

# Linear Quadrupole Cooling Channel

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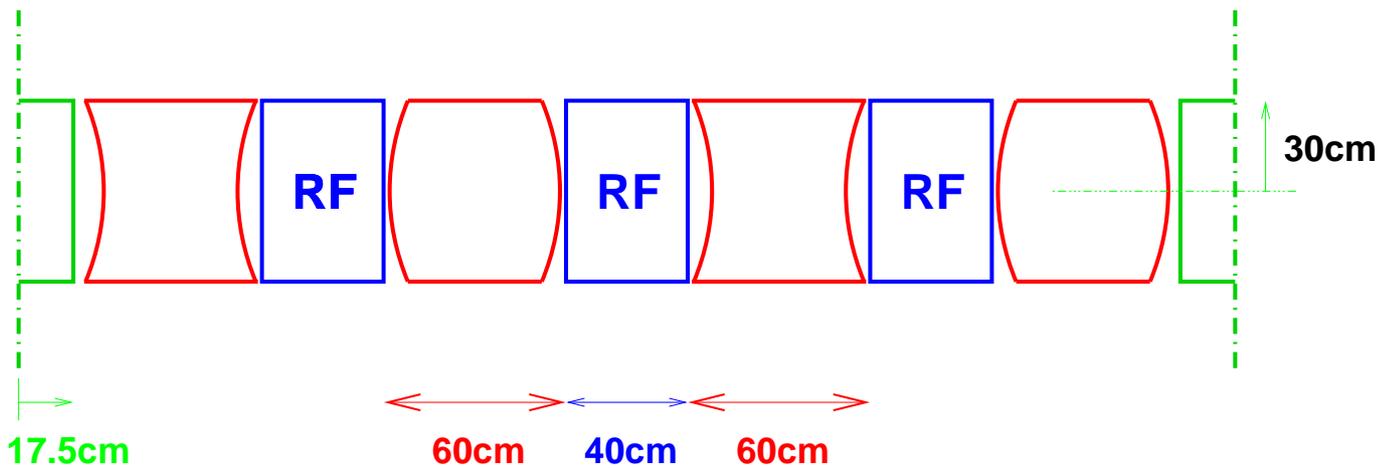
C. Johnstone (Fermilab)

M. Berz (MSU)

1. Quad Cooling Cell
2. COSY Simulation Technique
3. Some Results
4. Outlook

# Quad Cooling Cell

4m Cell



- Incoming Muons: 180 MeV/c to 245 MeV/c
- Magnetic Quadrupoles:  $k=2.88$
- 35cm Liquid H Absorber: Energy loss  $\approx 12$  MeV.  
The same design as Study II 2.75m sFOFO cell.
- RF Cavity: Energy gain to compensate the loss.  
200 MHz,  $\phi = 30^\circ$ .

# COSY Simulation

## Map Method

- The transfer map  $\mathcal{M}$  is the flow of the system ODE.

$$\vec{z}_f = \mathcal{M}(\vec{z}_i, \vec{\delta}),$$

where  $\vec{z}_i$  is the initial condition,  $\vec{z}_f$  is the final condition,  $\vec{\delta}$  is system parameters.

- For a repetitive system, only one cell transfer map has to be computed. Thus, it is much faster than ray tracing codes (i.e. tracing each individual particle through the system).
- The Differential Algebraic (DA) method allows a very efficient computation of high order Taylor transfer maps.

## Differential Algebra

- arbitrary order
- very transparent algorithms; effort independent of order
- can keep system parameters in map
- etc. etc.

## Field Description in Differential Algebra

There are various DA algorithms to treat the fields of beam optical systems efficiently.

For example, **DA PDE Solver**

- requires to supply only
  - the midplane field for a midplane symmetric element.
  - the on-axis potential for a solenoidal element, a quadrupole, and a higher multipole.
- treats arbitrary fields straightforwardly.
  - Magnet (or, Electrostatic) fringe fields:  
The Enge function fall-off model

