FODO-based Quadrupole Cooling Channel

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Quadrupole Cooling Channels: General Considerations

- Longitudinal and and Transverse Acceptance of Channel
  - Compare acceptances of different optical structures
- Insertion of Absorber
  - Location of minimal beam size, both planes
  - Calculate equilibrium emittance limit
- Physical Limitations
  - Quadrupole aperture and length constraints
  - Available space between magnets
- Match to emittance-exchange channel
  - Can same optical structure be used for emittance exchange

Choice of Optical Structure

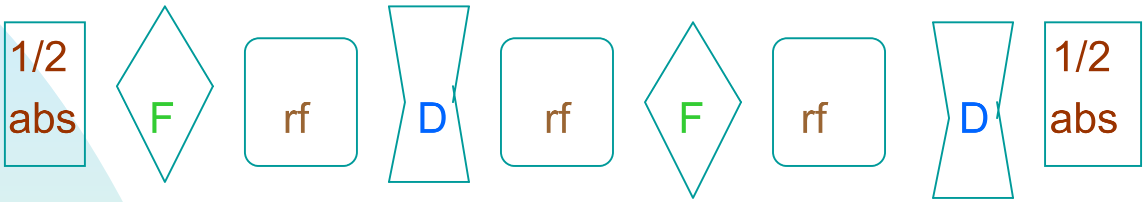
- FODO
  - Simplest: alternating focussing and defocussing (in one transverse plane) lenses
  - A minimum in beta or beam size cannot be achieved simultaneously in both planes
- Doublet or Triplet quadrupole system
  - 2 or 3 consecutive, alternating focussing and defocussing quadrupoles
  - required to form simultaneous low beta points in both planes (interaction regions of colliders, for example)

Acceptance of Quadrupole Channels

- Transverse Acceptance
  - Using only linear elements (quadrupoles and/or dipoles), the transverse dynamic aperture is normally larger than the physical aperture (unless a strong resonance is encountered in a long series of cooling cells).
  - Practically, FODO and doublet/triplet quadrupole channels have transverse acceptances or apertures limited only by poletip strength for a given gradient (≤8T for superconducting quads and ≤2T for normal conducting).
- Longitudinal Acceptance
  - The FODO cell is a simple lens system and has the largest chromatic acceptance of any quadrupole-based structure. Lattices based on FODO cells have been designed which transmit up to a factor of 4 change in momentum.
  - Doublet/triplet-based quadrupole structures are momentum limited to approximately ≤5% deviation from the central momentum of the channel.
- Logistics
  - Because the FODO cell cannot achieve a minimum beta point in both planes, its valid application is just after capture and phase rotation, where the transverse and longitudinal emittances are very large.
  - Conversely, the limited momentum acceptance of the triplet/doublet quadrupole channels restrict their implementation to after emittance exchange has occurred.

⇨ The rest of this talk will discuss the FODO cooling channel only

Construction of FODO Quad Cooling Cell



COOLING CELL PHYSICAL PARAMETERS:

Quad Length	0.6 m
Quad bore	0.6 m
Poletip Field	~1 T
Interquad space	0.4 - 0.5 m
Absorber length	0.35 m *
RF cavity length	0.4 - 0.7 m*
Total cooling cell length	2 m

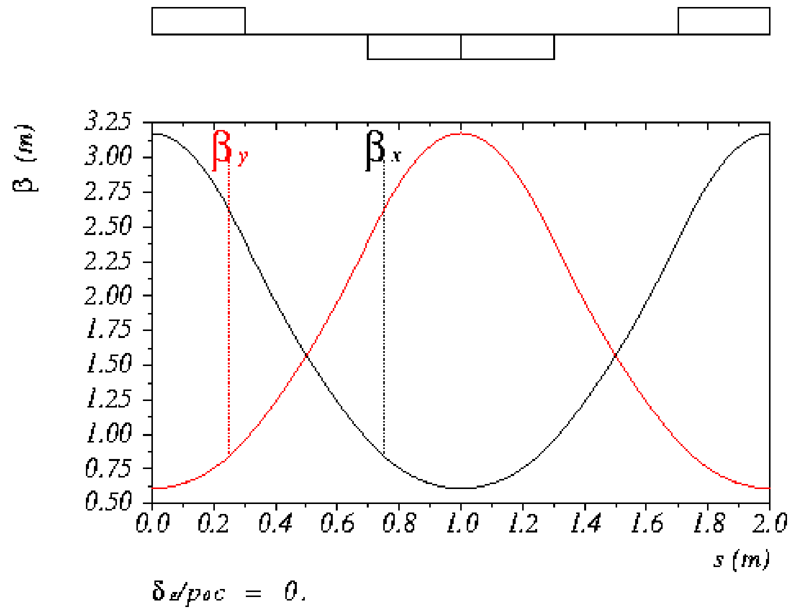
*The absorber and the rf cavity can be made longer if allowed to extend into the ends of the magnets.

