Ring Cooler Simulation in COSY. Maps of Short Solenoids.

Carlos O. Maidana , Martin Berz and Kyoko Makino
Michigan State University & University of Illinois at Urbana-Champaign

September 17, 2002

Abstract

Simulations of the recirculating muon cooling system presented by V. Balbekov (Fermilab) in McNote 0246 on May 6, 2002 is considered. The short and long solenoids were studied using COSY ∞ with the obtaining of linear and non-linear transfer maps until the fifth order computation and B field profiles. The computation was made working with 2 phase spaces (4 variables) and muons with kinetic energy of 250 MeV.

1 Short Solenoids

We have four short solenoids with axial magnetic fields that flips $\pm$ 2.7 T. There are different approaches to study the system. In these different cases we have studied solenoids sharp cut off and with kicks, with and without fringe fields. The different cases are presented in the next sections. The coordinate system used in COSY ∞ is:

x = x coordinate [m]

a = p_x/p_o coordinate where px is the x-component in momentum and p_0 is the total momentum

y = y coordinate [m]

b = p_y/p_o coordinate where p_y is the y-component in momentum

[p_x, p_y, p_o] = MeV

In Balbekov's system there are transverse momenta kicks (at constant energy) at the beginning and at the end.

$P_x = P_x + Y * B_z * C/2$

$P_y = P_y - X * B_z * C/2$

In COSY Infinity we had to transform the last equations to our coordinate system. The same kick's equations apply at the end with opposite sign of C, where C = 2.9979245.

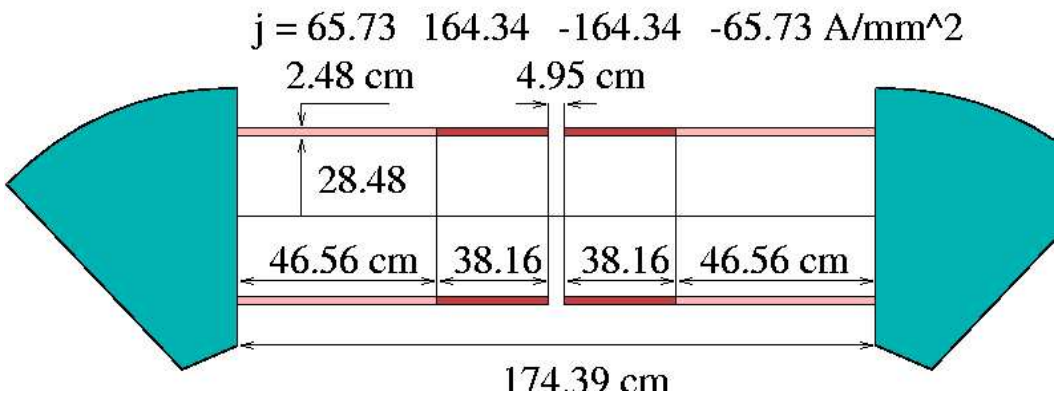

Figure 1: V. Balbekov's Short Solenoids

1.1 Case 1 : Superimposed with $\pm\infty$ length solenoids

We have used COSY Infinity's CMSTP command that compute a piece of thick magnetic solenoid. The computation was made for 1.74 m length (solenoid length).

1.1.1 Sharp cut off:

No fringe fields are included. The magnetic field profile for this system is shown in figure 2 and Balbekov's ring cooler B filed profile is shown in figure 3,

The study of the non-linearities of the system gives a transfer map which is shown below for a Kinetic energy of 250 MeV,

```
 -0.1467136     -1.796260     0.9193886    -0.2657072      0.000000     100000
   1.015304    -0.1467136     0.3060451E-11 0.9193886      0.000000     010000
 -0.9193886     0.2657072    -0.1467136     -1.796260      0.000000     001000
 -0.3060451E-11-0.9193886      1.015304    -0.1467136      0.000000     000100
  -5.150356      1.407434     -3.471527     -10.58868      0.000000     300000
  -6.411803     -5.982760     -1.425787     -8.650997      0.000000     210000
  -6.063072     -2.101315     -2.173004     -8.458362      0.000000     120000
  -2.226005     -3.298721     0.3628300     -2.185223      0.000000     030000
   3.471527      10.58868     -5.150356      1.407434      0.000000     201000
   8.867431      3.476400      1.049483      13.48336      0.000000     111000
   6.099573      13.79142     -5.180687      4.906295      0.000000     021000
  -7.441644      5.174597     -7.461286     -19.46612      0.000000     200100
  -3.926569     -5.333063    -0.8823853     -7.007610      0.000000     110100
 -0.3628300      2.185223     -2.226005     -3.298721      0.000000     020100
  -5.150356      1.407434     -3.471527     -10.58868      0.000000     102000
```

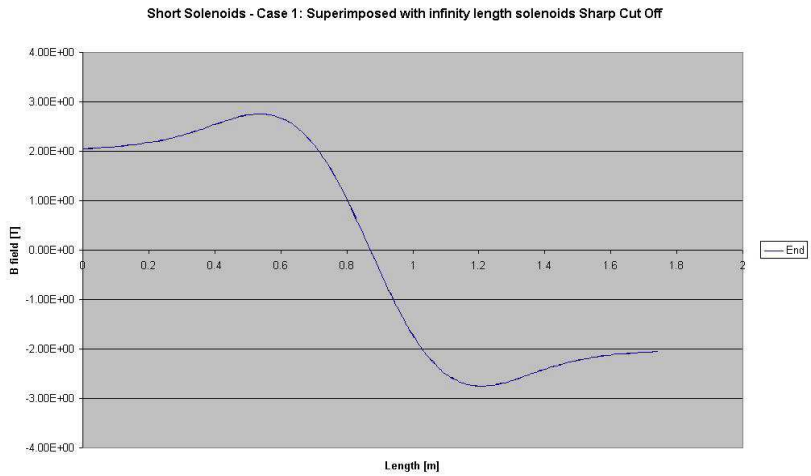

Figure 2: Short solenoid field by write 30.

