An Induction Kicker for Muon Cooling Rings

and other Applications Needing Very Large Apertures

DRAFT 4 2/1/02

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Abstract

The paper discusses the injection and extraction kicker requirements for the first cooling ring in a muon storage ring neutrino factory, or muon collider. It is shown that a kicker's current and single turn voltage are proportional to the normalized emittance of the beam; and that the stored energy is proportional to the square of that emittance. All three parameters are independent of the energy in the ring.

For a beam with $\epsilon_n = 10\ \ \pi$ mm, as in several current designs, the kicker energy and voltage are both far higher than in any conventional kicker. But a proposed 'induction kicker', powered by magnetic amplifiers similar to those in induction linacs, might meet the requirements.

1 Cooling Ring Parameters and required rise time

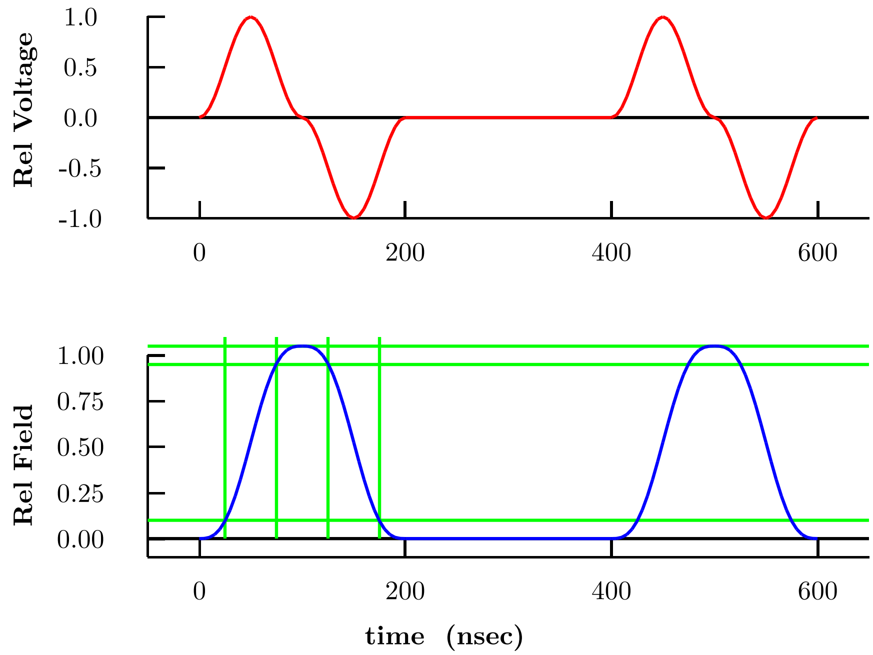
We will show below that the stored energy and other parameters of a kicker are strongly dependent on the transverse emittance of the beam. This being so, it is the first cooling ring that is the most difficult and will be considered here.

An attraction of cooling rings, as opposed to linear cooling, is the possible savings in total length, and thus in cost. But there will be no such reduction if the circumference of the ring is not significantly shorter than the linear cooling it replaces. With study 2 parameters, continuous cooling takes place in about 100 m. We may conclude that a cooling ring circumference should be small compared with this: say 30 m.

The rotation time in 30 m is about 100 nsec, and this must be divided between:

1. the length of the bunch, or bunch train being cooled;
2. the rise or fall time of the kicker pulse.

