



# FODO-based Quadrupole Cooling Channel

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## Quadrupole Cooling Channels: General Considerations

- Longitudinal and Transverse Acceptance of Channel
  - ◆ Compare acceptances of different optical structures
- Insertion of Absorber
  - ◆ Location of minimal beam size, both planes
  - ◆ Calculate equilibrium emittance limit
- Physical Limitations
  - ◆ Quadrupole aperture and length constraints
  - ◆ Available space between magnets
- Match to emittance-exchange channel
  - ◆ Can same optical structure be used for emittance exchange

## Choice of Optical Structure

- **FODO**
  - ◆ Simplest: alternating focussing and defocussing (in one transverse plane) lenses
  - ◆ A minimum in beta or beam size cannot be achieved simultaneously in both planes
- **Doublet or Triplet quadrupole system**
  - ◆ 2 or 3 consecutive, alternating focussing and defocussing quadrupoles
  - ◆ required to form simultaneous low beta points in both planes (interaction regions of colliders, for example)

# Acceptance of Quadrupole Channels

## ■ Transverse Acceptance

- ◆ Using only linear elements (quadrupoles and/or dipoles), the transverse dynamic aperture is normally larger than the physical aperture (unless a strong resonance is encountered in a long series of cooling cells).
- ◆ Practically, FODO and doublet/triplet quadrupole channels have transverse acceptances or apertures limited only by poletip strength for a given gradient ( $\leq 8T$  for superconducting quads and  $\leq 2T$  for normal conducting).

## ■ Longitudinal Acceptance

- ◆ The FODO cell is a simple lens system and has the largest chromatic acceptance of any quadrupole-based structure. Lattices based on FODO cells have been designed which transmit up to a factor of 4 change in momentum.
- ◆ Doublet/triplet-based quadrupole structures are momentum limited to approximately  $\leq 5\%$  deviation from the central momentum of the channel.

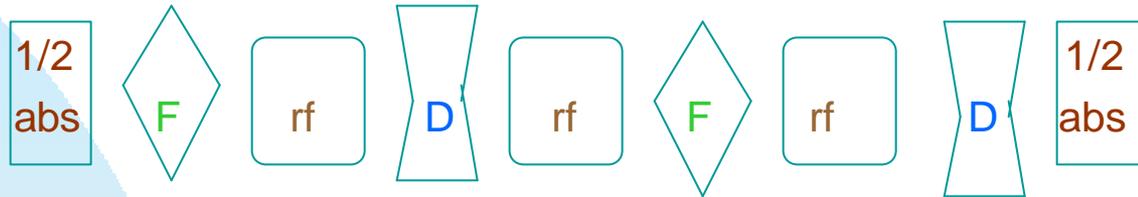
## ■ Logistics

- ◆ Because the FODO cell cannot achieve a minimum beta point in both planes, its valid application is just after capture and phase rotation, where the transverse and longitudinal emittances are very large.
- ◆ Conversely, the limited momentum acceptance of the triplet/doublet quadrupole channels restrict their implementation to after emittance exchange has occurred.



The rest of this talk will discuss the FODO cooling channel only

## Construction of FODO Quad Cooling Cell



### COOLING CELL PHYSICAL PARAMETERS:

Quad Length	0.6 m
Quad bore	0.6 m
Poletip Field	~1 T
Interquad space	0.4 - 0.5 m
Absorber length	0.35 m *
RF cavity length	0.4 - 0.7 m*
Total cooling cell length	4 m

\*The absorber and the rf cavity can be made longer if allowed to extend into the ends of the magnets.