$$F(s) = \frac{1}{1 + \exp(a_1 + a_2 \cdot (s/D) + \dots + a_6 \cdot (s/D)^5)}$$

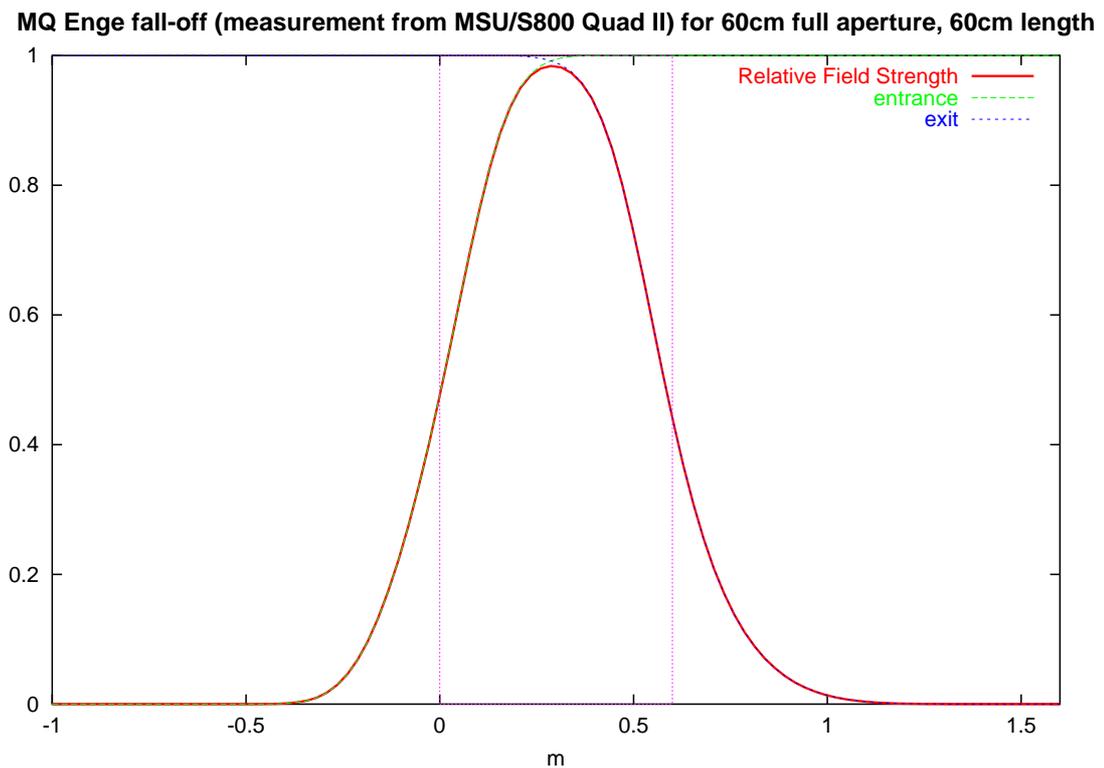
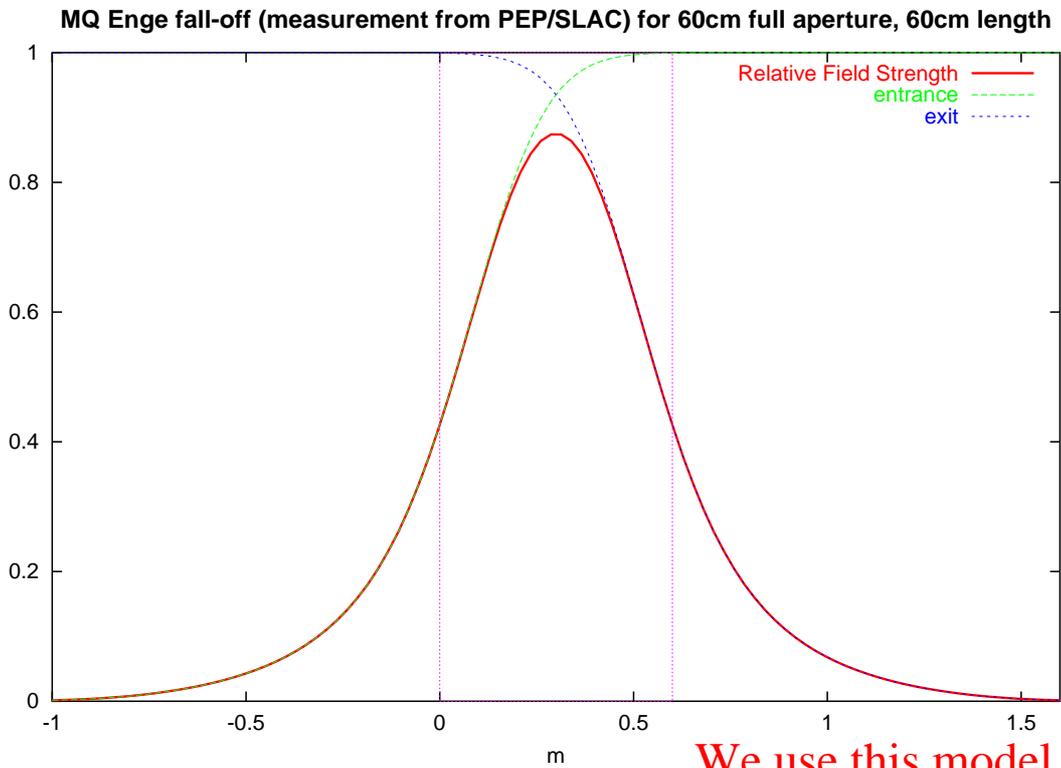
where  $D$  is the full aperture.

Or, any arbitrary model including the measured data representation.

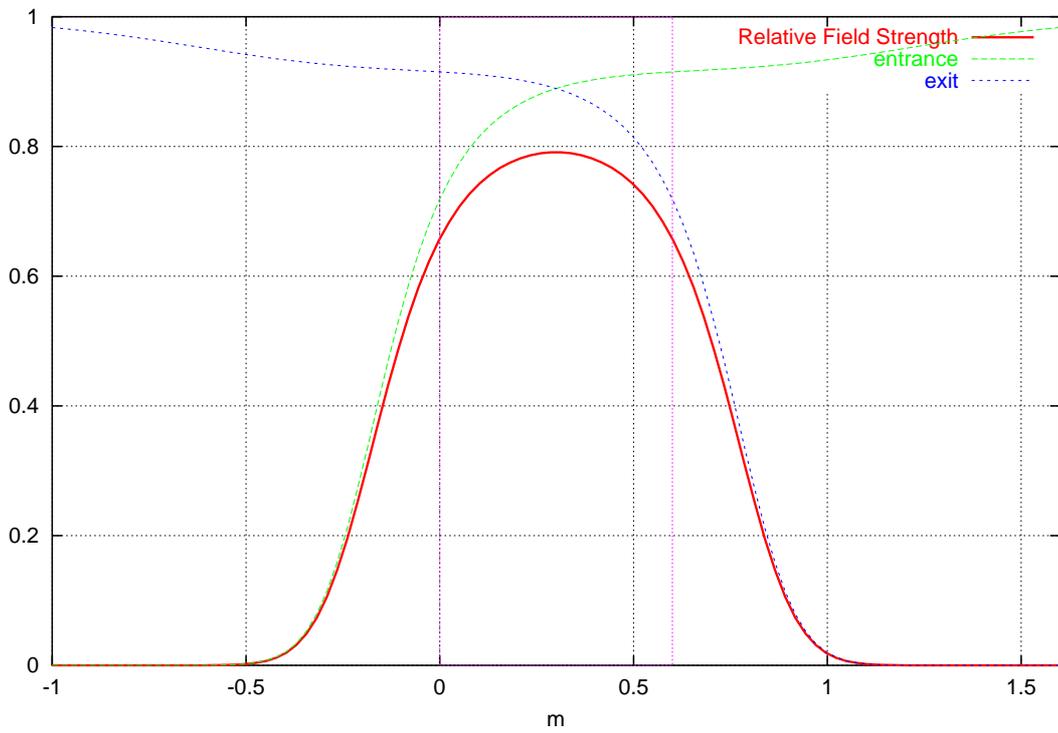
- Solenoid fields including the fringe
- Measured fields: E.g. Use Gaussian wavelet representation
- Etc. etc.