Or, more rf can be added by inserting another FODO cell between absorbers

In this design

→ For applications further upstream at larger emittances, this channel can support a 0.8 m bore, 0.8 m long quadrupole with no intervening drift without matching to the channel described here.

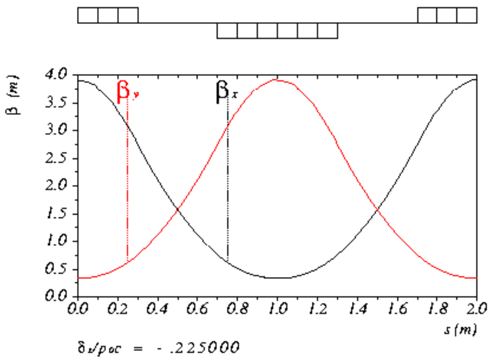
FODO LATTICE AND INSERTION OF ABSORBER

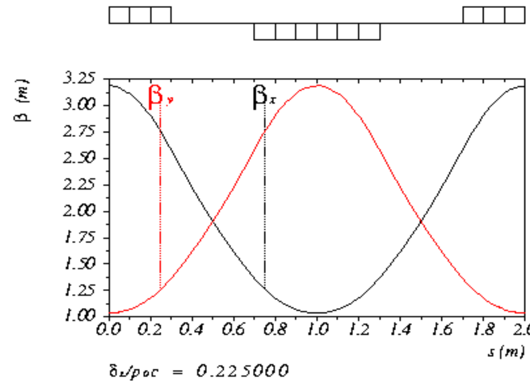


LATTICE FUNCTIONS OF A FODO CELL FOR A QUAD COOLING CHANNEL, P0=200 MeV/c

- In a FODO cell, the combined minimum for βx and βy is at their crossing point, halfway between quadrupoles.
- The achievable $\beta_{transverse}$ in the absorber is about 1.6 m, or a factor of 4 larger than the average transverse β in the 2.75 m SFOFO channel (0.4 m).
- This channel can accept a 3σ beam size for a transverse, ε_N(rms) of ~20 mm-rad at a p_0 of 200 MeV/c.

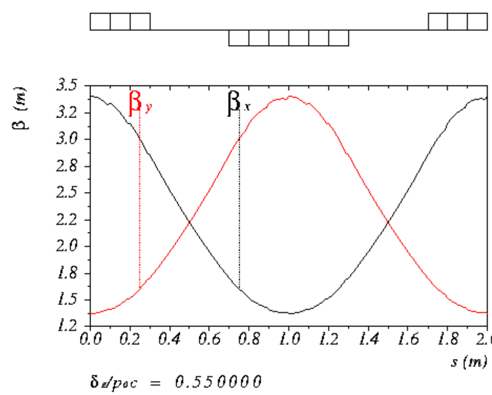
Longitudinal Acceptance and Lattice Stability at Different Momenta





LATTICE FUNCTIONS for 155, 245, and 300 MeV/c, clockwise.

- βaverage at the absorber ranges from 1.57 to 1.90 at 155 and 245 MeV/c, respectively, which represents the momentum range of the 2.75 m SFOFO channel.
- βmax is 3.90 m @155 MeV/c and 3.19 m @245 MeV./c
- The acceptance reach of this channel is clearly larger than 300 MeV/c; i.e. βavg is still only 2.3 m. and βmax is 3.5 m



Fodo Cell Properties as a Function of Momentum

P MeV/c	β at absorber (m)	ν per FODO cell	β max	β min
155	1.57 m	.33	3.85	0.35
200	1.63	.23	3.13	0.71
245	1.91 m	.18	3.20	1.10
300	2.28 m	.15	3.45	1.37

Equilibrium Stability Limits of the FODO Transverse Cooling Channel

The equilibrium emittance is given by

$$\varepsilon_{N,min} = \frac{\beta_\perp (14\ \mathrm{MeV})^2}{(2\beta m_\mu L_R \cdot dE/ds)}$$

where $\beta_\perp$ is the transverse beta function at the absorber, β the relativistic velocity, m_μ the mass of the muon, L_R the radiation length of the absorber material, and dE/ds the energy lost in the absorber.

The equilibrium emittance for this quad channel is 6.8 mm-rad @200 MeV/c ($\beta_\perp$~1.6 m). This is to be compared to an initial rms acceptance of 20 mm-rad.

The equilibrium emittance for the 2.75 m solenoidal channel is 1.7 mm-rad @200 MeV/c ($\beta_\perp$~0.4 m). This is to be compared to the initial rms acceptance of 12 mm-rad.

