1 Cooling Efficiency

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Introduction

It has become popular to judge cooling systems by their "Merit Factor":

$$\text{Merit} = \frac{\text{6D emittance In}}{\text{6D Emittance Out}} \times \text{Transmission} \tag{1}$$

- Merit depends on the initial beam size.
- A maximum Merit (Harold Kirk) starting with a uniformly fill
  but this is not a realistic distribution.
- Merit is also maximized by cooling for as long as possible. Will not give the best final performance for multiples
- A better criterion, perhaps, is a local parameter: "Cooling Efficiency" ($Q_6(z)$).

Definition of Efficiency Q_6

Define

$$Q_6(z) = \frac{d\epsilon_6/\epsilon_6}{dN/N} \tag{2}$$

Note, if $Q_6(z)$=constant, then

$$\int_o^n \frac{d\epsilon_6}{\epsilon_6} = Q_6 \int_o^n \frac{dN}{N}$$

$$Ln\left(\frac{\epsilon_6(n)}{\epsilon_6(o)}\right) = Q_6 \; Ln\left(\frac{N(n)}{N(o)}\right)$$

$$\frac{N(n)}{N(o)} = \left(\frac{\epsilon_6(n)}{\epsilon_6(o)}\right)^{1/Q_6} \tag{3}$$

This suggests a definition for an effective average $Q_6(ave)$:

$$Q_6(ave) = \frac{Ln\left(\frac{\epsilon_6(n)}{\epsilon_6(o)}\right)}{Ln\left(\frac{N(n)}{N(o)}\right)}$$

Requirement for a collider The Luminosity of a collider is given by

$$\mathcal{L} \propto \frac{\mathrm{N}^2\ \mathrm{f}}{\sigma_{\mathrm{x}}\ \sigma_{\mathrm{y}}} \propto \frac{\mathrm{N}^2}{\epsilon_\perp\ \beta} \ \propto\ \frac{\mathrm{N}^2}{\epsilon_6^{2/3}}$$

If N^2 and $\epsilon_6^{2/3}$, instead of N and ϵ_6, are taken as the criterion for cooling then:

$$\frac{N^2(n)}{N^2(o)} \ =\ \left(\frac{\epsilon_6(n)^{2/3}}{\epsilon_6(o)^{2/3}}\right)^{3/Q_6}$$

If no limit to how small the emittance, then $Q > 3$ will raise luminosity without limit.

For Collider Feasibility Study:

$$\frac{\epsilon_6(n)}{\epsilon_6(o)} = 10^{-6}, \qquad \frac{N(n)}{N(o)} \ =\ 0.5$$

Requiring $\qquad Q_6 \ =\ ln\left(\frac{10^{-6}}{0.5}\right) \ =\ 20$

Theoretical Expectation

With no heating:

$$\frac{d\epsilon_i}{\epsilon_i} = J_i \frac{dp}{p} = J_i \frac{dE}{E} \frac{1}{\beta_v^2}$$

If there are no wedges:

$$J_x = J_y = 1, \quad J_z \approx 0$$

With wedges: distribute the J's if

$$J_x + J_y + J_z \approx 2$$

$$Q_i = \frac{\frac{d\epsilon_i}{\epsilon_i}}{\frac{dn}{n}} = \frac{J_i \frac{dE}{E} \frac{1}{\beta_v^2}}{\frac{d\ell}{(c\gamma\beta_v\tau_\mu)}} \left(\frac{\text{Decay Loss}}{\text{All Losses}}\right)$$

$$Q_i = \left(J_i \frac{c\ \tau_\mu}{m_\mu}\right) \frac{dE}{d\ell} \frac{1}{\beta_v} \left(\frac{\text{Decay Loss}}{\text{All Losses}}\right) \tag{4}$$