# Field Profile of Various Magnets

## 60cm full aperture, 60cm length Magnets

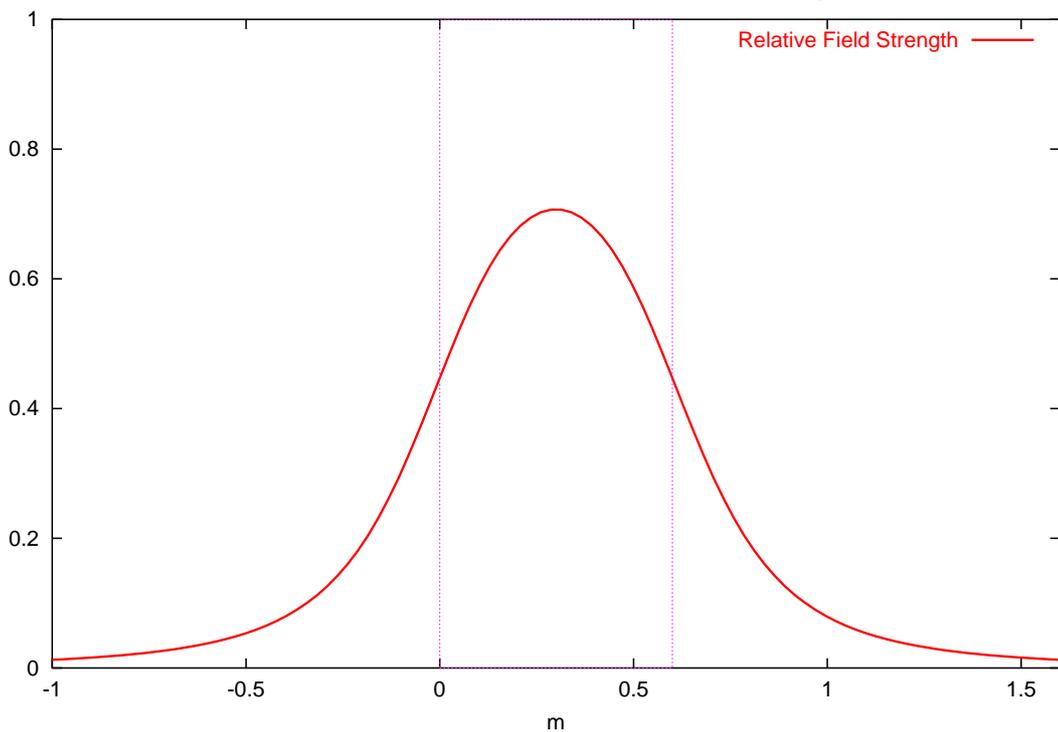


MQ Enge fall-off (measurement from LHC/HGQ lead end) for 60cm full aperture, 60cm length



Compare to: Ideal Solenoidal Magnet  
60cm full aperture, 60cm length Magnet

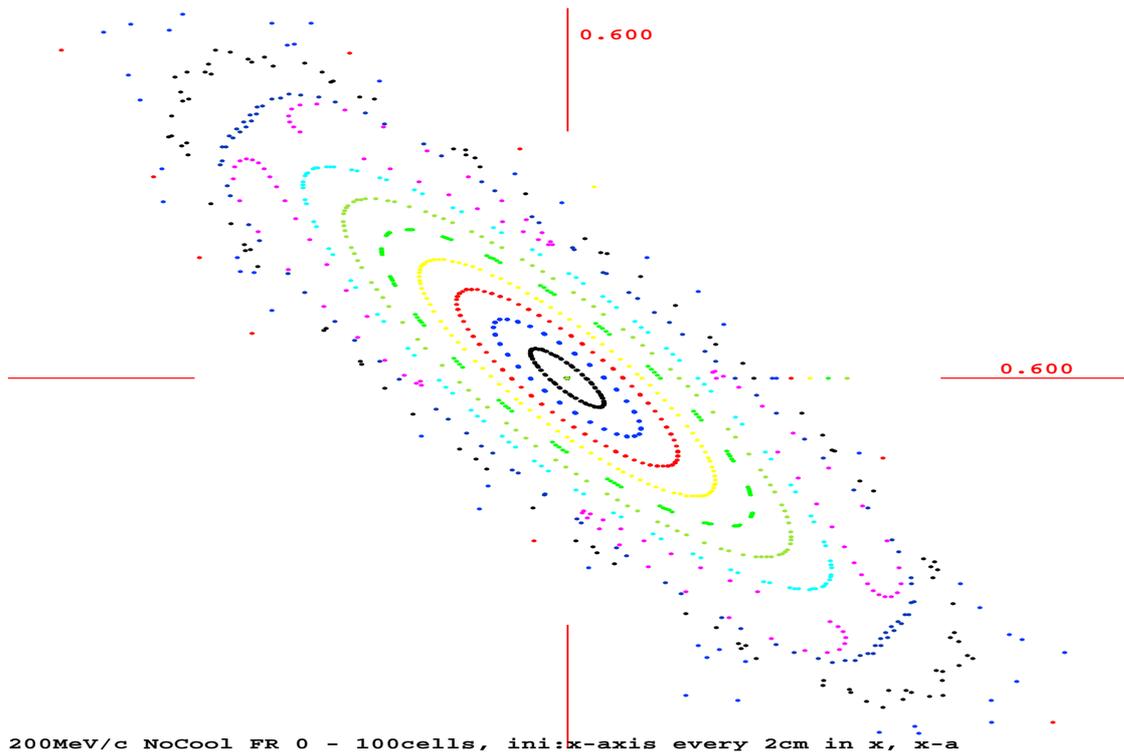
Ideal Solenoid for 60cm full aperture, 60cm length