Preliminary Tracking Studies of the FODO quad cooling channel

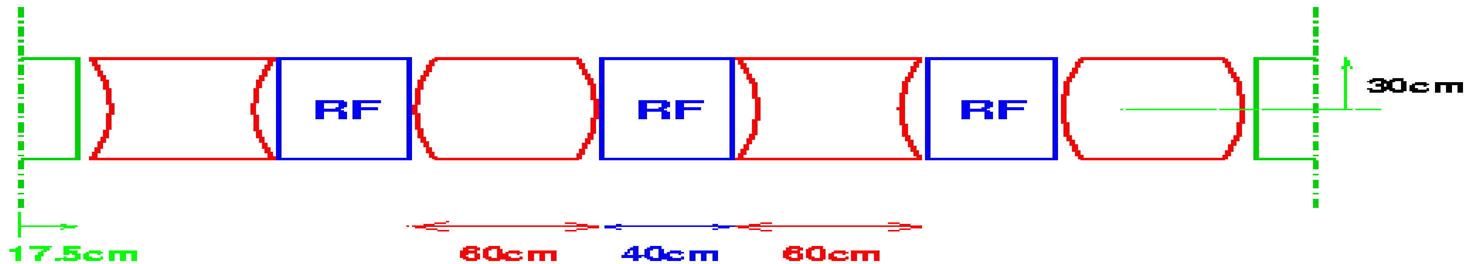
Tracking Studies were performed:

- with full nonlinear terms
- with/without quadrupole fringe fields
- with multiple scattering

Preliminary estimates of cooling were obtained by:

1. determining the invariant phase ellipse of the quad channel
2. tracking in 1 cm steps along the x axis to determine the dynamic aperture of the channel
3. particles were then launched on the outer stable invariant ellipse at various x,x' coordinates and on one inner phase ellipse near the calculated equilibrium emittance
4. particle positions were plotted for 10 cells, but at increasing distance down the length of the cooling channel (cells 21-30, 31-40, 101-110, for example) until the cooling converged.
5. the rough cooling factor was obtained by comparing the outermost stable ellipse with the final ellipse which clearly contained the majority of the particles.

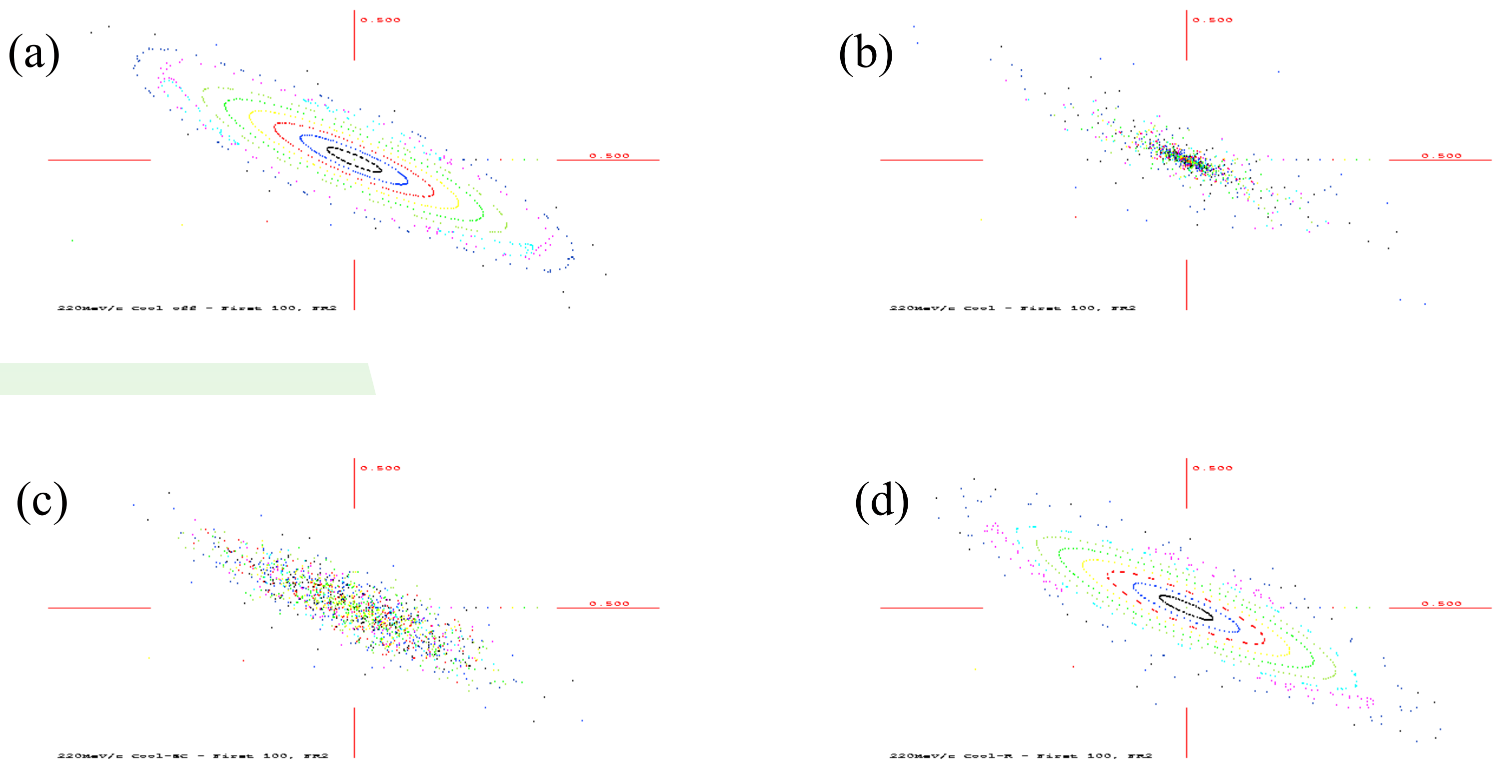
Quad Cooling Channel Simulation
by COSY Infinity