With heating:

$$Q_i = \left(1 - \frac{\epsilon_{\min}}{\epsilon}\right) \left(J_i \frac{c\ \tau_\mu}{m_\mu}\right) \frac{dE}{d\ell} \frac{1}{\beta_v} \left(\frac{\text{Decay Loss}}{\text{All Losses}}\right) \tag{5}$$

where for the **transverse directions**:

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$$\epsilon_{x,y}(min) \;=\; \frac{\beta_\perp}{J_i\ \beta_v}\ \ C(mat,E) \tag{6}$$

$$C(mat,E) \;=\; \frac{1}{2}\ \left(\frac{14.1\ 10^6}{(m_\mu)}\right)^2\ \frac{1}{L_R\ d\gamma/ds} \tag{7}$$

(For Hydrogen, $C(mat,E) \approx 38\ 10^{-4}$)

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and in the **longitudinal direction**:

$$(\epsilon_z)_{\rm min} \;=\; \beta_v \gamma\ \left(\frac{\sigma_p}{p}\right)_{\rm min}\ \ \sigma_z \tag{8}$$

where σ_z is set by the rf, and:

$$\left(\frac{\sigma_p}{p}\right)_{\rm min} \;=\; D(mat)\ \sqrt{\frac{\gamma}{\beta_v^2}\ \left(1-\frac{\beta_v^2}{2}\right)\ \frac{1}{J_z}} \tag{9}$$

$$D(mat) \;=\; \left(\left(\frac{m_e}{m_\mu}\right)\ \sqrt{\frac{0.06\ Z\ \rho}{2\ A\ (d\gamma/ds)}}\right)$$

(For Hydrogen, $D(mat,E) \approx 1.45\ \%$).

We see that:

- The efficiency Q_6 is strongly (linearly, for the same phase) dependent on the accelerating gradient dE/E.
- The cooling rate per unit of length is strongly dependent on the cooling momentum $\propto 1/(\gamma\beta^2)$, but Q_6 is only weakly dependent on the cooling momentum ($Q_6 \propto 1/\beta$): at higher momenta the cooling is slower but the decay rate is also slower.
- The efficiency is greatly reduced if the particle loss from scraping exceeds the decay loss.
- The efficiency is a function of the ratio of emittances to their equilibrium values

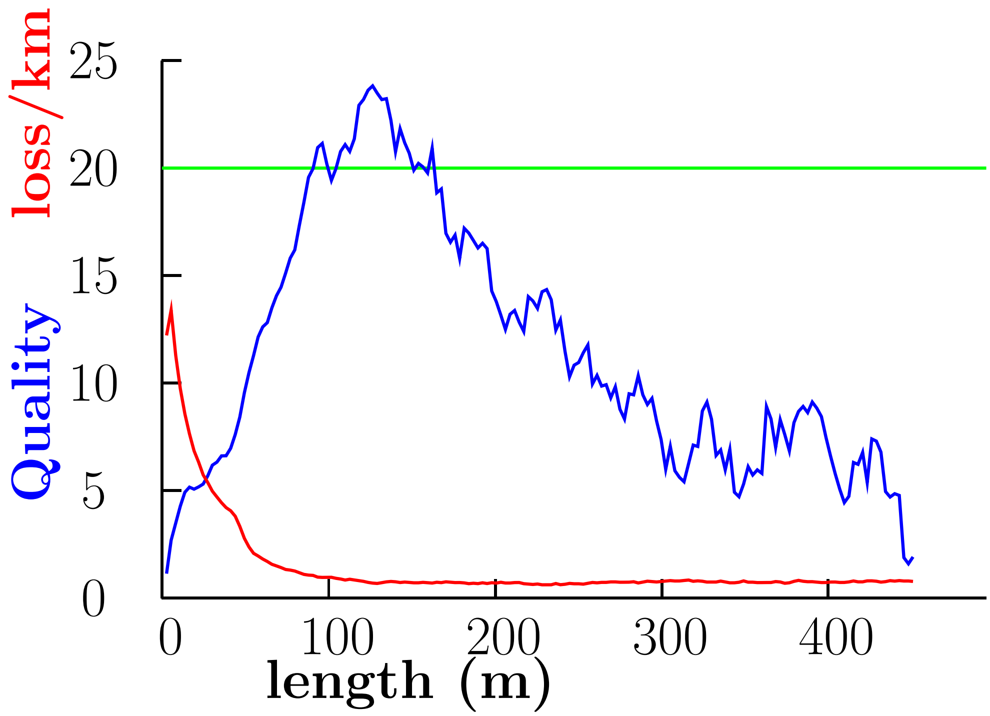
$$Q_i \propto \left(1 - \frac{\epsilon_{\min}}{\epsilon}\right)_{x,y,z}$$