# Fringe Field Effects

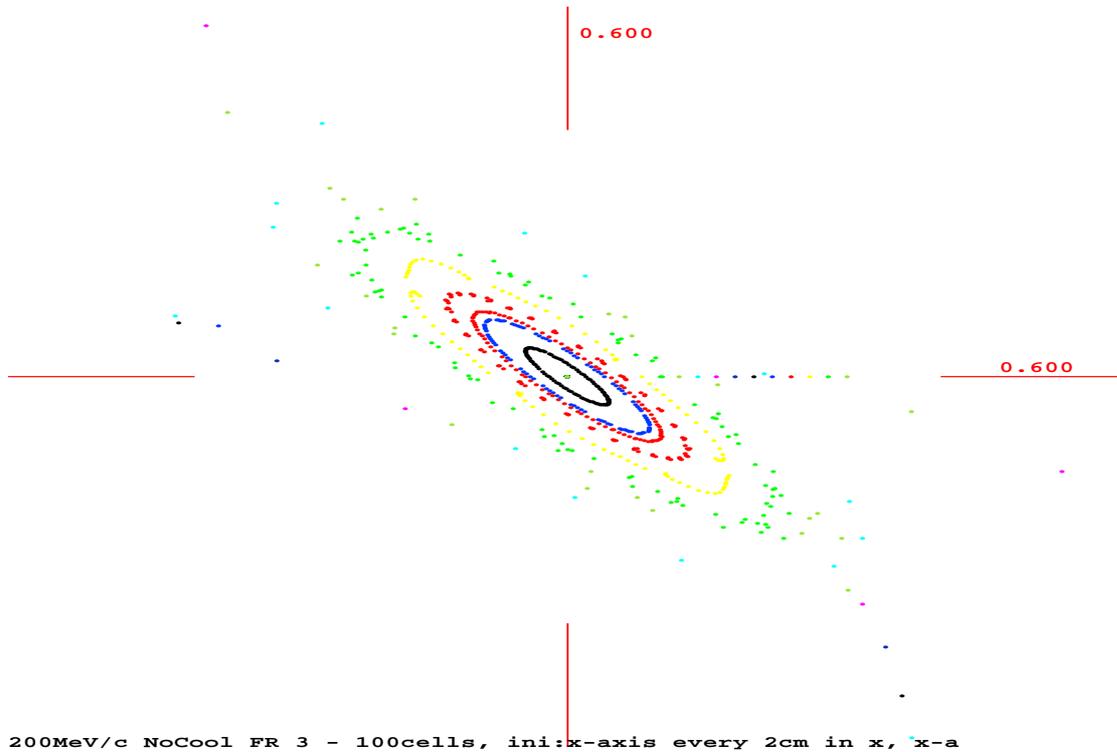
200 MeV/c Particles along the  $x$ -axis at every 2cm are tracked through 100 Quad cells without cooling, nor acceleration. Pictures: in the  $x - p_x/p_0$  plane (frame size: 60cm  $\times$  0.6 rad).

## Sharp Cut-off Fringe Field Model ( No Fringe Field Effect Consideration )

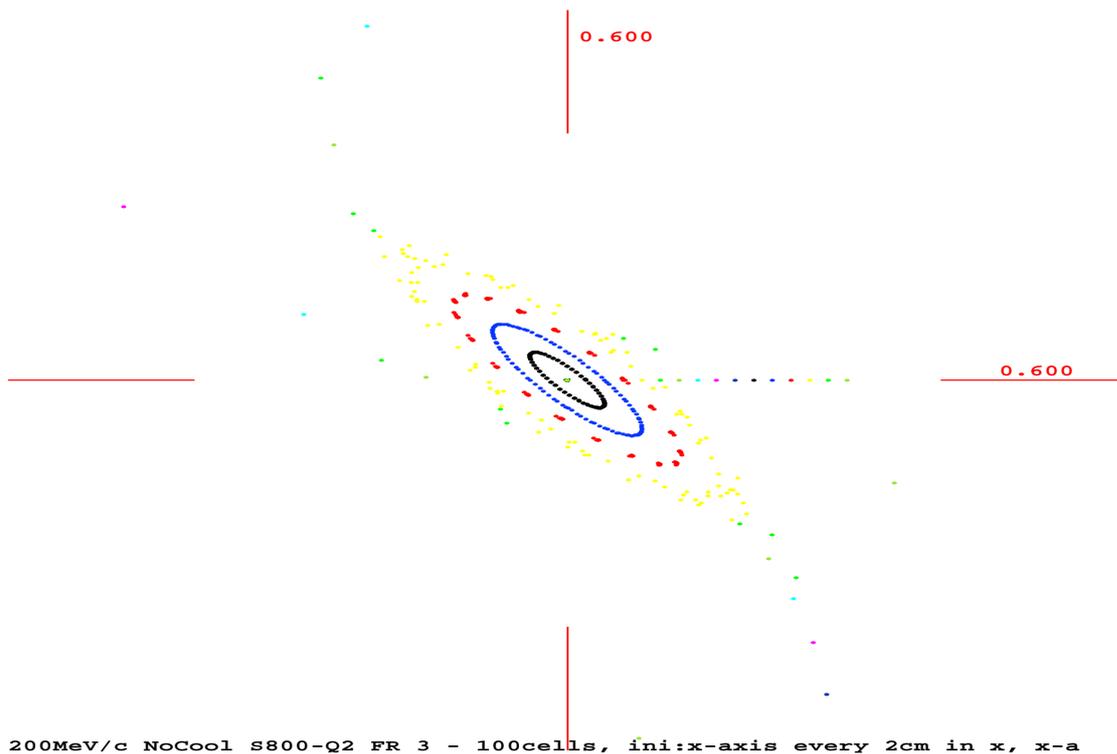


# PEP/SLAC Quad Fringe Field Fall-off Model

⇒ We use this model at the moment.