- Muons (180MeV/c to 245MeV/c)
- Magnetic Quadrupoles (k=2.88)
- Liquid H Absorber: -dE/dx = -12MeV/35cm
- Cavities: Energy gain +12MeV/Cell to compensate the loss in the absorber

Tracking the Quad Cooling Cells

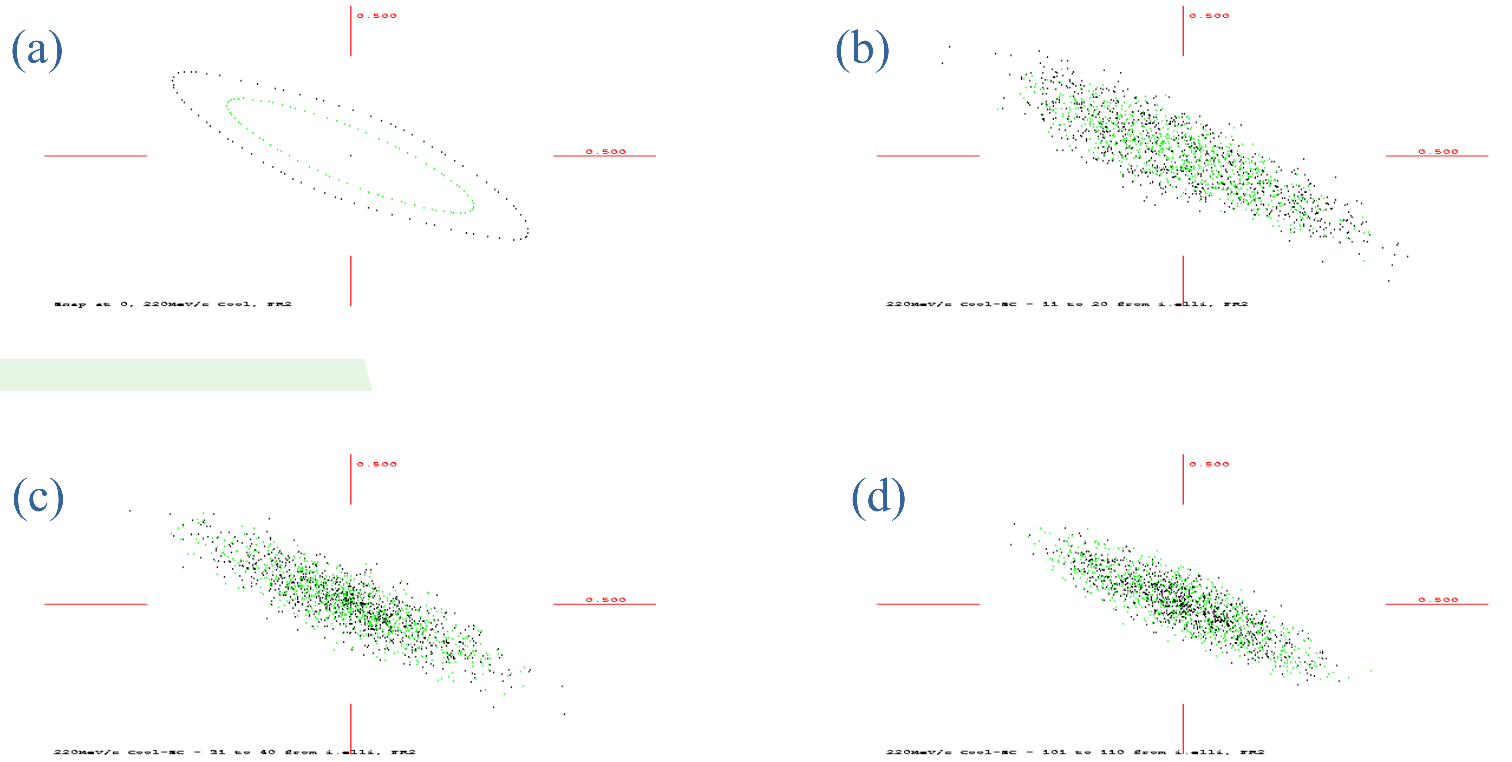
Momentum: 220 MeV/c, Starting from x=2cm,4cm,…,30cm, for 100 Cells
(a) Without Cooling (b) With Cooling (no scattering)
(c) With Cooling and Scattering
(d) Pseudo-Invariant Ellipses with Cooling (damping factor corrected)



Tracking the Quad Cooling Cells with Scattering

Momentum: 220 MeV/c, Starting from x=10cm, 15cm Pseudo-Invariant Ellipses

(a) Initial Ellipses (b) for 11-20 Cells (c) for 31-40 Cells (d) for 101-110 Cells



Preliminary Results

Approximately:

The quadrupole channel was found to cool from 12.5 cm to ≤8 cm on the invariant ellipses.