-7.461286	-19.46612	7.441644	-5.174597	0.000000	012000
1.049483	13.48336	-8.867431	-3.476400	0.000000	101100
-0.8823853	-7.007610	3.926569	5.333063	0.000000	011100
-5.180687	4.906295	-6.099573	-13.79142	0.000000	100200
-2.226005	-3.298721	0.3628300	-2.185223	0.000000	010200
3.471527	10.58868	-5.150356	1.407434	0.000000	003000
1.425787	8.650997	-6.411803	-5.982760	0.000000	002100
2.173004	8.458362	-6.063072	-2.101315	0.000000	001200
-0.3628300	2.185223	-2.226005	-3.298721	0.000000	000300
-15.71585	27.23532	-23.72338	-33.63860	0.000000	500000
-36.02640	5.753202	-21.46530	-55.14127	0.000000	410000
-38.39075	16.29484	-33.88081	-55.16902	0.000000	320000
-33.14696	-7.116152	-14.53667	-36.65477	0.000000	230000
-14.27604	2.840011	-11.47237	-15.47623	0.000000	140000
-7.567237	-3.099152	-1.462962	-6.833978	0.000000	050000
23.72338	33.63860	-15.71585	27.23532	0.000000	401000
55.74263	-36.40506	46.08522	99.46462	0.000000	311000
93.84505	43.66483	10.39065	121.6189	0.000000	221000
46.89272	-13.36919	35.57062	65.34311	0.000000	131000
37.16991	20.41435	0.8954823	38.14645	0.000000	041000
-34.27733	91.54633	-82.11162	-93.71142	0.000000	400100
-59.96424	11.50418	-48.78140	-105.3240	0.000000	310100
-32.35605	50.02397	-68.71758	-72.45926	0.000000	220100
-25.69754	-4.938121	-15.17153	-35.30644	0.000000	130100
1.462962	6.833978	-7.567237	-3.099152	0.000000	040100

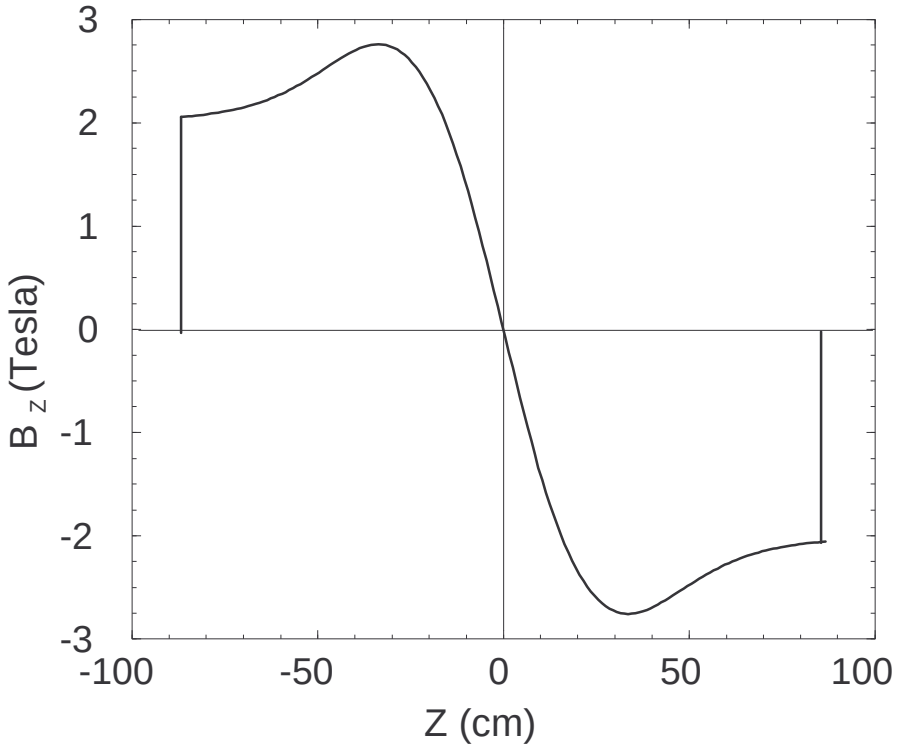

Figure 3: B field profile shown in V. Balbekov's notes

-31.43170	54.47063	-47.44675	-67.27721	0.000000	302000
-118.1380	-87.95822	12.81203	-146.6876	0.000000	212000
-78.94206	25.16981	-57.27854	-120.5226	0.000000	122000
-90.46738	-65.21267	9.060995	-97.77821	0.000000	032000
46.08522	99.46462	-55.74263	36.40506	0.000000	301100
32.32123	-123.0740	106.7597	119.2030	0.000000	211100
79.07023	50.84993	-0.3026055	108.8777	0.000000	121100
-15.17153	-35.30644	25.69754	4.938121	0.000000	031100
-30.16067	130.4938	-117.2428	-109.0184	0.000000	300200
-54.89676	0.1304368	-37.83173	-84.40901	0.000000	210200
-13.38056	40.98646	-48.64227	-35.89058	0.000000	120200
-15.13447	-6.198304	-2.925925	-13.66796	0.000000	030200
47.44675	67.27721	-31.43170	54.47063	0.000000	203000
55.74263	-36.40506	46.08522	99.46462	0.000000	113000
117.2428	109.0184	-30.16067	130.4938	0.000000	023000
-12.81203	146.6876	-118.1380	-87.95822	0.000000	202100
-106.7597	-119.2030	32.32123	-123.0740	0.000000	112100
37.83173	84.40901	-54.89676	0.1304368	0.000000	022100
57.27854	120.5226	-78.94206	25.16981	0.000000	201200
0.3026055	-108.8777	79.07023	50.84993	0.000000	111200
48.64227	35.89058	-13.38056	40.98646	0.000000	021200
-9.060995	97.77821	-90.46738	-65.21267	0.000000	200300
-25.69754	-4.938121	-15.17153	-35.30644	0.000000	110300
2.925925	13.66796	-15.13447	-6.198304	0.000000	020300
-15.71585	27.23532	-23.72338	-33.63860	0.000000	104000
-82.11162	-93.71142	34.27733	-91.54633	0.000000	014000
46.08522	99.46462	-55.74263	36.40506	0.000000	103100
-48.78140	-105.3240	59.96424	-11.50418	0.000000	013100
10.39065	121.6189	-93.84505	-43.66483	0.000000	102200
-68.71758	-72.45926	32.35605	-50.02397	0.000000	012200
35.57062	65.34311	-46.89272	13.36919	0.000000	101300
-15.17153	-35.30644	25.69754	4.938121	0.000000	011300
0.8954823	38.14645	-37.16991	-20.41435	0.000000	100400
-7.567237	-3.099152	-1.462962	-6.833978	0.000000	010400
23.72338	33.63860	-15.71585	27.23532	0.000000	005000
21.46530	55.14127	-36.02640	5.753202	0.000000	004100
33.88081	55.16902	-38.39075	16.29484	0.000000	003200
14.53667	36.65477	-33.14696	-7.116152	0.000000	002300
11.47237	15.47623	-14.27604	2.840011	0.000000	001400
1.462962	6.833978	-7.567237	-3.099152	0.000000	000500