- In the transverse direction the smallness of $\epsilon_{\min}/\epsilon$ will be limited by the lattice angular acceptance ($\sigma_\theta \propto \sqrt{\epsilon/\beta} \propto \sqrt{\epsilon/\epsilon_{\rm min}}$) needed to assure negligible scraping.

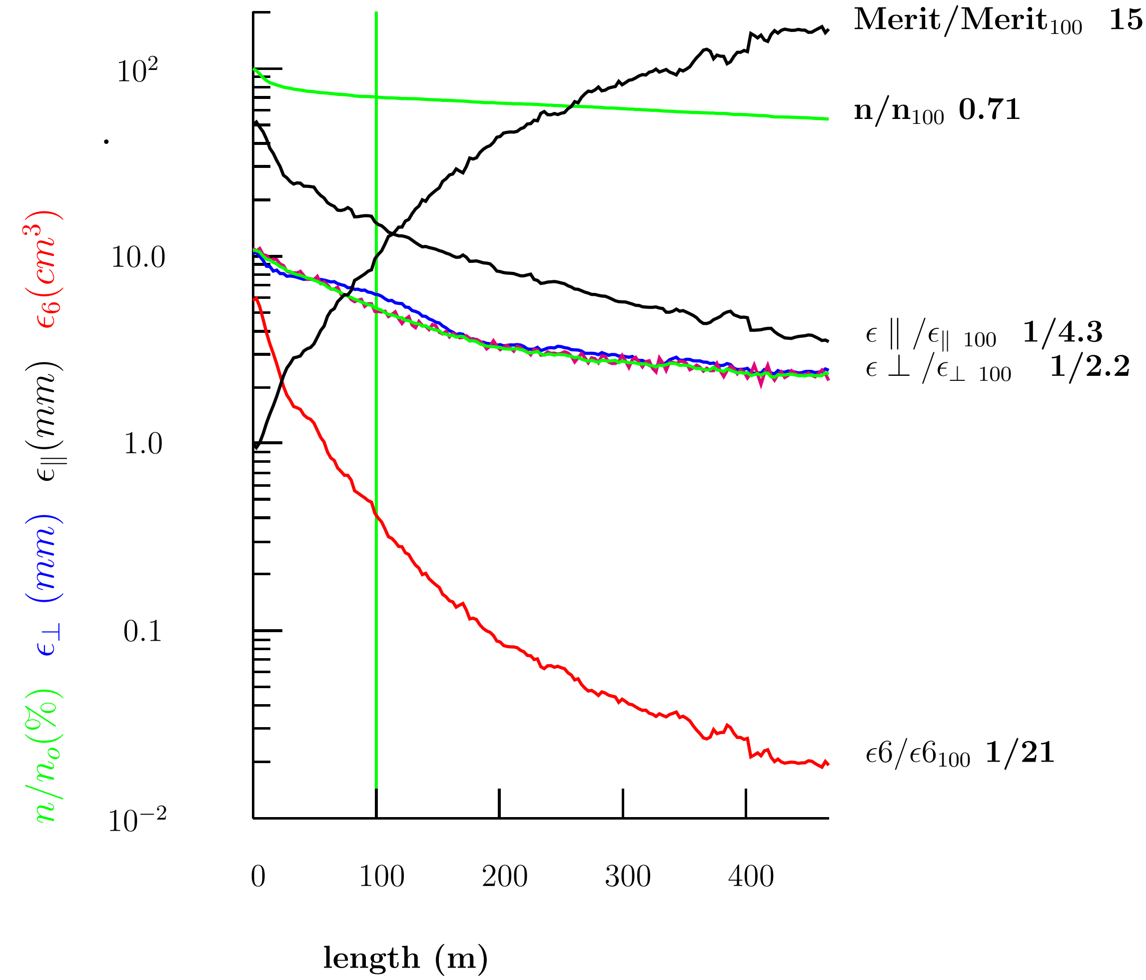
- In the longitudinal direction, $\epsilon_{\rm min}/\epsilon$ will be limitted by the bucket size needed to avoid significant losses.
- maintaining $\epsilon/\epsilon(min.)$ in the transverse direction implies tapering down the transverse beta with distance.
- maintaining $\epsilon/\epsilon(min.)$ in the longitudinal direction implies tapering down the rf frequency so as to compress the bunch and maintain a high rms dp/p.