# S800 Q II Quad Fringe Field Fall-off Model



# Treatment of Dynamics through Material

## Deterministic Effects

- The mean energy loss.  
⇒ Compensate it by RF cavities.
- Include the effects in the transfer map  $\mathcal{M}$ .

## Nondeterministic Random Effects

- Multiple scattering.  $\mathcal{R}_{\text{MS}}$ : A random kick in  $p_x$  and  $p_y$ .  
Gaussian distribution.
- Straggling.  $\mathcal{R}_{\text{St}}$ : A random kick in energy.  
In the absorbers under consideration, the distribution follows Vavilov's theory.

A set of particles is tracked via the map for the cell  $\mathcal{M}$ ; then the Monte Carlo kicks  $\mathcal{R}_{\text{MS}}$  and  $\mathcal{R}_{\text{St}}$  are added.

$$\vec{z}_f = (\mathcal{R}_{\text{MS}} \circ \mathcal{R}_{\text{St}} \circ \mathcal{M})(\vec{z}_i)$$

This procedure is iterated for the next cell, and so on.

# Mean Energy Loss through Material

The Bethe-Bloch Formula

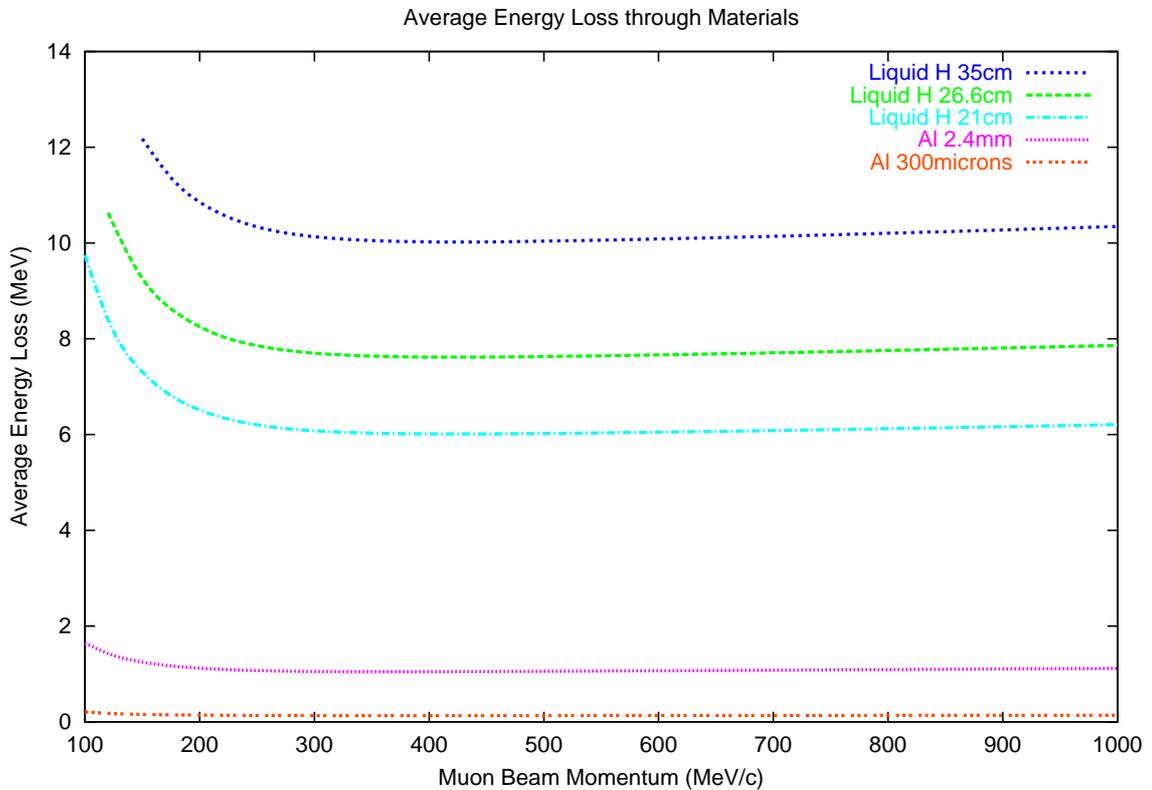
$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z z^2}{A \beta^2} \left[ \ln \left( \frac{2m_e \gamma^2 v^2 W_{max}}{I^2} \right) - 2\beta^2 - \delta - 2\frac{C}{Z} \right]$$

$r_e$  : classical electron radius,  $I$  : mean excitation potential,

$W_{max}$  : maximum energy transfer in a single collision,

$\delta$  : density correction,  $C$  : shell correction.