Symplectic Error : 1.159670981749786

1.2 With kicks :

If we include kicks to the system we obtain the following map's components for one kinetic energy of 250 MeV,

-0.1467136	-0.9637261	-0.2022731E-03	0.5845774E-04	0.000000	100000
1.015304	-0.1467136	0.3060451E-11	-0.2022730E-03	0.000000	010000
0.2022731E-03	-0.5845774E-04	-0.1467136	-0.9637261	0.000000	001000
-0.3060451E-11	0.2022730E-03	1.015304	-0.1467136	0.000000	000100
-2.390608	-0.9355901	-0.6343269E-01	0.3448516	0.000000	300000
-4.681487	-4.156470	-0.3289389	0.1435238	0.000000	210000
-5.734446	-4.222595	-0.1568456	-0.2767571	0.000000	120000
-2.226005	-2.970095	0.3628300	-0.1690648	0.000000	030000
0.6343269E-01	-0.3448516	-2.390608	-0.9355901	0.000000	201000
-0.7210790	-0.9625503E-01	-2.791046	-0.6036380	0.000000	111000
0.5109928E-01	1.028817	-4.194809	-1.077648	0.000000	021000
1.050018	-0.4726879E-01	-1.890442	-3.552832	0.000000	200100
0.1057464	-0.7520603	-1.539637	-3.144947	0.000000	110100
-0.3628300	0.1690648	-2.226005	-2.970095	0.000000	020100
-2.390608	-0.9355901	-0.6343269E-01	0.3448516	0.000000	102000
-1.890442	-3.552832	-1.050018	0.4726879E-01	0.000000	012000
-2.791046	-0.6036380	0.7210790	0.9625503E-01	0.000000	101100
-1.539637	-3.144947	-0.1057464	0.7520603	0.000000	011100
-4.194809	-1.077648	-0.5109928E-01	-1.028817	0.000000	100200
-2.226005	-2.970095	0.3628300	-0.1690648	0.000000	010200
0.6343269E-01	-0.3448516	-2.390608	-0.9355901	0.000000	003000
0.3289389	-0.1435238	-4.681487	-4.156470	0.000000	002100
0.1568456	0.2767571	-5.734446	-4.222595	0.000000	001200
-0.3628300	0.1690648	-2.226005	-2.970095	0.000000	000300
-2.968705	1.388108	1.284507	1.099203	0.000000	500000
-12.74829	-1.191336	1.970436	4.189763	0.000000	410000
-22.23560	-5.817135	-0.2996768	5.761626	0.000000	320000
-22.28744	-10.62269	-3.195642	4.297247	0.000000	230000
-15.60109	-7.532831	-4.618497	1.461128	0.000000	140000
-7.567237	-4.424200	-1.462962	0.1989097E-01	0.000000	050000
-1.284507	-1.099203	-2.968705	1.388108	0.000000	401000
-2.289681	-2.937748	-5.381652	5.573426	0.000000	311000
1.573128	-2.095154	-8.995122	12.02961	0.000000	221000
4.112552	2.454551	-4.436828	11.24604	0.000000	131000
2.900562	1.189778	-5.729758	4.570654	0.000000	041000
0.3192453	-1.252015	-7.366638	-6.764762	0.000000	400100
-1.273452	-3.666472	-13.24048	-17.84675	0.000000	310100
-0.9169098	-6.751799	-17.85061	-21.86873	0.000000	220100
1.717935	-2.650906	-9.871333	-12.10349	0.000000	130100
1.462962	-0.1989097E-01	-7.567237	-4.424200	0.000000	040100
-5.937411	2.776216	2.569013	2.198405	0.000000	302000
-20.11493	-7.956098	1.651191	5.441778	0.000000	212000