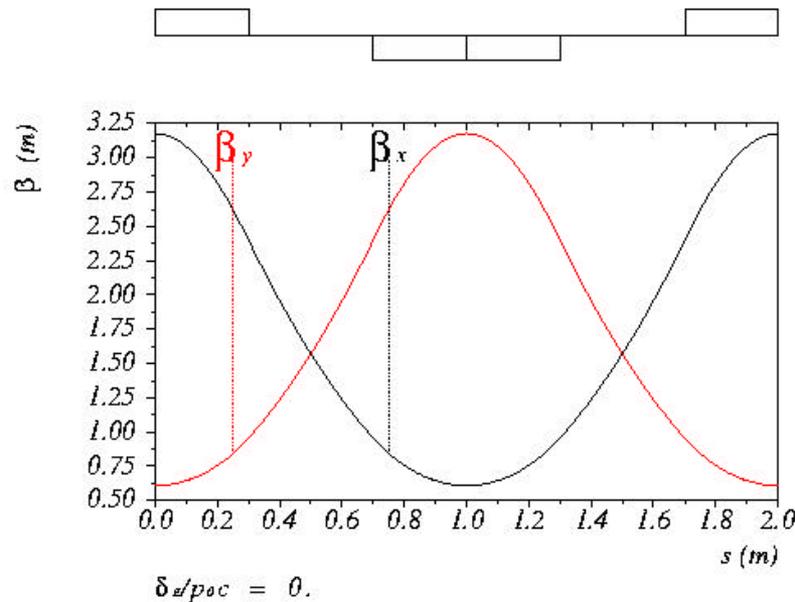
Or, more rf can be added by inserting another FODO cell between absorbers

In this design



For applications further upstream at larger emittances, this channel can support a 0.8 m bore, 0.8 m long quadrupole with no intervening drift without matching to the channel described here.

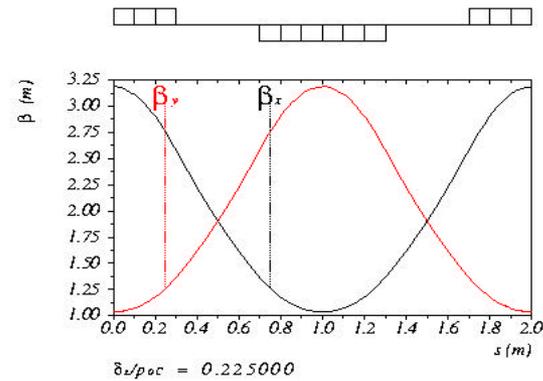
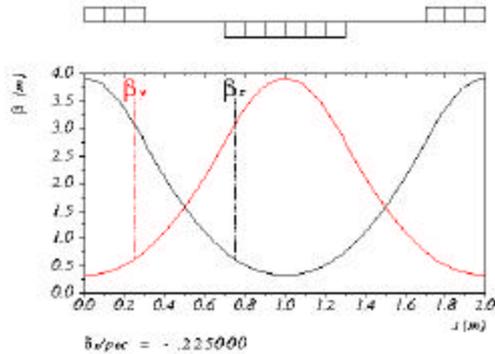
# FODO LATTICE AND INSERTION OF ABSORBER



LATTICE FUNCTIONS OF A FODO CELL FOR A QUAD COOLING CHANNEL,  $P_0=200$  MeV/c

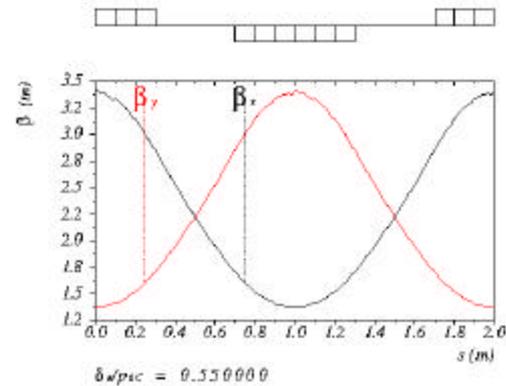
- In a FODO cell, the combined minimum for  $\beta_x$  and  $\beta_y$  is at their crossing point, halfway between quadrupoles.
- The achievable  $\beta_{\text{transverse}}$  in the absorber is about 1.6 m, or a factor of 4 larger than the average transverse  $\beta$  in the 2.75 m SFOFO channel (0.4 m).
- This channel can accept a  $3\sigma$  beam size for a transverse,  $\epsilon_N(\text{rms})$  of  $\sim 20$  mm-rad at a  $p_0$  of 200 MeV/c.

# Longitudinal Acceptance and Lattice Stability at Different Momenta



LATTICE FUNCTIONS for 155, 245, and 300 MeV/c, clockwise.