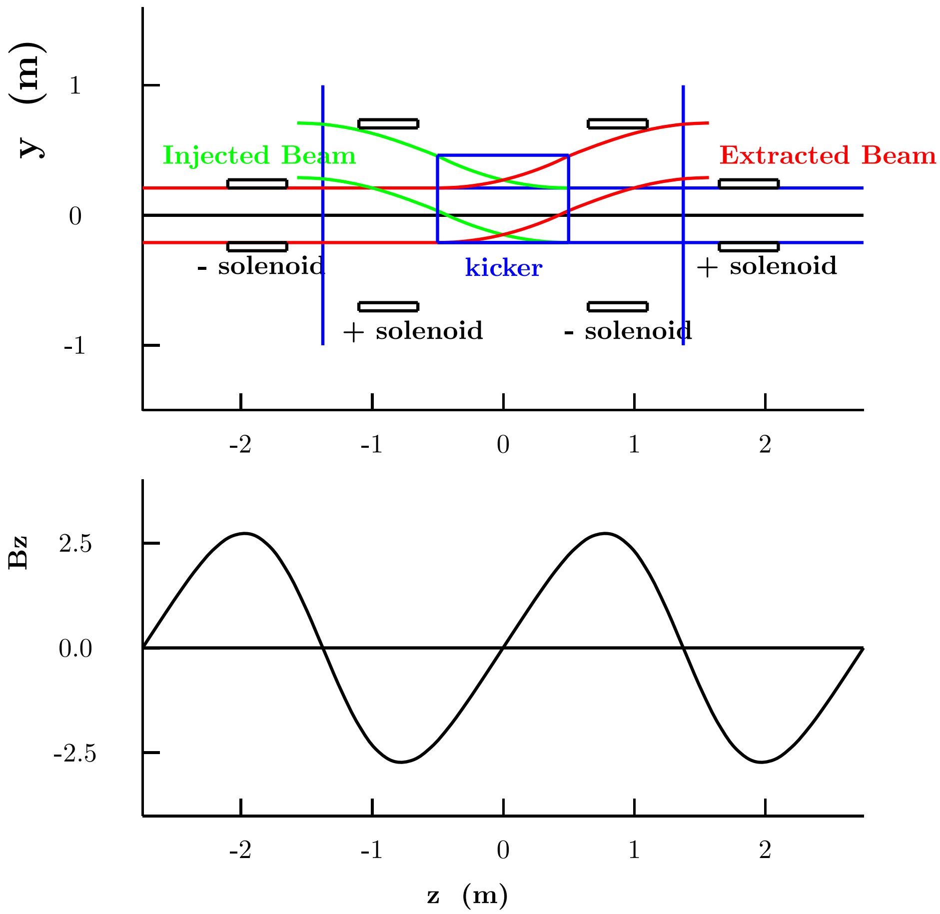
Let us consider dividing the time equally between them: 50 nsec each: a 50 nsec (12 m) bunch train and a field rise time of 50 nsec. Plausible magnetic and voltage pulse shapes are shown below. It is assumed that field errors of up to ± 5% are allowable, so the required rise in the 50 nsec is from 10 to 90 %. With this requirement, and a sin wave voltage pulse shape, then the maximum rate of field rise is seen to be approximately $dB/dt \approx B_{\max}/50nsec$. Symmetric shapes are shown that allow the same kicker to be used for injection and extraction (in a small ring, this is a significant efficiency advantage). The second extraction pulse is shown following 400 nsec later (4 turns), but might be required somewhat later.



2 extraction lattice

The following figures shows a conceptual layout of such a kicker located within the RFOFO lattice[1] of one possible cooling ring design. The standard cell axial field profile is also shown. Maintaining this profile in the extraction section avoids matching problems and is the best way of assuring good acceptance. The layout shown is similar to that proposed by Valeri Balbakov for his cooling ring.

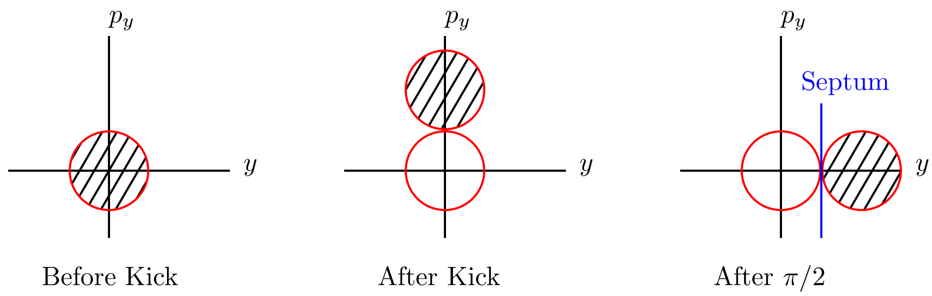
The $\beta_\perp$ in the center of the cell is approximately 1 m. The kicker should not be much longer than this. For a kicker length $L = \beta$ the increased height of the kicker, to accommodate the deflected beam, is approximately 50% and is rising as the square of the length.