-25.24306	-14.37981	3.119961	5.260326	0.000000	122000
-17.88139	-16.40166	0.3039041	0.5566886	0.000000	032000
-5.381652	5.573426	2.289681	2.937748	0.000000	301100
-7.225557	-0.7213951	-5.565824	4.669072	0.000000	211100
-4.375287	0.3119036	-2.886541	9.935669	0.000000	121100
-9.871333	-12.10349	-1.717935	2.650906	0.000000	031100
-12.00258	3.466936	1.846510	1.593854	0.000000	300200
-22.31821	-5.155619	-3.808648	-1.897863	0.000000	210200
-21.33085	-2.962178	-7.519059	0.2713506	0.000000	120200
-15.13447	-8.848400	-2.925925	0.3978193E-01	0.000000	030200
-2.569013	-2.198405	-5.937411	2.776216	0.000000	203000
-2.289681	-2.937748	-5.381652	5.573426	0.000000	113000
-1.846510	-1.593854	-12.00258	3.466936	0.000000	023000
-1.651191	-5.441778	-20.11493	-7.956098	0.000000	202100
5.565824	-4.669072	-7.225557	-0.7213951	0.000000	112100
3.808648	1.897863	-22.31821	-5.155619	0.000000	022100
-3.119961	-5.260326	-25.24306	-14.37981	0.000000	201200
2.886541	-9.935669	-4.375287	0.3119036	0.000000	111200
7.519059	-0.2713506	-21.33085	-2.962178	0.000000	021200
-0.3039041	-0.5566886	-17.88139	-16.40166	0.000000	200300
1.717935	-2.650906	-9.871333	-12.10349	0.000000	110300
2.925925	-0.3978193E-01	-15.13447	-8.848400	0.000000	020300
-2.968705	1.388108	1.284507	1.099203	0.000000	104000
-7.366638	-6.764762	-0.3192453	1.252015	0.000000	014000
-5.381652	5.573426	2.289681	2.937748	0.000000	103100
-13.24048	-17.84675	1.273452	3.666472	0.000000	013100
-8.995122	12.02961	-1.573128	2.095154	0.000000	102200
-17.85061	-21.86873	0.9169098	6.751799	0.000000	012200
-4.436828	11.24604	-4.112552	-2.454551	0.000000	101300
-9.871333	-12.10349	-1.717935	2.650906	0.000000	011300
-5.729758	4.570654	-2.900562	-1.189778	0.000000	100400
-7.567237	-4.424200	-1.462962	0.1989097E-01	0.000000	010400
-1.284507	-1.099203	-2.968705	1.388108	0.000000	005000
-1.970436	-4.189763	-12.74829	-1.191336	0.000000	004100
0.2996768	-5.761626	-22.23560	-5.817135	0.000000	003200
3.195642	-4.297247	-22.28744	-10.62269	0.000000	002300
4.618497	-1.461128	-15.60109	-7.532831	0.000000	001400
1.462962	-0.1989097E-01	-7.567237	-4.424200	0.000000	000500

Symplectic Error : 0.1396560810794969E-002

Like in Balbekov's coordinate system we obtain zero, or almost zero, in certain characteristics elements. This fact indicate the simulation is right for certain elements.

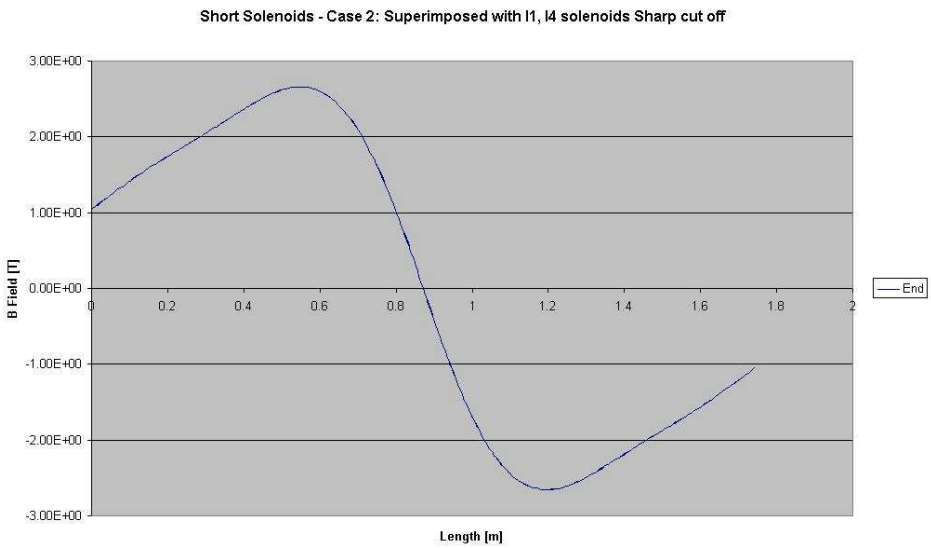

Figure 4: B field profile in case 2

1.3 Case 2: Superimposed with finite length solenoids

Again the simulation was made using CMSTP command but in this case we didn't use the approximation made in the first case by Balbekov. Now the computation is made for finite length solenoids. Kinetic Energy = 250 MeV.

The magnetic field profile is shown in figure 4,

1.3.1 Sharp Cut Off:

In this system we don't include the fringe field but we make the integration of the B Field over a real solenoid (no infinity length approximation). The transfer map is

0.3762572E-01	-1.146453	0.5033401	0.3468572E-01	0.000000	100000
1.092008	0.3762572E-01	0.1216993E-10	0.5033401	0.000000	010000
-0.5033401	-0.3468572E-01	0.3762572E-01	-1.146453	0.000000	001000
-0.1216993E-10	-0.5033401	1.092008	0.3762572E-01	0.000000	000100
-3.823132	-1.926509	-1.388275	-3.800984	0.000000	300000
-7.309344	-6.926245	-1.623955	-5.544482	0.000000	210000
-7.330362	-6.368015	-1.092651	-5.709820	0.000000	120000
-2.497266	-4.260799	0.5651303	-1.279245	0.000000	030000
1.388275	3.800984	-3.823132	-1.926509	0.000000	201000
4.536294	5.904955	-3.549300	2.046974	0.000000	111000