THIS corresponds to an initial emittance of 9.8 mm-rad (ε_N = 18.6 mm-rad) and a final emittance of 4.0 mm-rad (ε_N = 7.6 mm-rad).

→ THIS RESULTS SUPPORT A COOLING FACTOR OF 2.5

HOWEVER, the results are very preliminary

The Next Step is to Add:

- dE/ds as a function of energy
- straggling
- realistic rf bucket

Full Simulation now includes:

- dE/dx as a function of energy

  dE/dx curves have now been loaded as a function of energy into COSY; i.e. energy lost in each absorber depends on the particle's energy.

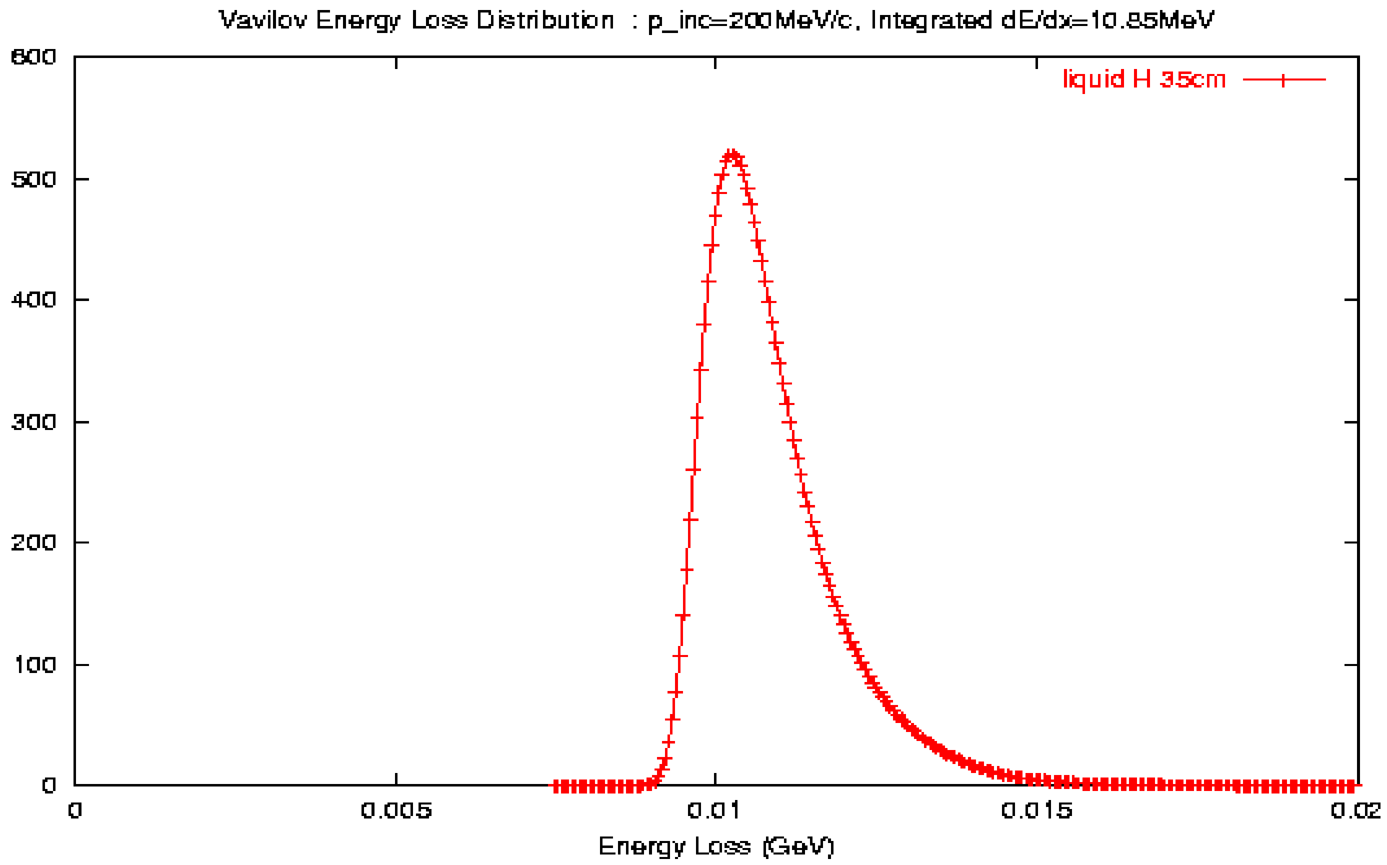
- Straggling

  A. Van Ginneken's energy loss function has been interfaced to COSY. Low energy tail of the straggling function is believed to be an important loss mechanism in the early channels AND the spin of the particle affects the average energy of the distribution. In this simulation the energy of the reference particle is calculated with full straggling and spin. The energy loss of other particles are calculated relative to this reference particle following the dE/dx curve.