3 Kickers

3.1 Introduction

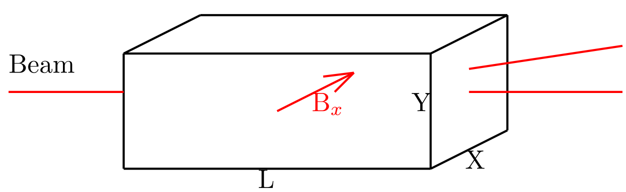
In order to kick a beam into or out of a ring we need to displace it's phase space so that it is separate from that of the stored beam, and jumps past a septum of some sort. This can be done in transverse momentum (with a conventional kicker) or longitudinal momentum (with acceleration as discussed by Neuffer) directions. The beam can then be manipulated by a transverse 90 deg. phase space advance, or with dispersion, so that the kicked beam is transversely displaced from the stored beam, and the septum can be introduced between them. This note will consider only transverse kickers.



If the momentum spread were small then the ejection kick might be distributed around the ring at n locations spaced by any numbers of half integers of the betatron tune. This, as suggested for FFAG injection and extraction at KEK, would reduce the required fields in the kickers. But the wide variations in phase advance as a function of the large momentum spread in an initial cooling ring makes this impractical in this case. The deflection must thus be given in a single location and must be given in a length of the order of, or smaller than, the betatron parameter $\beta_\perp$ (we assume $\beta_x = \beta_y$).

3.2 Formulae

Consider a kicker with horizontal field B_x, length L, height Y, and depth X.



With the transverse twiss parameter in the kick direction β_y, relativistic parameters $\beta\gamma$, normalized emittance ϵ_n (assumed equal in x and y), the half acceptances in sigmas f_σ, the ratio of beam size in the y over x directions R, the muon mass in Volts m_μ, and the velocity of light c: the required minimum transverse momentum kick is:

$$\Delta p_y \;=\; B_x\ L\ c \;=\; m_\mu\ 2\ f_\sigma\ \sqrt{\frac{\epsilon_n\ \beta\gamma}{\beta_y}}$$

We use this to determine B_x for a given L, set at a fixed fraction of β_y. No allowance has been given here for the finite thickness of the following septum. For the moment, this is assumed to be taken out of the relatively generous 3 sigma acceptance that we will be using.

Assuming no dispersion at the kicker location, the x aperture of the kicker X is set to contain the beams up to f_σ of the rms beam size.

$$X \; = \; 2 \; f_\sigma \; \sqrt{\frac{\epsilon_n \; \beta_x}{\beta\gamma}}$$

The y aperture has to be larger to accommodate the deflected beam. If $\beta_x = \beta_y$ then:

$$R \; = \; \frac{Y}{X} \; \approx \; \left(1 \; + \; \frac{L}{2 \; \beta_y}\right)$$

Defining f_Φ so that the total flux $\Phi \; = \; f_\Phi \; B_x \; L \; Y$ to allow for leakage flux, and f_μ so that $\int Bdl/\mu \; = \; f_\mu \; B \; X$ to allow for finite μ's in the flux return, then the current I, single turn Voltage V, and total kicker stored energy J, are given by:

$$I \; = \; \frac{f_\mu \; B \; X}{\mu_o} \; = \; \frac{f_\mu \; 4 \; f_\sigma^2}{\mu_o \; c} \quad \frac{\epsilon_n}{L}$$

$$V \; = \; \frac{f_\Phi \; B \; Y \; L}{t_{\rm rise}} \; = \; \frac{f_\Phi \; 4 \; f_\sigma^2 \; m_\mu \; R}{c} \quad \frac{\epsilon_n}{t_{\rm rise}}$$

$$J \; = \; f_\mu \; f_\Phi \quad \frac{B^2 \; L \; X \; Y}{2 \; \mu_o} \; = \; f_\mu \; f_\Phi \quad \frac{m_\mu^2 \; 8 \; f_\sigma^4 \; R}{\mu_o \; c^2} \quad \frac{\epsilon_n^2}{L}$$

where $t_{\rm rise}$ is the linear rise time of the pulse, as defined in section 1. We see that, for a fixed L and $t_{\rm rise}$, neither the stored energy, current or total Voltage are dependent on the beam energy or directly on β_x. But if L is set equal to β, as is required for a reasonably optimized R, then the current and stored energy fall with β, while the Voltage remains independent even of this.

3.3 Examples

Consider the case of a first cooling ring with circumference $\approx$30 m and initial normalized emittance of 10 π mm (acceptance at 3 sigma of 90 mm), momentum of 215 MeV/c, and $\beta_\perp$= 1m, as in Study 2).