3.433730	9.304556	-5.091437	-0.6175929	0.000000	021000
-2.912339	-0.3604728	-3.760043	-8.973219	0.000000	200100
-2.341078	-3.594736	-2.238926	-5.750423	0.000000	110100
-0.5651303	1.279245	-2.497266	-4.260799	0.000000	020100
-3.823132	-1.926509	-1.388275	-3.800984	0.000000	102000
-3.760043	-8.973219	2.912339	0.3604728	0.000000	012000
-3.549300	2.046974	-4.536294	-5.904955	0.000000	101100
-2.238926	-5.750423	2.341078	3.594736	0.000000	011100
-5.091437	-0.6175929	-3.433730	-9.304556	0.000000	100200
-2.497266	-4.260799	0.5651303	-1.279245	0.000000	010200
1.388275	3.800984	-3.823132	-1.926509	0.000000	003000
1.623955	5.544482	-7.309344	-6.926245	0.000000	002100
1.092651	5.709820	-7.330362	-6.368015	0.000000	001200
-0.5651303	1.279245	-2.497266	-4.260799	0.000000	000300
-7.024981	3.068808	-3.212673	-5.204727	0.000000	500000
-20.85751	0.1145394E-01	-7.808336	-10.41922	0.000000	410000
-31.77869	1.148110	-16.81883	-15.72985	0.000000	320000
-31.26774	-3.142224	-16.56386	-14.27044	0.000000	230000
-20.16655	0.2965039E-02	-12.46469	-12.47953	0.000000	140000
-8.419566	-5.004937	-1.776100	-4.647193	0.000000	050000
3.212673	5.204727	-7.024981	3.068808	0.000000	401000
21.74434	1.433363	-0.2345324E-01	25.16933	0.000000	311000
45.93245	19.18062	-1.688297	40.50234	0.000000	221000
40.63174	9.860645	8.145171	38.31300	0.000000	131000
23.94901	20.39823	-3.924540	16.36480	0.000000	041000
-13.93601	8.985857	-20.83406	-25.15787	0.000000	400100
-29.11362	-3.450772	-30.09039	-39.35423	0.000000	310100
-24.06788	4.409796	-39.41291	-41.45522	0.000000	220100
-11.48432	-7.918705	-16.24201	-16.36184	0.000000	130100
1.776100	4.647193	-8.419566	-5.004937	0.000000	040100
-14.04996	6.137617	-6.425347	-10.40945	0.000000	302000
-41.69157	-25.14642	6.127670	-19.40508	0.000000	212000
-52.72772	-19.49162	-7.323765	-39.49546	0.000000	122000
-42.71906	-42.71872	10.87427	-25.34622	0.000000	032000
-0.2345324E-01	25.16933	-21.74434	-1.433363	0.000000	301100
11.80768	1.925221	10.12350	50.98200	0.000000	211100
14.75747	40.84000	-14.24453	32.01220	0.000000	121100
-16.24201	-16.36184	11.48432	7.918705	0.000000	031100
-22.63733	19.86262	-36.43739	-42.94624	0.000000	300200
-34.57389	-4.405723	-29.75746	-35.20686	0.000000	210200
-24.09109	16.36777	-36.41369	-32.87776	0.000000	120200
-16.83913	-10.00987	-3.552201	-9.294386	0.000000	030200
6.425347	10.40945	-14.04996	6.137617	0.000000	203000
21.74434	1.433363	-0.2345324E-01	25.16933	0.000000	113000
36.43739	42.94624	-22.63733	19.86262	0.000000	023000
-6.127670	19.40508	-41.69157	-25.14642	0.000000	202100

```
  -10.12350     -50.98200      11.80768      1.925221      0.000000     112100
   29.75746      35.20686     -34.57389     -4.405723      0.000000     022100
   7.323765      39.49546     -52.72772     -19.49162      0.000000     201200
   14.24453     -32.01220      14.75747      40.84000      0.000000     111200
   36.41369      32.87776     -24.09109      16.36777      0.000000     021200
  -10.87427      25.34622     -42.71906     -42.71872      0.000000     200300
  -11.48432     -7.918705     -16.24201     -16.36184      0.000000     110300
   3.552201      9.294386     -16.83913     -10.00987      0.000000     020300
  -7.024981      3.068808     -3.212673     -5.204727      0.000000     104000
  -20.83406     -25.15787      13.93601     -8.985857      0.000000     014000
 -0.2345324E-01  25.16933     -21.74434     -1.433363      0.000000     103100
  -30.09039     -39.35423      29.11362      3.450772      0.000000     013100
  -1.688297      40.50234     -45.93245     -19.18062      0.000000     102200
  -39.41291     -41.45522      24.06788     -4.409796      0.000000     012200
   8.145171      38.31300     -40.63174     -9.860645      0.000000     101300
  -16.24201     -16.36184      11.48432      7.918705      0.000000     011300
  -3.924540      16.36480     -23.94901     -20.39823      0.000000     100400
  -8.419566     -5.004937     -1.776100     -4.647193      0.000000     010400
   3.212673      5.204727     -7.024981      3.068808      0.000000     005000
   7.808336      10.41922     -20.85751     0.1145394E-01  0.000000     004100
   16.81883      15.72985     -31.77869      1.148110      0.000000     003200
   16.56386      14.27044     -31.26774     -3.142224      0.000000     002300
   12.46469      12.47953     -20.16655     0.2965039E-02  0.000000     001400
   1.776100      4.647193     -8.419566     -5.004937      0.000000     000500
 -------------------------------------------------------------------------------
Symplectic error :  0.4439028201745663
```

1.3.2 With Kicks:

Like in the case 1 with the incorporation of the kicks we obtain,

```
  0.3762572E-01-0.9144481     0.1324007E-03 0.9123892E-05  0.000000     100000
   1.092008     0.3762572E-01 0.1216993E-10 0.1324008E-03  0.000000     010000
 -0.1324007E-03-0.9123892E-05 0.3762572E-01-0.9144481      0.000000     001000
 -0.1216993E-10-0.1324008E-03  1.092008     0.3762572E-01  0.000000     000100
  -3.506944     -2.081410    -0.1403881     0.3911380      0.000000     300000
  -6.760836     -6.392128    -0.4722334    -0.5081493E-01  0.000000     210000
  -7.069945     -6.930725     0.5811285E-01-0.4885030      0.000000     120000
  -2.497266     -4.000382     0.5651303    -0.1284806      0.000000     030000
  0.1403881    -0.3911380     -3.506944     -2.081410      0.000000     201000
 -0.9477851     0.3464533E-01 -4.574534     -2.625524      0.000000     111000
 -0.1856353E-01  1.428129     -4.310184     -2.377504      0.000000     021000
   1.420018     0.1616960E-01 -2.186302     -3.766604      0.000000     200100
 -0.3954932E-01-0.9396256     -2.759761     -4.553221      0.000000     110100
 -0.5651303     0.1284806     -2.497266     -4.000382      0.000000     020100
```