Through Typical Muon Beam Absorber Material

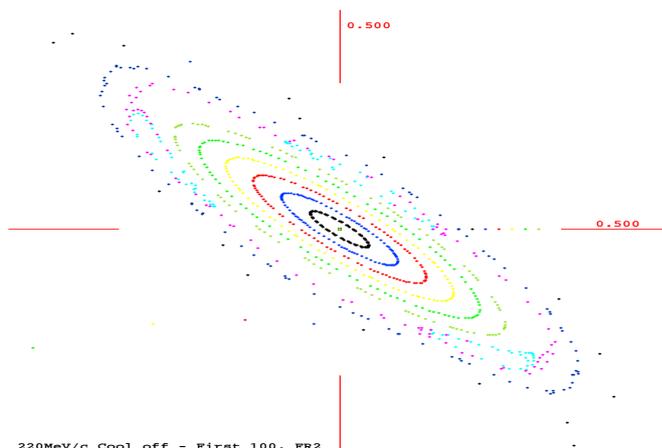


# Pseudo Invariant Ellipses of Quad Cooling Cells

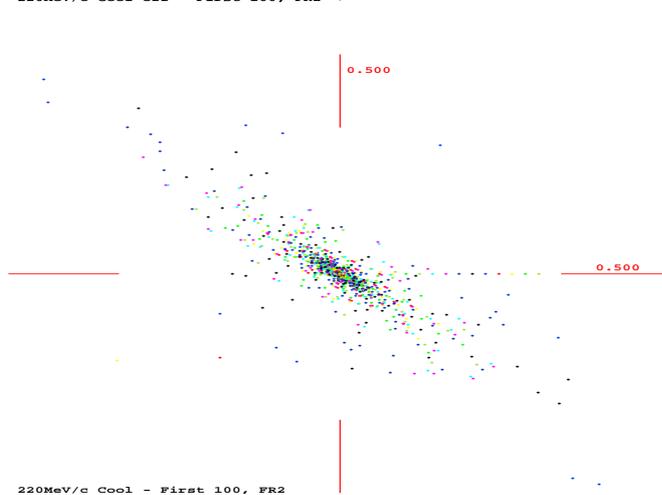
The Quad Cooling channel is a decoupled damped system.  
By scaling the transfer map by  $\sqrt{D}$ , ellipses can be restored.  
(  $D$ : the determinant of the linear map. )

