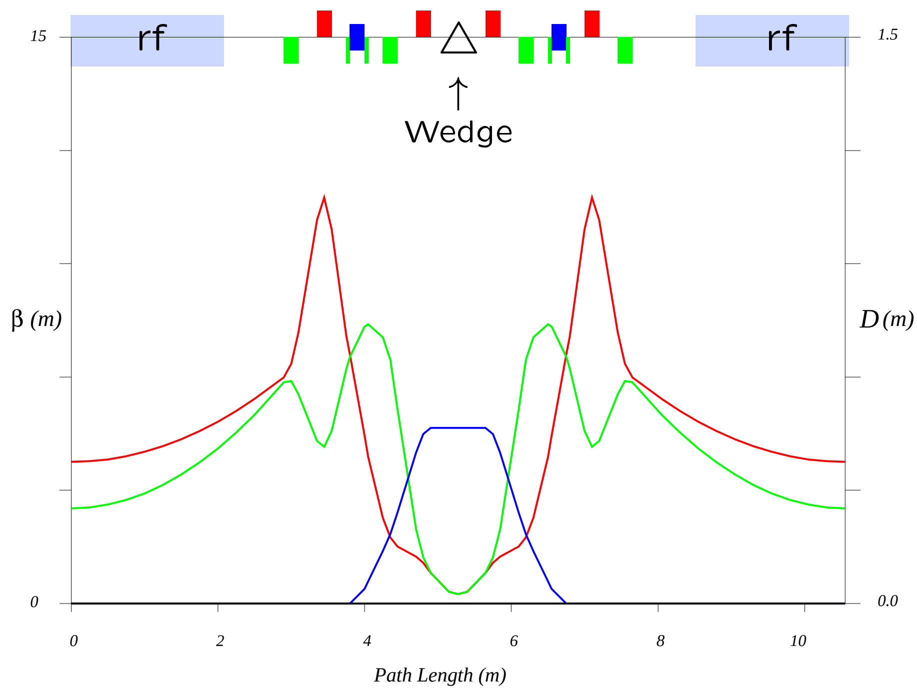


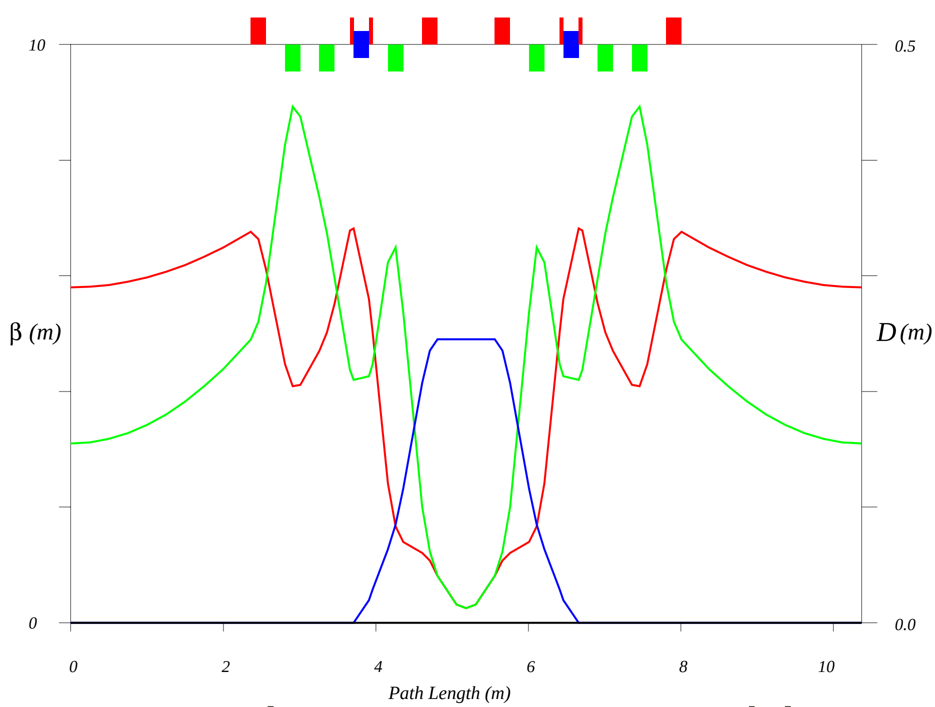
Half Section of Bending Cell

Quadrupole rings/Old and new lattice

Muon Cooling Ring

Bending Cell





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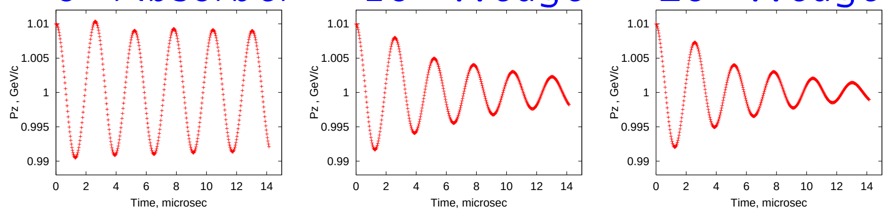
Cooling Ring: No Stochastic Processes