-3.506944	-2.081410	-0.1403881	0.3911380	0.000000	102000
-2.186302	-3.766604	-1.420018	-0.1616960E-01	0.000000	012000
-4.574534	-2.625524	0.9477851	-0.3464533E-01	0.000000	101100
-2.759761	-4.553221	0.3954932E-01	0.9396256	0.000000	011100
-4.310184	-2.377504	0.1856353E-01	-1.428129	0.000000	100200
-2.497266	-4.000382	0.5651303	-0.1284806	0.000000	010200
0.1403881	-0.3911380	-3.506944	-2.081410	0.000000	003000
0.4722334	0.5081493E-01	-6.760836	-6.392128	0.000000	002100
-0.5811285E-01	0.4885030	-7.069945	-6.930725	0.000000	001200
-0.5651303	0.1284806	-2.497266	-4.000382	0.000000	000300
-4.559878	2.194351	1.925709	2.734348	0.000000	500000
-14.03917	1.790312	1.247951	8.100432	0.000000	410000
-26.15120	-0.5027561	-4.741542	9.421870	0.000000	320000
-29.55137	-6.150214	-9.833677	4.913186	0.000000	230000
-20.98499	-6.094498	-8.584871	-0.5031165	0.000000	140000
-8.419566	-5.823382	-1.776100	-0.7673755	0.000000	050000
-1.925709	-2.734348	-4.559878	2.194351	0.000000	401000
-3.184145	-6.348066	-7.048422	8.618150	0.000000	311000
6.011687	-1.270187	-13.83428	14.42301	0.000000	221000
9.435860	7.481453	-12.97088	11.56420	0.000000	131000
4.549919	5.172413	-8.016761	3.560799	0.000000	041000
1.936194	-1.752366	-6.990751	-6.827839	0.000000	400100
-1.270145	-8.151683	-12.31692	-14.92577	0.000000	310100
0.3978174	-12.39464	-16.58049	-17.71442	0.000000	220100
4.034952	-4.669297	-12.96823	-9.655297	0.000000	130100
1.776100	0.7673755	-8.419566	-5.823382	0.000000	040100
-9.119756	4.388702	3.851418	5.468696	0.000000	302000
-21.02992	-5.037527	-0.6882433	9.852798	0.000000	212000
-26.65942	-13.64111	1.614651	9.778743	0.000000	122000
-16.45390	-15.80807	-0.1310618	2.532026	0.000000	032000
-7.048422	8.618150	3.184145	6.348066	0.000000	301100
-11.30046	11.35094	-11.44224	7.437937	0.000000	211100
-13.22406	7.751500	-9.969371	12.24377	0.000000	121100
-12.96823	-9.655297	-4.034952	4.669297	0.000000	031100
-14.34250	1.284657	0.3445058	1.627060	0.000000	300200
-29.42478	-4.243863	-9.566922	-4.949428	0.000000	210200
-29.00175	-2.533699	-13.13479	-5.675530	0.000000	120200
-16.83913	-11.64676	-3.552201	-1.534751	0.000000	030200
-3.851418	-5.468696	-9.119756	4.388702	0.000000	203000
-3.184145	-6.348066	-7.048422	8.618150	0.000000	113000
-0.3445058	-1.627060	-14.34250	1.284657	0.000000	023000
0.6882433	-9.852798	-21.02992	-5.037527	0.000000	202100
11.44224	-7.437937	-11.30046	11.35094	0.000000	112100
9.566922	4.949428	-29.42478	-4.243863	0.000000	022100
-1.614651	-9.778743	-26.65942	-13.64111	0.000000	201200
9.969371	-12.24377	-13.22406	7.751500	0.000000	111200

```
  13.13479      5.675530     -29.00175     -2.533699      0.000000     021200
 0.1310618     -2.532026     -16.45390     -15.80807      0.000000     200300
  4.034952     -4.669297     -12.96823     -9.655297      0.000000     110300
  3.552201      1.534751     -16.83913     -11.64676      0.000000     020300
 -4.559878      2.194351      1.925709      2.734348      0.000000     104000
 -6.990751     -6.827839     -1.936194      1.752366      0.000000     014000
 -7.048422      8.618150      3.184145      6.348066      0.000000     103100
 -12.31692     -14.92577      1.270145      8.151683      0.000000     013100
 -13.83428      14.42301     -6.011687      1.270187      0.000000     102200
 -16.58049     -17.71442    -0.3978174      12.39464      0.000000     012200
 -12.97088      11.56420     -9.435860     -7.481453      0.000000     101300
 -12.96823     -9.655297     -4.034952      4.669297      0.000000     011300
 -8.016761      3.560799     -4.549919     -5.172413      0.000000     100400
 -8.419566     -5.823382     -1.776100    -0.7673755      0.000000     010400
 -1.925709     -2.734348     -4.559878      2.194351      0.000000     005000
 -1.247951     -8.100432     -14.03917      1.790312      0.000000     004100
  4.741542     -9.421870     -26.15120    -0.5027561      0.000000     003200
  9.833677     -4.913186     -29.55137     -6.150214      0.000000     002300
  8.584871     0.5031165     -20.98499     -6.094498      0.000000     001400
  1.776100     0.7673755     -8.419566     -5.823382      0.000000     000500
-------------------------------------------------------------------------------

Symplectic error :  0.1625025668560141E-002
```

The symplectic error goes down in this simulation what means a good approximation to the real behave. Again we have approximately zero in the elements (x,y); (x,b); (a,y); (a,b); (y,x); (y,a); (b,x) and (b,a).

1.4 Case 3: Superimposed with finite length solenoids and negative drifts

In this case fringe fields are included automatically.