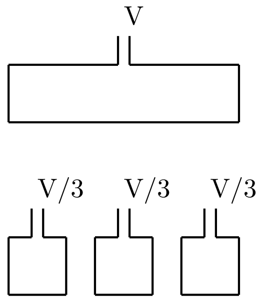
		μ Cooling	CERN $\bar{p}$	Ind Linac
f_Φ		1.05		
f_μ		1.05		
f_σ		3		
m_μ	V	1.05 10^8		
c	m/s	3 10^8		
ϵ_n	π mm	10		
β_x	m	1.0		
$\int Bd\ell$	Tm	.43	.088	
L	m	1.0	$\approx$5	5.0
$t_{\rm rise}$ (5-95)	ns	50	90	40
t	ns	100	500	100
$\beta\gamma$			2	
B	T	.42	$\approx$0.018	0.6
X	m	.42	.08	
Y	m	.63	.25	
$\mathrm{J}_{\rm magnetic}$	J	8200	$\approx$13	8000
I	kA	150		73
$\mathrm{V}_{\rm 1\ turn}$	kV	5,700	800	
$\mathrm{n}_{\rm units}$		30	10	50
$\mathrm{V}_{\rm p.s.}$	kV	190	80	190

3.4 Practicality

The table includes some parameters of a large conventioinal kicker (that used for the CERN antiproton accumulator [2]), and for a 5 m section of the second induction linac in Feasibility Study 2[3].

It is seen that the required field is higher and the stored energy almost three orders of magnitude greater than that of the CERN antiproton kicker. This could be reduced somewhat by increasing $\beta_\perp$ and L by perhaps 1.5, but further increases will reduce the cooling rate unacceptably. However, the stored energy is of the same order as that supplied by magnetic amplifiers to a few meters of the induction linac. The peak power is higher, but the pulse length is correspondingly shorter. Magnetic amplifiers with more pulse compression should be possible that would provide the needed peak power. But the Voltage (5 MV) is far too high for any plausible pulser.

In the CERN $\bar{p}$ case, the voltage is reduced by dividing the kicker into 10 segments, separately powered, thus reducing the voltage by this factor. In our case, the kicker length is $\approx$3 times as long as its width, one could break it into only 3 parts, as shown below, but each part would still require almost 3 MV, which is still far too high. However, the comparison with induction linacs suggests a better solution.

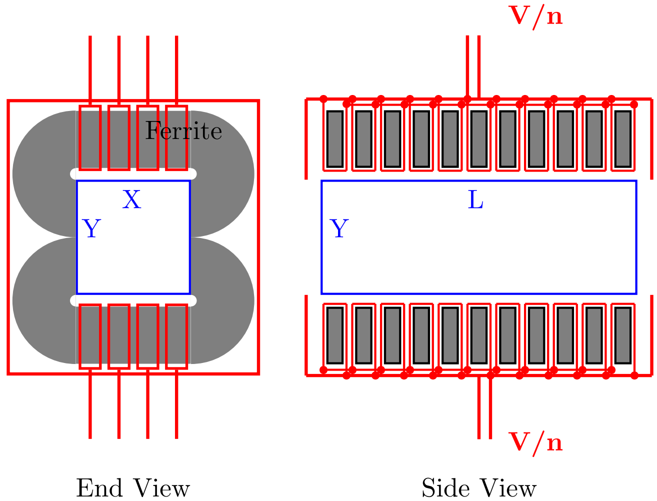


3.5 Induction Kicker

In this concept, the pulsed conductors are wound around the flux return yokes, instead of around the beam aperture its self. The windings can now be broken into n separate loops, each about a sub section of the yoke. The windings are connected in parallel, so the current increases, but the voltage is reduced by $1/n$ of the single turn value, and can be chosen to match a magnetic amplifier driver (typical Voltage 190 kV, requiring $n \approx 30$: 15 on each side, each 7 cm thick).

In a DC magnet, such a system w ould generate a large stray field and the stored energy would be higher than that in the aperture alone. However, as Lou Reginato pointed out, a pulsed magnet must anyway be contained in a conducting box for shielding reasons, and currents will be induced in this box such as to remove the stray fields and restore the full efficiency.

If such a kicker has not been named before, I would name it an 'Induction Kicker'.