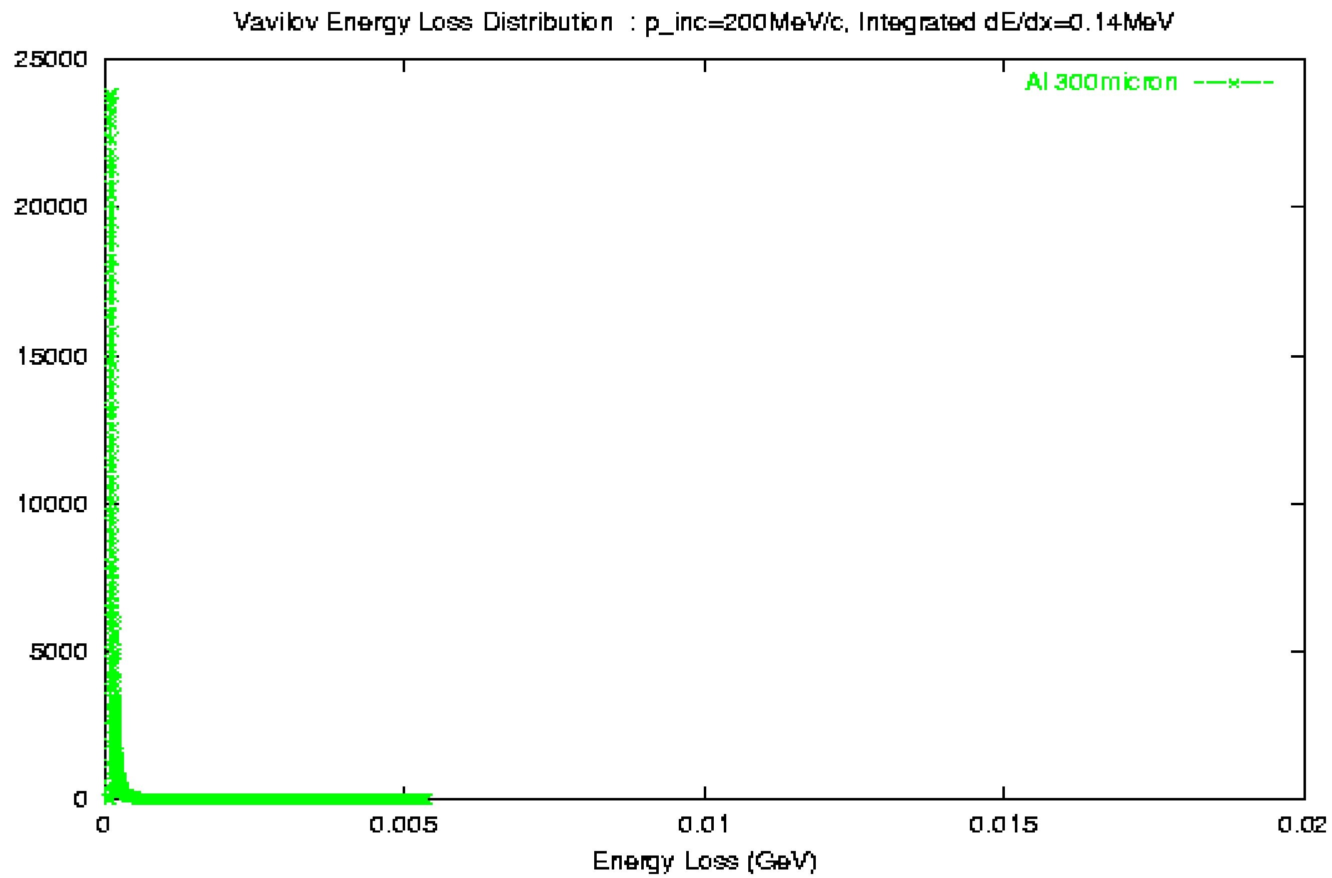
- rf bucket

  A sinusoidal 200-MHz rf waveform has been implemented assuming a gradient of 10MV/m.