The graphic in figure 5 represent the magnetig field in the solenoid (Bz). We only show the main part of the graphic, due the system can represent the magnetic field outside the solenoid (fringe field). If we compare this picture with the one in case 2 we can see like the B field goes beyond 1.7 m and before zero.

```
 0.9113584E-02-0.9101484     0.2707143E-04-0.8741688E-06  0.000000     100000
  1.098631     0.9113584E-02-0.1597370E-05 0.2707097E-04  0.000000     010000
-0.2707143E-04 0.8741688E-06 0.9113584E-02-0.9101484      0.000000     001000
 0.1597370E-05-0.2707097E-04  1.098631     0.9113584E-02  0.000000     000100
 -3.150921     -1.837470    -0.1834700E-02 0.1854774      0.000000     300000
 -6.500820     -5.842104    -0.2235787    -0.5519444E-02  0.000000     210000
 -6.921392     -6.536973     0.7051571E-02-0.2237857      0.000000     120000
 -2.569483     -3.759954     0.2704996     0.2268403E-02  0.000000     030000
```

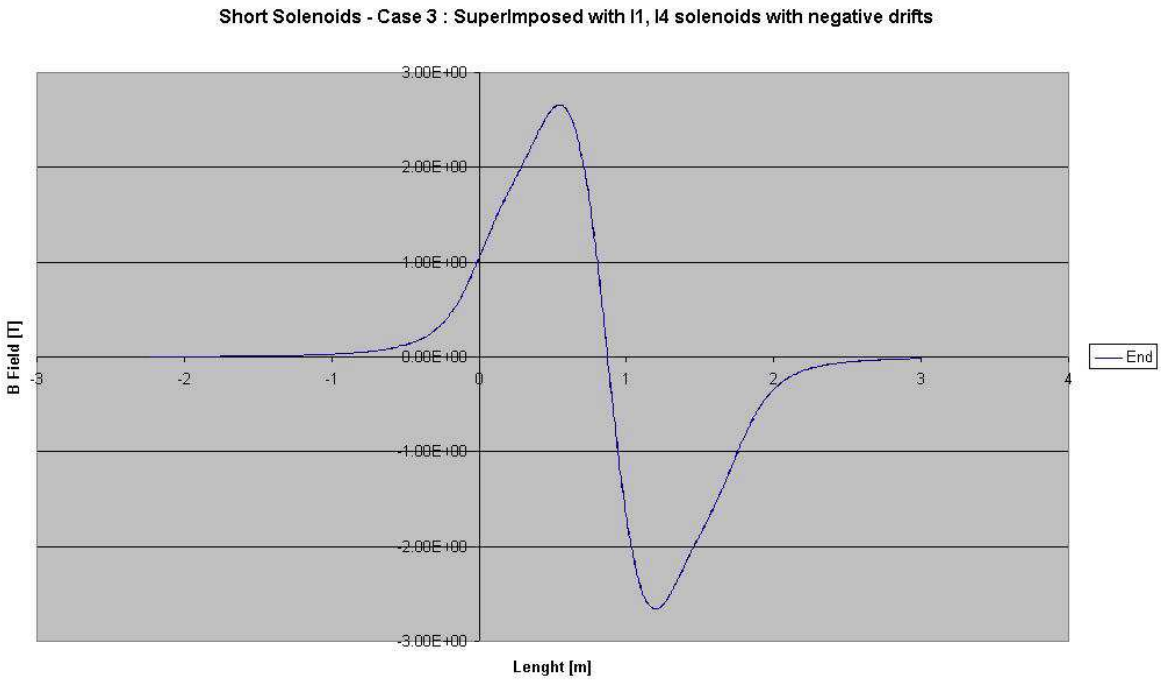

Figure 5: B field in case #3

0.1834700E-02	-0.1854774	-3.150921	-1.837470	0.000000	201000
-0.4477300	0.3828194E-02	-4.350800	-2.205016	0.000000	111000
-0.2176700E-02	0.6719272	-4.346775	-2.195911	0.000000	021000
0.6713086	0.1691251E-02	-2.150020	-3.637088	0.000000	200100
-0.4874880E-02	-0.4481415	-2.574617	-4.341062	0.000000	110100
-0.2704992	-0.2268410E-02	-2.569487	-3.759954	0.000000	020100
-3.150921	-1.837470	-0.1834700E-02	0.1854774	0.000000	102000
-2.150020	-3.637088	-0.6713086	-0.1691251E-02	0.000000	012000
-4.350800	-2.205016	0.4477300	-0.3828194E-02	0.000000	101100
-2.574617	-4.341062	0.4874880E-02	0.4481415	0.000000	011100
-4.346775	-2.195911	0.2176700E-02	-0.6719272	0.000000	100200
-2.569487	-3.759954	0.2704992	0.2268410E-02	0.000000	010200
0.1834700E-02	-0.1854774	-3.150921	-1.837470	0.000000	003000
0.2235787	0.5519444E-02	-6.500820	-5.842104	0.000000	002100
-0.7051571E-02	0.2237857	-6.921392	-6.536973	0.000000	001200
-0.2704996	-0.2268402E-02	-2.569483	-3.759954	0.000000	000300
-4.919054	1.095873	0.6296333	1.603693	0.000000	500000
-14.58589	0.6387705	-0.2037065	4.302134	0.000000	410000
-24.12332	-1.227331	-3.197581	3.442199	0.000000	320000
-26.81068	-7.088700	-4.268833	-0.1562313	0.000000	230000
-19.59752	-9.127604	-2.390249	-2.335453	0.000000	140000
-8.224955	-7.060173	0.1989972	-0.9178104	0.000000	050000
-0.6296333	-1.603693	-4.919054	1.095873	0.000000	401000
-4.056742	-4.350419	-7.643146	7.928397	0.000000	311000
-1.032520	0.1320169	-11.92493	14.53062	0.000000	221000
4.303828	8.546711	-10.46203	11.70051	0.000000	131000
3.723242	7.234822	-7.672192	2.976700	0.000000	041000
4.260448	0.4828497E-01	-6.942749	-7.289627	0.000000	400100
4.230101	-3.574215	-12.19839	-15.75795	0.000000	310100
-0.3502340E-01	-8.390479	-16.34866	-18.78921	0.000000	220100
-1.330578	-4.899385	-11.92471	-12.10429	0.000000	130100
-0.2597511	0.9180567	-8.251046	-7.060358	0.000000	040100
-9.838109	2.191747	1.259267	3.207385	0.000000	302000
-21.52864	-6.650856	-4.464155	4.253849	0.000000	212000
-26.15939	-13.86911	-2.975168	3.529459	0.000000	122000
-16.87910	-16.24962	0.1222064E-01	1.885221	0.000000	032000
-7.643146	7.928397	4.056742	4.350419	0.000000	301100
-8.126244	9.525617	-4.674927	3.399695	0.000000	211100
-9.401110	6.621329	-4.258269	4.463806	0.000000	121100
-11.92579	-12.10429	1.332459	4.899363	0.000000	031100
-13.96100	1.888834	1.254934	-0.4475678E-01	0.000000	300200
-27.34113	-4.549109	-4.291627	-6.661490	0.000000	210200
-27.27074	-6.150883	-6.112953	-9.570270	0.000000	120200
-16.43574	-14.12087	0.4056194	-1.835593	0.000000	030200
-1.259267	-3.207385	-9.838109	2.191747	0.000000	203000
-4.056742	-4.350419	-7.643146	7.928397	0.000000	113000

```
 -1.254934     0.4475678E-01 -13.96100      1.888834      0.000000     023000
  4.464155     -4.253849     -21.52864     -6.650856      0.000000     202100
  4.674927     -3.399695     -8.126244      9.525617      0.000000     112100
  4.291627      6.661490     -27.34113     -4.549109      0.000000     022100