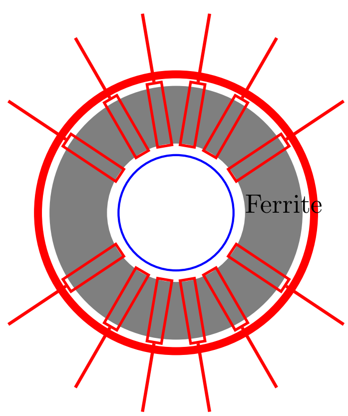


3.6 Cos Theta Design

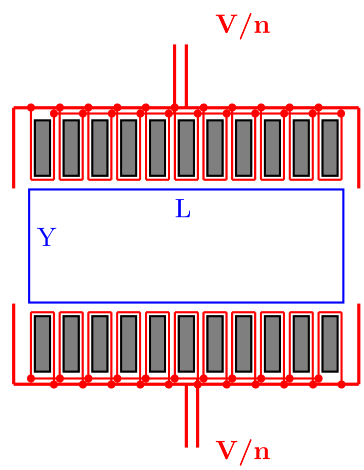
Some stored energy can be saved if, instead of a picture frame magnet, we use a $\cos\Theta$ designs as shown below. The energy is reduced by $\pi/4$. In this figure, 6 separate loops are located on each side. Their locations are such that, given equal currents in each loop, the central field is dipole, with sextupole and decapole fields set to zero. The lowest non uniform multipole is thus the 14 pole, that is unlikely to be a problem. With more loops, even higher multipoles could be removed.

Note that the coupled flux, and thus the required voltages, are not the same for all loops. Those nearer the ends have less voltage, while those in the center (mid plane) have the same voltage as in the rectangular case.

In this design, the field will remain good even without the ferrite yoke. This could be important if we are unable to shield the kicker from the strong solenoidal focus fields. The Voltage and stored energies are increased by a little over 2.



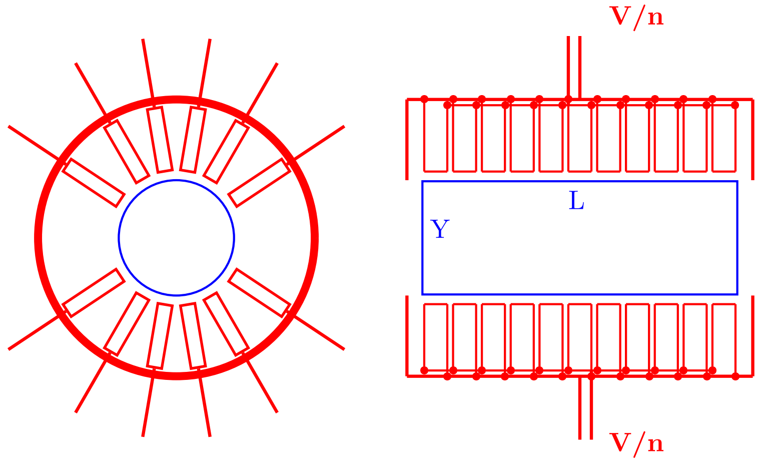
End View



Side View

3.7 Without Ferrite

This cos theta version will still work with no ferrite present. The fields induced by the radial currents in each loop are cancelled by the returning currents in the next loop. But the axial currents on the inside and outside ends, add to form cos theta currents on the cylindrical surfaces formed by the inside and outside limits of the loops. The axial currents on the inside of each loop add to generate a uniform field, as in a conventional cos theta dipole magnet. Those on the outside of each loop add to generate a uniform field of the opposite sign, but weaker than that from the inner sides by the ratio r_1/r_2. Thus the field, for the same currents, would reduced to $(1 - r_1/r_2)$, and the currents have to be approptiately higher. The Voltages, however, are the same, since they are related to the enclosed fluxes which are unchanged.