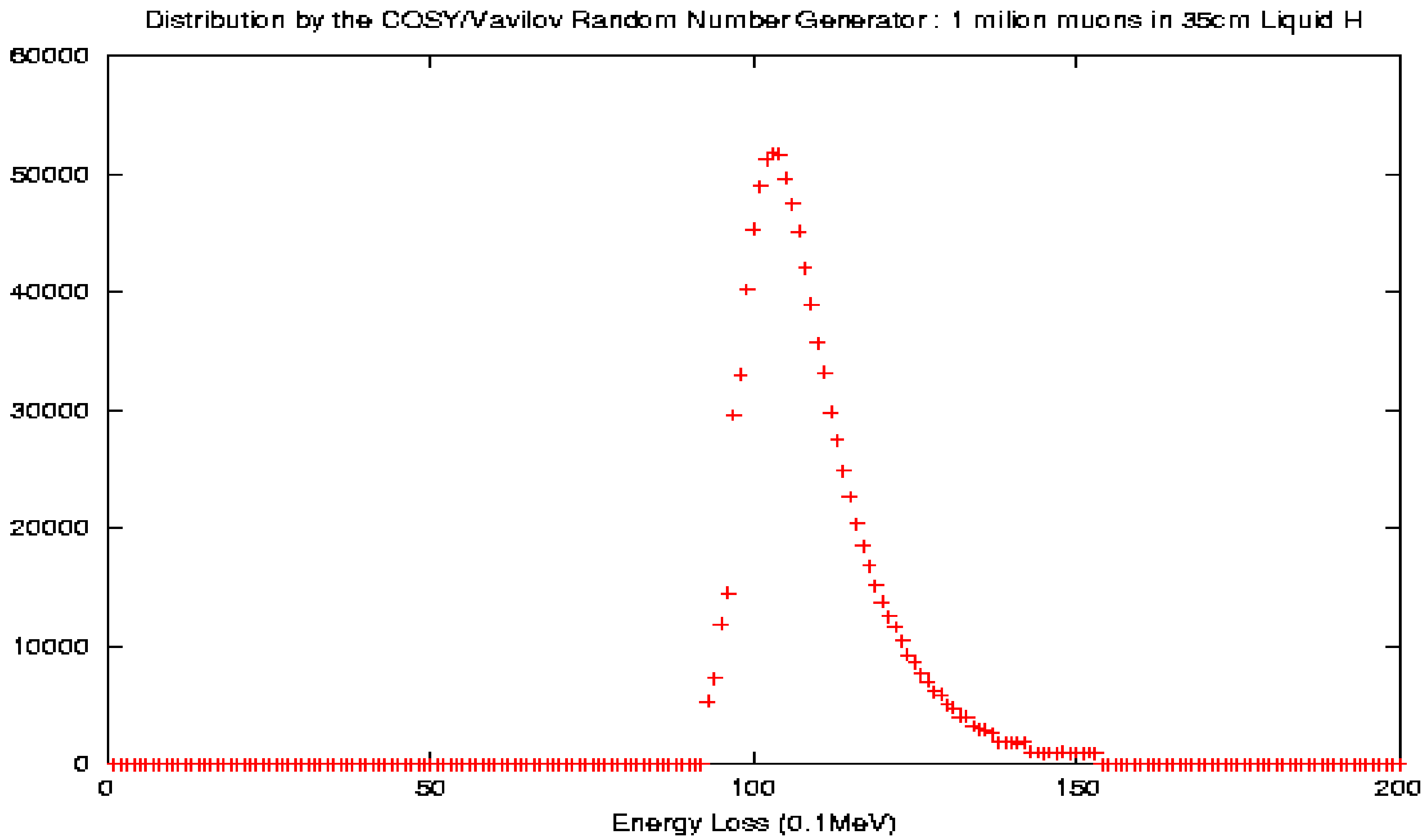
Straggling: Energy distribution after hydrogen absorber



Straggling: Energy distribution after Al window



Generation of particle energy distribution after absorber



Quantifying the Cooling: Description of a Merit Factor

TRANSVERSE

Load an elliptical distribution in x,x' which corresponds to the stable phase ellipses of a quad channel without cooling. The exact shape will be a Gaussian whos rms width is 1/3 the half aperture of the quadrupoles, which is 30cm (βmax =3.1 m, $\sigma \cong 20\pi$ mm-rad @p=200MeV/c). An initial to final rms ratio can be calculated for the transverse cooling factor.

LONGITUDINAL

Two cases will be loaded: one with little longitudinal loss, and one "filling" the bucket for comparison.