RFOFO Ring Example

e.g. for RFOFO Ring with Hydrogen wedges, without windows or Injection/extraction



We see thsat the losses are only small compared with decay loss after about 100 m of cooling in this ring.



.	len m	trans %	$\epsilon_\perp$ mm	$\epsilon_\parallel$ mm	ϵ_6 cm^3	max Q	merit
final	468	54	2.3	3.5	0.019	24	
after 100 m	100		5.0	15	0.375		
ratio 100m→			2.2	4.3	21	24	**15**
initial			10.7	50.1	5.787		
ratio from start			4.6	14.4	302.0		**162**

We Note:

- Initially, the emittances are falling, but Q is low because particles are being lost by scraping or falling out of the bucket.
- After about 100 m, particle loss, other than decay, has become negligible and Q reaches a maximum that exceeds the above collider specification of 20.
- At this "negligible loss" point $\epsilon_\perp/\epsilon_{\perp\mathrm{min}} = 2.2$ and $\epsilon_\parallel/\epsilon_{\parallel\mathrm{min}} = 4.3$. These will be reduced when rf and absorber windows are included.
- Later, as the emittances approach their equilibria, the efficiency falls again. But if the lattice were tapered, with a falling transverse beta, then the efficiency could presumably be maintained for a significant length.

Required Acceptances for low loss

At 100 m, **Transverse**:

$$\sigma_r \;=\; \sqrt{2}\,\sigma_\perp \;=\; 4.58cm$$

cf 18 cm: **3.9 sigma**.

$$A_r \;=\; \beta_v\gamma\, r^2\,\beta_\perp \;=\; 52\ \mathrm{mm}$$

10.2 $\times\ \epsilon_\perp$.

At 100 m, **Longitudinal**:

$$\sigma_z \;=\; 10\ \mathrm{cm}$$

cf bucket $\pm$ 32 cm: **3.2 sigma**.

$$A_\parallel \;=\; \beta_v\gamma\,\sigma_z\,\sigma_p/p \;=\; 2\times.32\times0.24 \;=\; 153\,\mathrm{mm}$$

10.2 $\times\ \epsilon_\parallel$.

Scattering & straggling non-Gaussian

Conclusion

- Efficiency Q_6 is a useful criterion for judging cooling systems
- Its maximum value does not depend on unrealistic initial beams
- It can be applied to linear or ring systems equally
- A goal of $Q_6 \geq 20$ needed for the 'Collider Feasibility Study' parameter, provides a useful benchmark
- It is shown that this value is achieved briefly in the RFOFO Ring without windows.
- Continuous cooling with tapering, and very thin windows, would appear needed to meet this requirement.