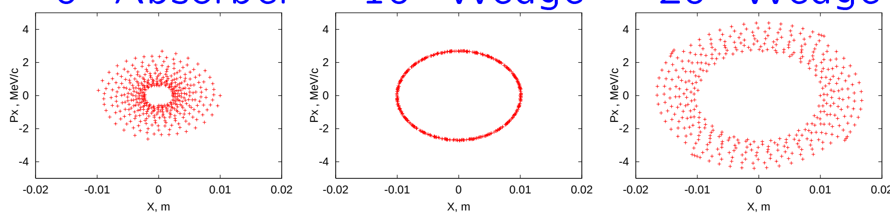
Quadrupole ring

0° Absorber 10° Wedge 20° Wedge

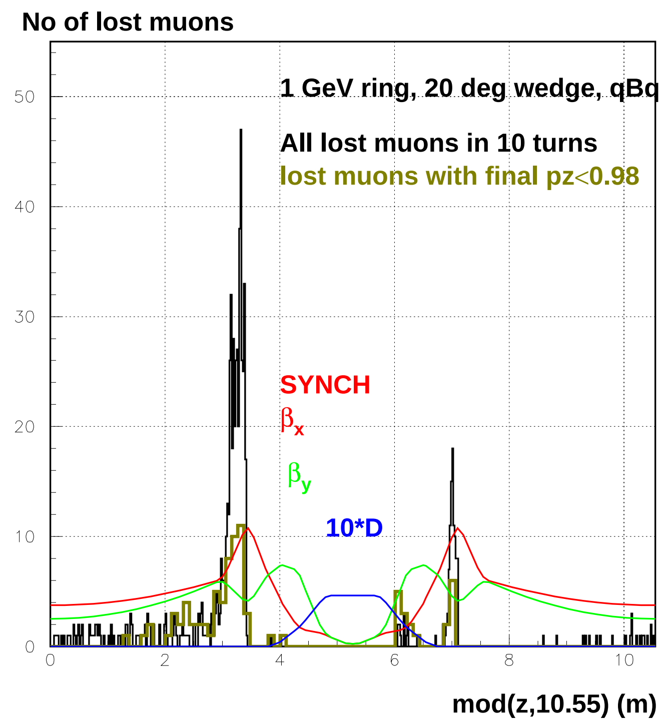


Longitudinal Momentum

0° Absorber 10° Wedge 20° Wedge



Horizontal Phase-space

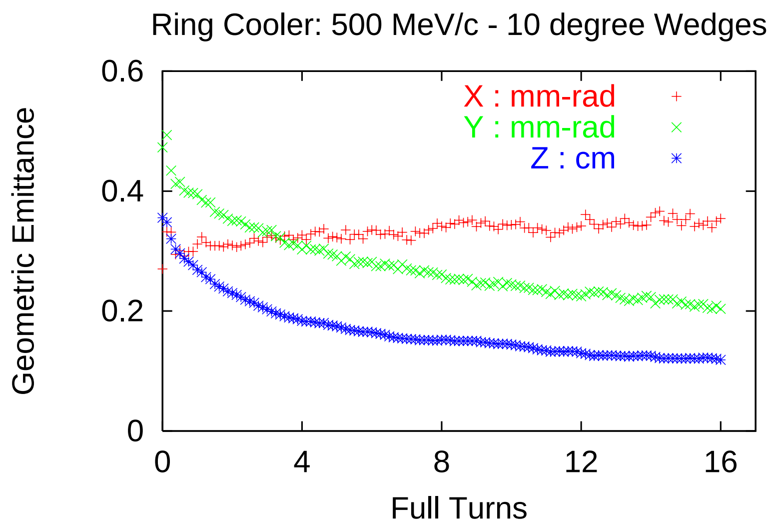


Quadrupole Ring

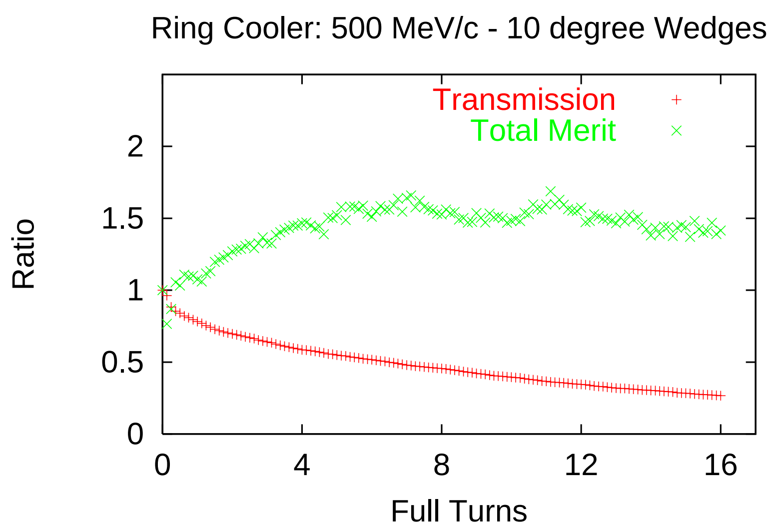
500 MeV/c Cooling Ring: 10° Wedges

With Multiple Coulomb Scattering and Straggling

$$\gamma\beta\epsilon_{x_i} = 1.3mm;\ \gamma\beta\epsilon_{y_i} = 2.2mm\ ;\ \gamma\beta\epsilon_{z_i} = 16mm$$



$$\text{Total Merit} = \text{Transmission} \times \frac{\epsilon_{x_i}}{\epsilon_{x_f}}\frac{\epsilon_{y_i}}{\epsilon_{y_f}}\frac{\epsilon_{z_i}}{\epsilon_{z_f}}$$



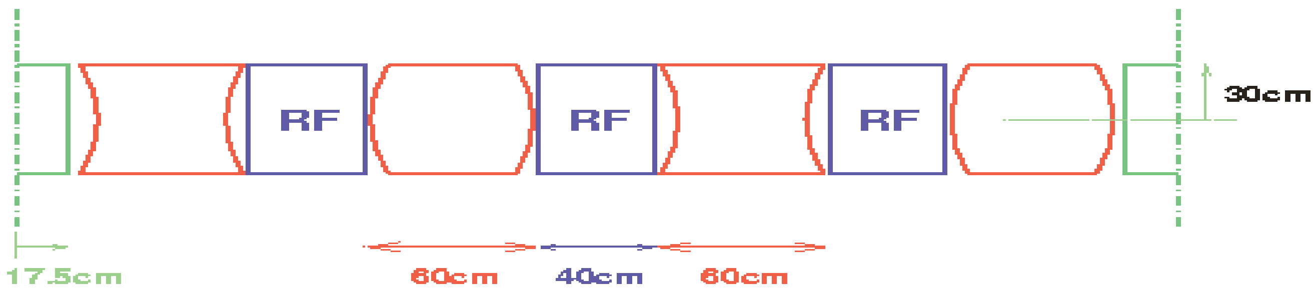