End View Side View

For this case, we can write the 2D fields in the long magnet limit. Let the inner and outer radii of the coils be r_1 and r_2, let $r_2/r_1 \ = \ \alpha$ Then if i_1 is the current density on the inside of the loops. Then if i_2 is the current density on the outside of the loops. Then if i_3 is the current density on the shield can.

$$i_1 \ = \ \frac{I}{2 \ r_1} \ \cos(2\theta)$$

$$i_2 \ = \ -\frac{I \ \alpha}{2 \ r_1} \cos(2\theta)$$

$$i_3 \ = \ \frac{I \ (\alpha - \alpha^2)}{2 \ r_1} \ \cos(2\theta)$$

The fields at $r < r_1$, including all three currents, are:

$$B_o \ = \ \frac{\mu_o \ i_1}{2} \ (1 \ - \ \alpha^2)$$

The fields for $r > r_1 < r_2$ are:

$$B_y \ = \ \frac{\mu_o \ i_1}{2} \ \left(\frac{r_1^2 \ \sin(2\theta)}{r^2} \ - \ \alpha^2 \right)$$

$$B_x \ = \ \frac{\mu_o \ i_1}{2} \ \left(\frac{r_1^2 \ \cos(2\theta)}{r^2} \right)$$

and there are no fileds for $r > r_2$

The stored energy for $r < r_1$:

$$J(r < r_1) \ = \ \pi \ r_1^2 \ L \ \frac{B^2}{2 \ \mu_o}$$

and for $r_2 > r > r_1$:

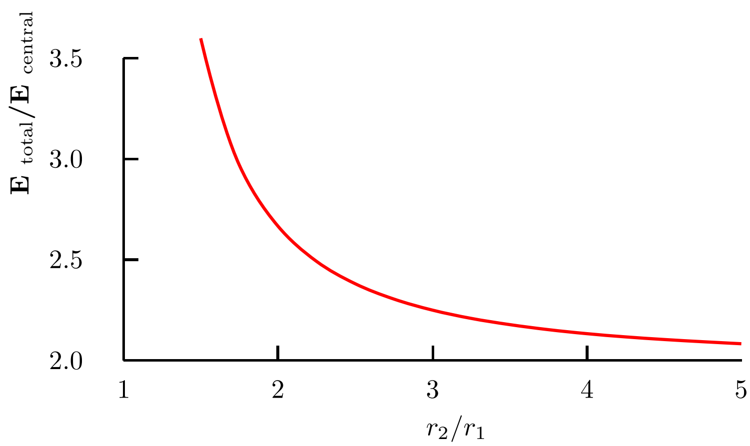
$$J(r_2 > r > r_1) \;=\; \frac{\mu_o\; i_1^2\; L}{4} \int_{r_1}^{r_2} \int_{o}^{2\pi} \left[\left(\frac{r_1^2\;\;\sin(2\theta}{r^2} \;-\; \alpha^2 \right)^2 \;+\; \left(\frac{r_1^2\;\;\cos(2\theta}{r^2} \right)^2 \right] r\; d\theta\; dr$$

$$= \frac{\mu_o\; i_1^2}{4}\; \pi\; r_1^2\; L\; \left(1 \;+\; \frac{1-\alpha^4}{(1-\alpha^2)^2}\right)$$

The total stored energy divided by that within the inner conductors is:

$$\frac{J_{\text{total}}}{J_{\text{central}}} \;=\; 1 \;+\; \frac{1-\alpha^4}{(1-\alpha^2)^2}$$

This is plotted:



It is seen that, for a ratio of $r_2/r_1 = 3$ the stored energy is increased by a factor of 2.25.

Despite the higher currents and stored energy, this solution has the advantages that 1) there is no rise time limit from the ferrite, and 2) the kicker will work in the stray fields from the focus solenoids.

4 Conclusion

We have found that the kicker requirements for an initial cooling ring are a few orders of magnitude beyond those of even the largest kickers now existing. However, by using drivers and other concepts from induction linacs, it appears that these requirements may be attainable. But much work remains. For instance:

- A realistic lattice for the injection and extraction must be designed, and its matching into the rest of the ring established.

- The kicker must be engineered with realistic insulation, cooling and structural integrity.
- The field abberations in the kicker must be determined and controlled.
- The driving circuit needs to be defined. If a damping resistor is employed to stop ringing, then significantly more energy must be supplied by the drivers. If no resistor is employed, then an appropriate driving waveform can be chosen to provide the required pulse shape, but there will be a reflected signal that must be damped at the source. This needs study.

The proposed kicker could also have application in FFAG acceleration of large emittance beams for neutrino factories.

5 Acknowledgments

The authors wish to thank Glen Lambertson, Thomas Roser and Alvin Tollestrup for their useful discussions.

References

[1] my ring

[2] CERN kicker

[3]