  2.975168     -3.529459     -26.15939     -13.86911      0.000000     201200
  4.258269     -4.463806     -9.401110      6.621329      0.000000     111200
  6.112942      9.570271     -27.27074     -6.150883      0.000000     021200
-0.1222064E-01 -1.885221     -16.87910     -16.24962      0.000000     200300
 -1.332469     -4.899363     -11.92579     -12.10429      0.000000     110300
-0.4056751      1.835609     -16.43575     -14.12087      0.000000     020300
 -4.919054      1.095873     0.6296333      1.603693      0.000000     104000
 -6.942749     -7.289627     -4.260448    -0.4828497E-01  0.000000     014000
 -7.643146      7.928397      4.056742      4.350419      0.000000     103100
 -12.19839     -15.75795     -4.230101      3.574215      0.000000     013100
 -11.92493      14.53062      1.032520    -0.1320169      0.000000     102200
 -16.34866     -18.78921     0.3502362E-01  8.390479      0.000000     012200
 -10.46203      11.70051     -4.303828     -8.546711      0.000000     101300
 -11.92472     -12.10429      1.330561      4.899385      0.000000     011300
 -7.672191      2.976700     -3.723244     -7.234822      0.000000     100400
 -8.251015     -7.060343     0.2604491    -0.9180558      0.000000     010400
-0.6296333     -1.603693     -4.919054      1.095873      0.000000     005000
 0.2037065     -4.302134     -14.58589     0.6387705      0.000000     004100
  3.197581     -3.442199     -24.12332     -1.227331      0.000000     003200
  4.268833     0.1562313     -26.81068     -7.088700      0.000000     002300
  2.390273      2.335452     -19.59755     -9.127604      0.000000     001400
-0.1994733     0.9178381     -8.224912     -7.060156      0.000000     000500
-------------------------------------------------------------------------------
```

Symplectic error : 0.2036645962560155E-004

2 Comparison between V. Balvekov's Maps and COSY ∞

```
Matrices in V. Balbekov's study make use of (X,dX/dZ,Y,dY/dZ) (cm, rad).
Length of solenoids 174.39736 and 667.79 cm, hard edges appx.
Three energies were considered by Valeri Balbekov: Etotal = 200, 250, 300 MeV.
```

Short solenoid

Balbekov's Matrix (Total Energy = 200 MeV)

```
-.9555563  -9.8735123    .0000000    .0000000
 .0088026   -.9555563    .0000000    .0000000
 .0000000    .0000000   -.9555563  -9.8735123
 .0000000    .0000000    .0088026   -.9555563
```

COSY ∞ 's matrix (Total Energy = 200 MeV)

```
 -0.9564893     0.8698287     0.3899243E-04 0.7621680E-03  100000
 -0.9786779E-01-0.9564893    -0.6900079E-10 0.3899249E-04  010000
 -0.3899243E-04-0.7621680E-03-0.9564893     0.8698287      001000
  0.6900079E-10-0.3899249E-04-0.9786779E-01-0.9564893      000100

Symplectic Error :  0.2103333055282110E-003
```

Balbekov's Matrix (Total Energy 250 MeV - K.E. = 144.342 MeV)

```
 -.8496877  38.7790486    .0000000    .0000000
 -.0071696   -.8496877    .0000000    .0000000
  .0000000    .0000000   -.8496877  38.7790486
  .0000000    .0000000   -.0071696   -.8496877
```

COSY ∞ 's matrix (Total Energy 250 MeV - K.E. = 144.342 MeV)

```
 -0.8482881    -0.7207622    -0.1161705E-03 0.5066076E-03  100000
  0.3890428    -0.8482881    -0.8252680E-10-0.1161704E-03  010000
  0.1161705E-03-0.5066076E-03-0.8482881    -0.7207622      001000
  0.8252680E-10 0.1161704E-03 0.3890428    -0.8482881      000100

Symplectic Error :  0.1889487657397328E-003
```

Balbekov's Matrix (Total Energy 300 MeV)

```
 -.4892819  74.8901796    .0000000    .0000000
 -.0101562   -.4892819    .0000000    .0000000
  .0000000    .0000000   -.4892819  74.8901796
  .0000000    .0000000   -.0101562   -.4892819
```

COSY ∞ 's matrix (Total energy 300 MeV)

```
 -0.4874776     -1.016476    -0.1807232E-03 0.2349266E-03  100000
  0.7500085    -0.4874776    -0.9316218E-11-0.1807232E-03  010000
  0.1807232E-03-0.2349266E-03-0.4874776     -1.016476      001000
  0.9316218E-11 0.1807232E-03 0.7500085    -0.4874776      000100

Symplectic Error :  0.1525071377607198E-003
```