Minimal bucket loss

σ_E = 12 MeV; rms bunch length = 7.5 cm

ϕ_s = 60°; $\Delta\phi$ = ±54° for a 3σ distribution

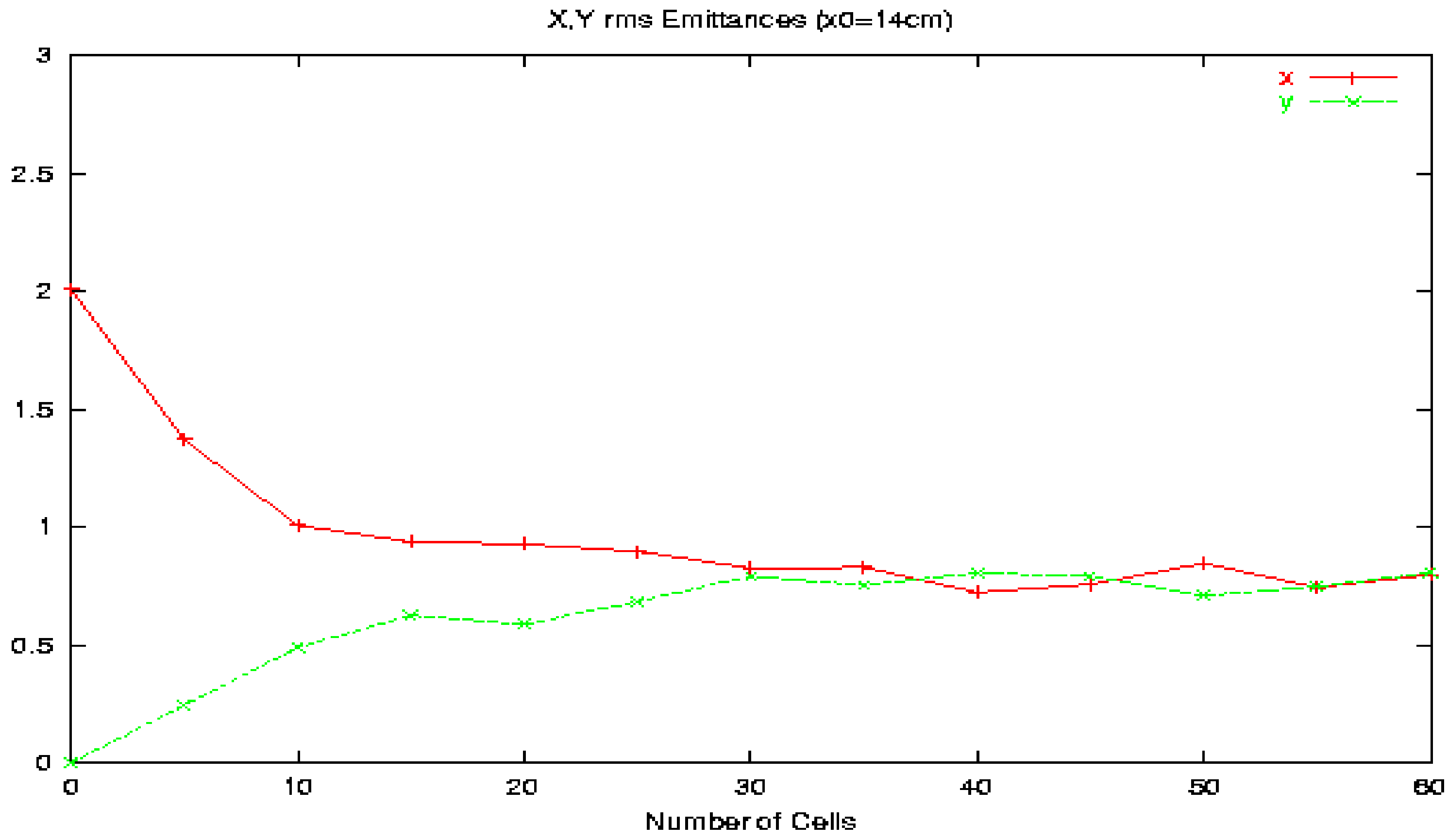
Maximul bucket loss

σ_E = 24 MeV; rms bunch length = 15 cm

ϕ_s = 60°; $\Delta\phi$ = ±108° for a 3σ distribution

Longitudinal and transverse losses will form the overall transmission factor. The product of the transmission and cooling factors combined with the decay losses will provide the merit factor for this emittance range.

Full simulation: transverse cooling along channel



Goals Achieved with a the FODO-based Quad Cooling Channel

- Cool transversely by a factor of 2 in the emittance of each plane: from an rms normalized emittance of about 20 mm-rad to 10 mm-rad.
- Inject cleanly (without matching) into an emittance exchange channel using the same FODO-based optical cell.
- Recool transversely with the same channel.
- After this channel, inject into more sophisticated cooling channels such as solenoidal ones to cool to the final required emittances

Future Design Studies:

- Move the central energy of the channel to the minimum in the total strength of the reheating terms (~300-400 MeV/c) by increasing the poletip strength of the quadrupoles (they are still normal conducting).
- Although this, in principle, halves the momentum spread, this quad channel will still accept more than a 40% total momentum bite due to its naturally high longitudinal acceptance.
- Compare cooling rates, final emittances, and losses at the two different momenta
- Load "exact" particle distributions from target/capture/buncher string
- Investigate superconducting rf and increased absorber lengths. (The quadrupole end fields fall much more quickly than solenoidal ones.)
- Evaluate other absorbers: He?, LiH (rotating drum target)