220 MeV/c.  $x - p_x/p_0$  plane.  $x_0=2, 4, \dots, 30$ cm. For 100 cells.

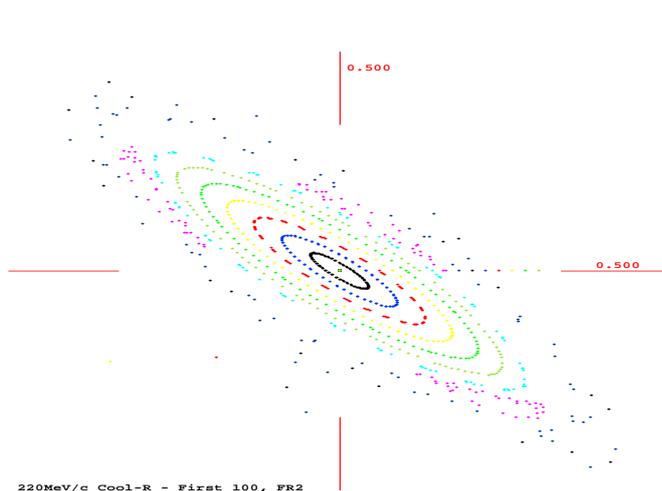
No Cooling



With Cooling  
No Scattering



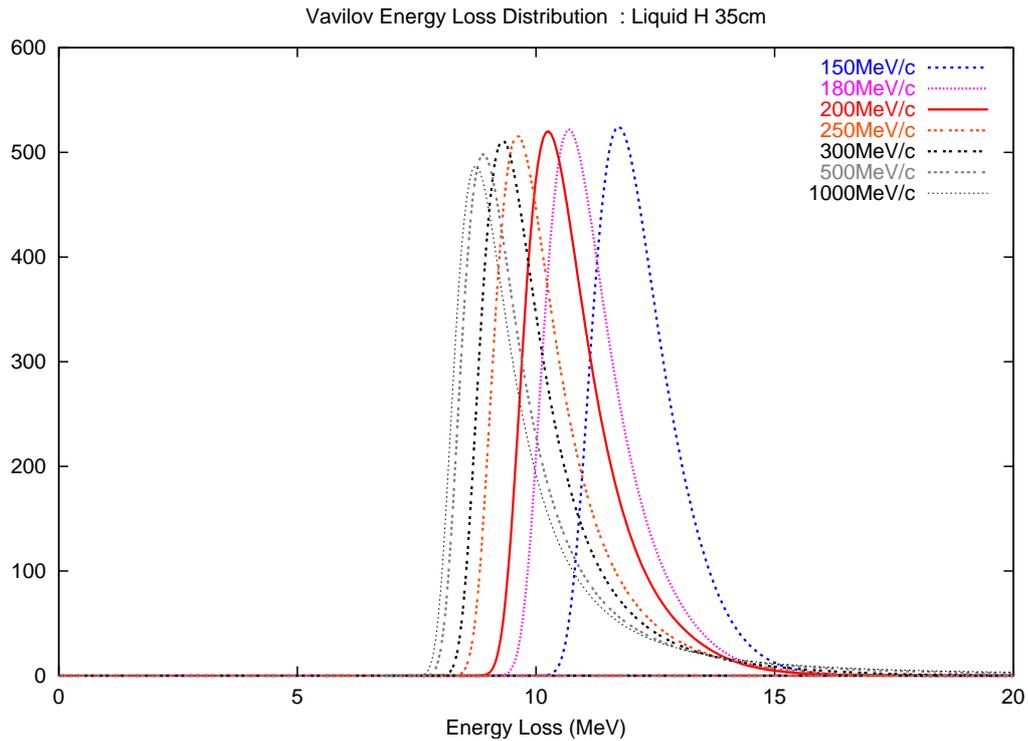
Pseudo Invariant  
Ellipses w Cooling



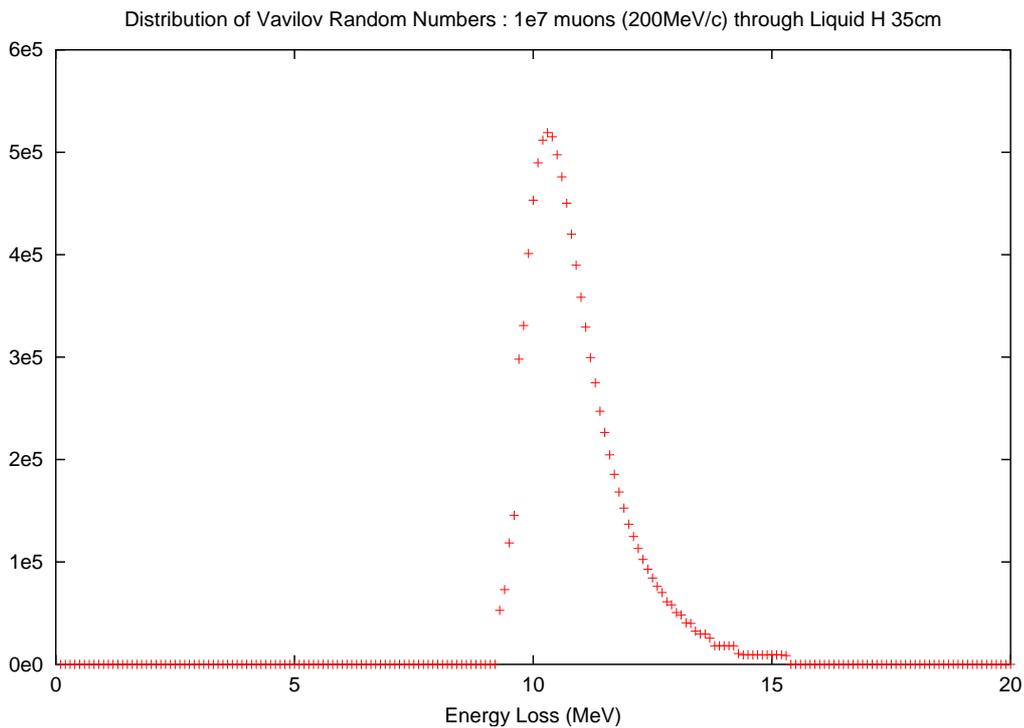
# Energy Straggling

## – Vavilov Energy Loss Distribution –

Through 35cm thick Liquid H



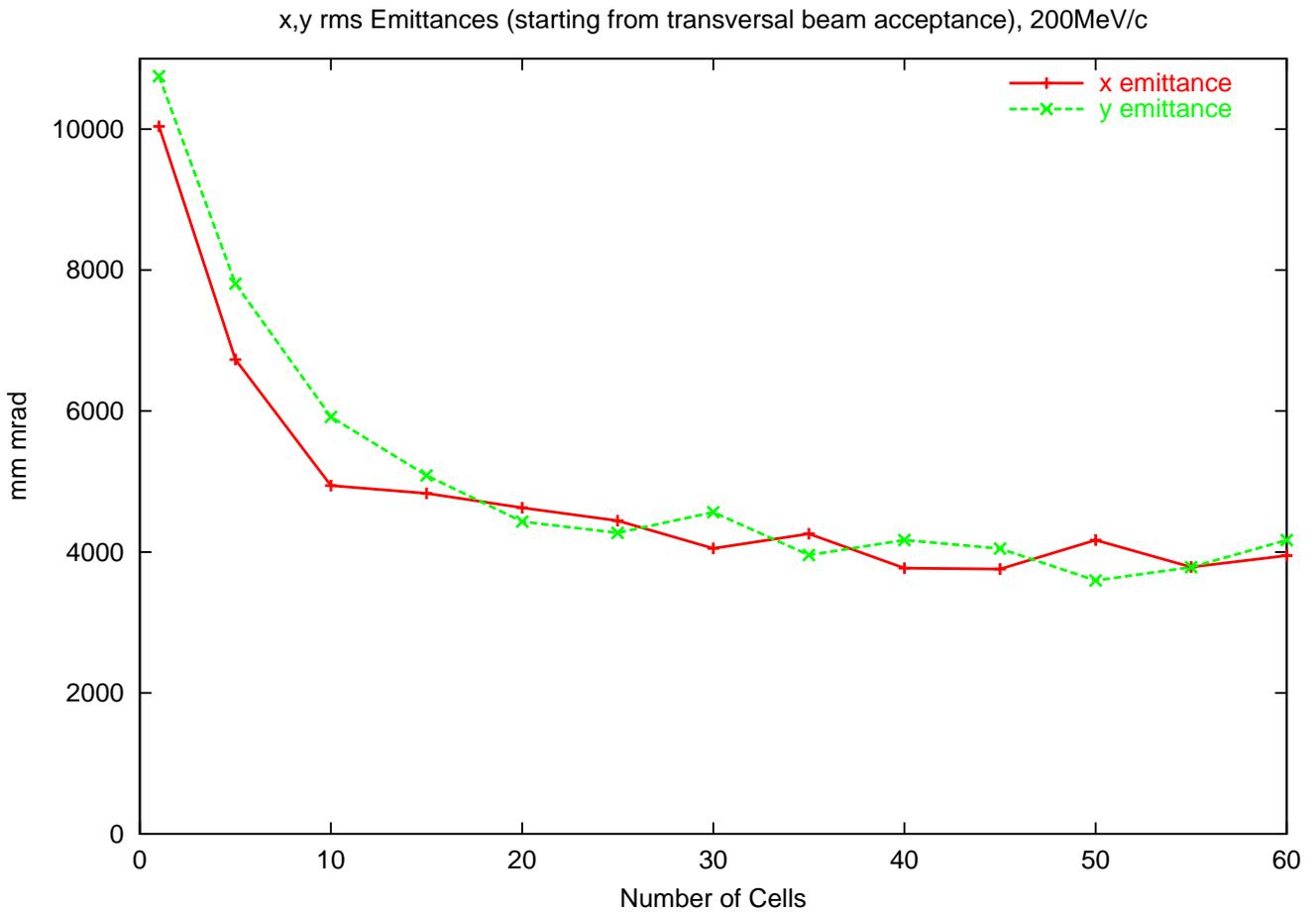
The distributions are computed using the code vavidis by A. van Ginneken.