Quadrupole Ring

Summary

- SYNCH $\rightarrow$ ICOOL strategy works
- Emittance exchange demonstrated
- Transmission losses need study
- Need to consider realistic quads
- Need to incorporate soft edges

Quadrupole Cooling Channel

- Berz,Errede,Johnstone,Makino
- Can perhaps inject into Quadrupole Ring cooler. Use COSY to simulate.



- Muons (180MeV/c to 245MeV/c)
- Magnetic Quadrupoles (k=2.88)
- Liquid H Absorber: -dE/dx = -12MeV/35cm
- Cavities: Energy gain +12MeV/Cell to compensate the loss i the absorber

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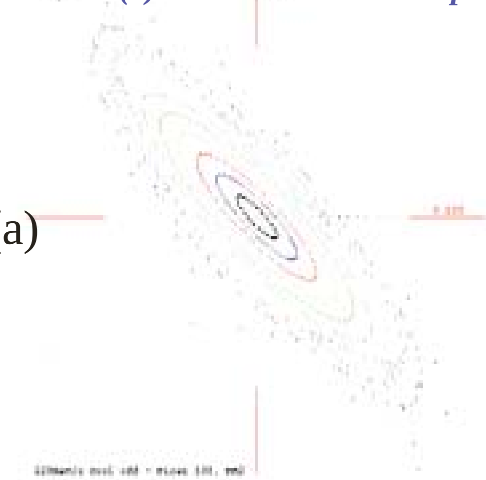
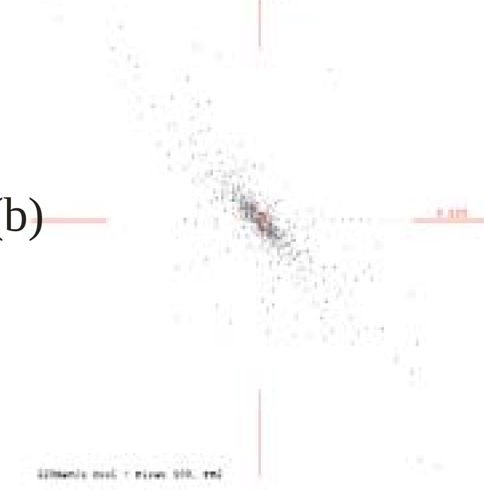
Tracking the Quad Cooling Cells