- $\beta$ average at the absorber ranges from 1.57 to 1.90 at 155 and 245 MeV/c, respectively, which represents the momentum range of the 2.75 m SFOFO channel.
- $\beta_{max}$  is 3.90 m @155 MeV/c and 3.19 m @245 MeV/c
- The acceptance reach of this channel is clearly larger than 300 MeV/c; i.e.  $\beta_{avg}$  is still only 2.3 m. and  $\beta_{max}$  is 3.5 m



## Fodo Cell Properties as a Function of Momentum

P MeV/c	$\beta$ at absorber (m)	$\nu$ per FODO cell	$\beta$ max	$\beta$ min
155	1.57 m	.33	3.85	0.35
200	1.63	.23	3.13	0.71
245	1.91 m	.18	3.20	1.10
300	2.28 m	.15	3.45	1.37

# Equilibrium Stability Limits of the FODO Transverse Cooling Channel

The equilibrium emittance is given by

$$\varepsilon_{N,\min} = \frac{\beta_{\perp} (14 \text{ MeV})^2}{(2\beta m_{\mu} L_R \cdot dE/ds)}$$

where  $\beta_{\perp}$  is the transverse beta function at the absorber,  $\beta$  the relativistic velocity,  $m_{\mu}$  the mass of the muon,  $L_R$  the radiation length of the absorber material, and  $dE/ds$  the energy lost in the absorber.

The equilibrium emittance for this quad channel is **6.8 mm-rad** @200 MeV/c ( $\beta_{\perp} \sim 1.6$  m). This is to be compared to an initial rms acceptance of 20 mm-rad.

The equilibrium emittance for the 2.75 m solenoidal channel is **1.7 mm-rad** @200 MeV/c ( $\beta_{\perp} \sim 0.4$  m). This is to be compared to the initial rms acceptance of 12 mm-rad.

# Current Status of Tracking Studies of the FODO quad cooling channel

## Tracking Studies were performed:

- ◆ with full nonlinear terms
- ◆ with/without quadrupole fringe fields
- ◆ with multiple scattering

## Preliminary estimates of cooling were obtained by:

1. determining the invariant phase ellipse of the quad channel
2. tracking in 1 cm steps along the x axis to determine the dynamic aperture of the channel
3. particles were then launched on the outer stable invariant ellipse at various  $x, x'$  coordinates and on one inner phase ellipse near the calculated equilibrium emittance
4. particle positions were plotted for 10 cells, but at increasing distance down the length of the cooling channel (cells 21-30, 31-40, 101-110, for example) until the cooling converged.
5. the rough cooling factor was obtained by comparing the outermost stable ellipse with the final ellipse which clearly contained the majority of the particles.

## Preliminary Results

Approximately:

The quadrupole channel was found to cool from 12.5 cm to  $\leq 8$  cm on the invariant ellipses.

THIS corresponds to an initial emittance of 9.8 mm-rad ( $\epsilon_N = 18.6$  mm-rad) and a final emittance of 4.0 mm-rad ( $\epsilon_N = 7.6$  mm-rad).

 THIS RESULTS SUPPORT A COOLING FACTOR OF 2.5



HOWEVER, further study is warranted

The Next Step is to Add:

- agree on a measure of emittance (rms emittance)
- realistic rf bucket
- $dE/ds$  as a function of energy
- straggling

## Goals of the FODO-based Quad Cooling Channel

- Cool transversely by a factor of 2 in the emittance of each plane: from an rms normalized emittance of about 20 mm-rad to 10 mm-rad.
- Inject cleanly (without matching) into an emittance exchange channel using the same FODO-based optical cell.
- Recool transversely with the same channel.
- After this channel, inject into more sophisticated cooling channels such as solenoidal ones to cool to the final required emittances

## Future Design Studies:

- Move the central energy of the channel to the minimum in the total strength of the reheating terms ( $\sim 300\text{-}400 \text{ MeV}/c$ ) by increasing the poletip strength of the quadrupoles (they are still normal conducting).
- Although this, in principle, halves the momentum spread, this quad channel will still accept more than a 40% total momentum bite due to its naturally high longitudinal acceptance.
- Compare cooling rates, final emittances, and losses at the two different momenta
- Investigate superconducting rf and increased absorber lengths. (The quadrupole end fields fall much more quickly than solenoidal ones.)