# Muons through the Quad Cooling Cells

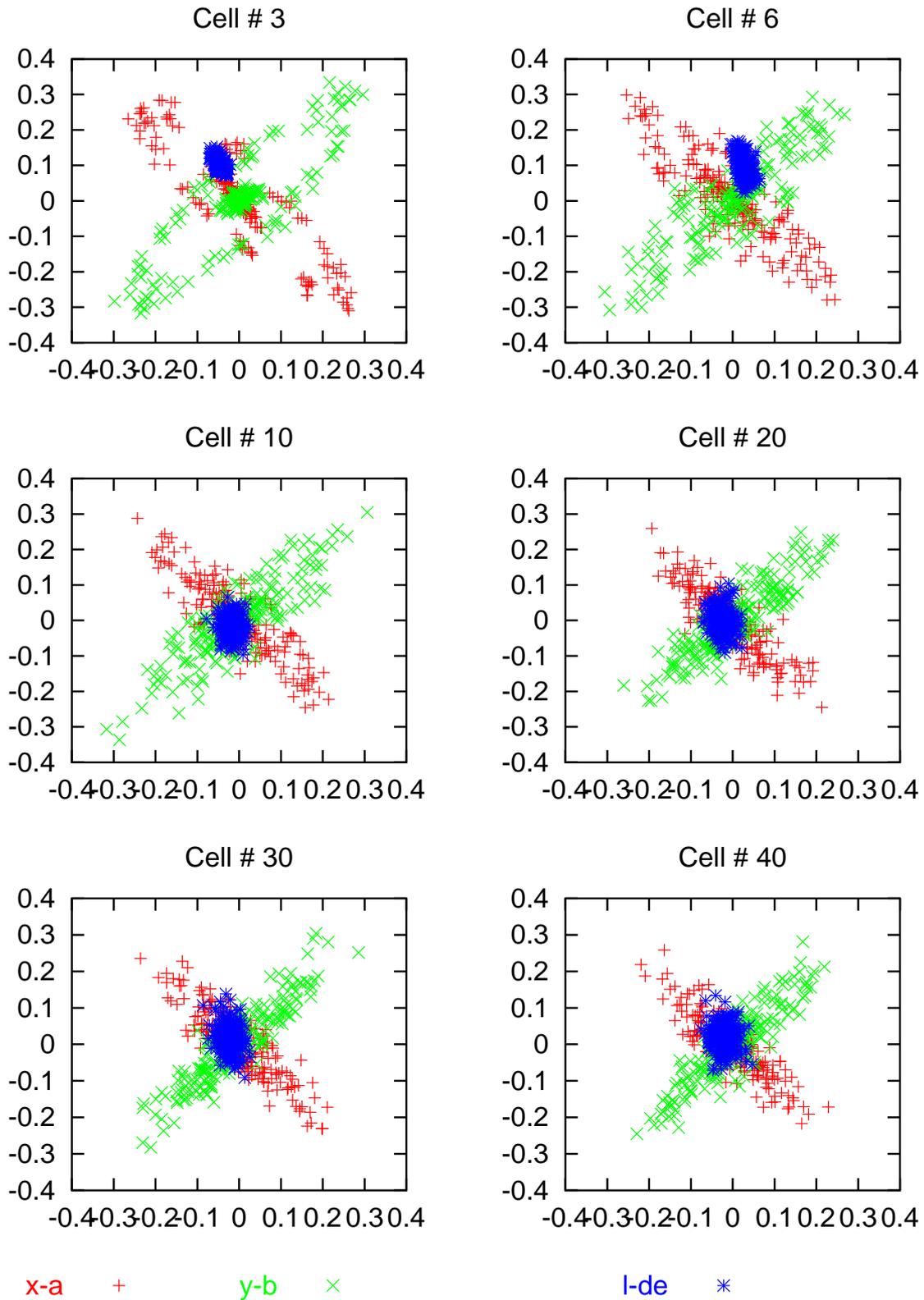
## 6D Simulation

Transversal Emittance starting from the beam acceptance.  
 $p_0 = 200$  MeV/c. The initial longitudinal emittance: 0.  
The simulation includes multiple scattering and straggling.



# Transversal Cooling and Equilibrium in the Quad Cooling Channel

Refer to the file pout-steps.ps to see the behavior after each cell.



# Outlook

- Design the matching section.
- Propagate the realistic beam distribution.
- Simulate through from the capture through the cooling channel.
- ...
- Treat the detailed absorber shape.
- ...
- Optimize the system.
- More details of RF cavities.
- More details of magnets.
- ...

## References

- [1] C. J. Johnstone, M. Berz, D. Errede, and K. Makino. Muon Beam Ionization Cooling in a Linear Quadrupole Channel. In *Proceedings of 2002 International Charged Particle Optics Conference CPO-6. To appear in NIM-A.*
- [2] D. Errede, M Berz, C. J. Johnstone, K. Makino, and A. van Ginneken. Stochastic Processes in Muon Ionization Cooling. In *Proceedings of 2002 International Charged Particle Optics Conference CPO-6. To appear in NIM-A.*
- [3] K. Makino, M. Berz, D. Errede, and C. J. Johnstone. High Order Map Treatment of Superimposed Cavities, Absorbers, and Magnetic Multipole and Solenoid Fields. In *Proceedings of 2002 International Charged Particle Optics Conference CPO-6. To appear in NIM-A.*