Momentum: 220 MeV/c, Starting from x=2cm,4cm,…,30cm, for 100 Cells
(a) Without Cooling (b) With Cooling (no scattering)
(c) With Cooling and Scattering
(d) Pseudo-Invariant Ellipses with Cooling (damping factor corrected)

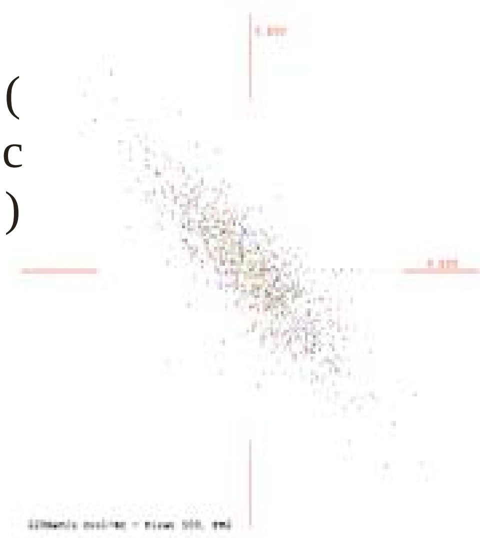
(a)



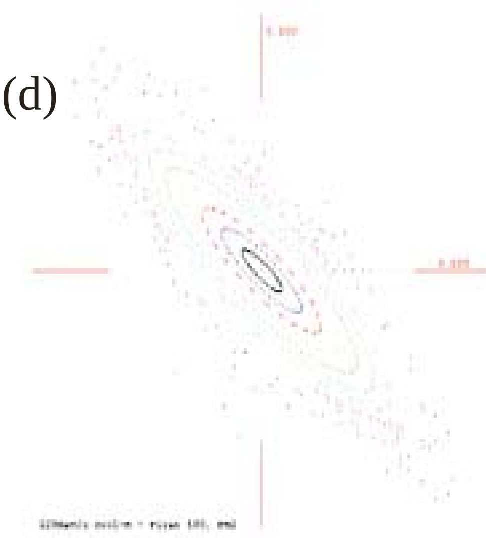
(b)



(c)



(d)



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Summary of Rest of Workshop

- 26 People attended. Productive Workshop.
- RF manipulations-S.Koscielniak
- Frictional Cooling(R.Galea)
- Helical Coolers(G.Penn)
- Code Management Issues
- Merit Factors discussion

Work to do

- Work out injection schemes into ring coolers
- Investigate Spirals as opposed to rings. Spirals can have a changing, tapered lattice as they spiral up and cooling proceeds.
- Put in realistic field and complete Geant simulations for tetragonal, octagonal and snake coolers
- Develop Quadrupole and rfofo rings further.
- Lithium lens coolers?
- Acquire more simulation manpower

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Conclusions

- Analytic theory has been developed (Including solenoids, wedges, foils). This provides a solid basis for design.
- Linear Analysis has been developed and applied. (Using codes such as Synch). Thus a cooling structure can be designed in linear approximation.
- Non-Linear Analysis has been done using tracking and transformations. (ICOOL, GEANT, COSY). Thus a structure's dynamic aperture can be determined, as well as its behavior when stochastic processes are included.
- Further tool development and simulations are needed and are underway. Nevertheless, with tools we already have in hand have allowed us to make significant progress.
- Two separate problems have been studied and significant advances have been made on both.
- 1)Improving the performance of a Neutrino Factory. The best simulation of a FOFO ring cooler to date, achieved at the workshop, has a dipole field over every thing, (and some other non-realistic but hopefully unimportant approximations) and taking the beam after phase rotation and bunching section gives cooling in all three dimensions. It suggests that we might improve the neutrino factory performance by as much as a factor of two.

Conclusions

- 2)Towards a Muon Collider- The best simulation to date takes the beam just after the low frequency phase rotation, as in the Status Report, (where there is one bunch of length ~7 meters and an energy spread ~150MeV). With two (possibly one) ring , (with some unrealistic features such as hard edge magnets, soon to be removed), there is cooling in all 3 dimensions with the final longitudinal emittance lower than is needed for a low energy collider. The transverse emittance is not yet what is needed, but experience suggests that a channel with lithium lenses (not yet designed) would be adequate to reach collider requirements.
- Further work is needed, and will be done, on 6D cooling, with rings, spirals, snakes. Injecting into rings needs further study. But we are making good progress and can reasonably be expected to soon have designs on paper that do the two jobs. Subsequently, we will need to do serious engineering and cost analysis, but the first task is to have a design on paper and it is not unduly optimistic to expect that soon.