Methods for Pseudo Invariants

Decoupled Damped Systems

- Restore "Determinant=1" by scaling the transfer map by the damping factor

$$\sqrt{D}$$

  where D is the determinant of the linear map of the decoupled damped system.

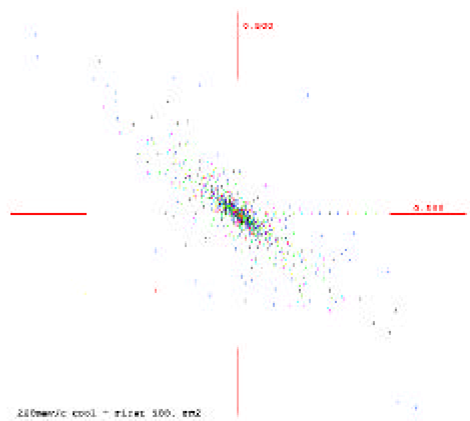
Coupled Systems

- First, decouple the system to 2 x 2 blocks by taking real and imaginary parts of eigen vectors.

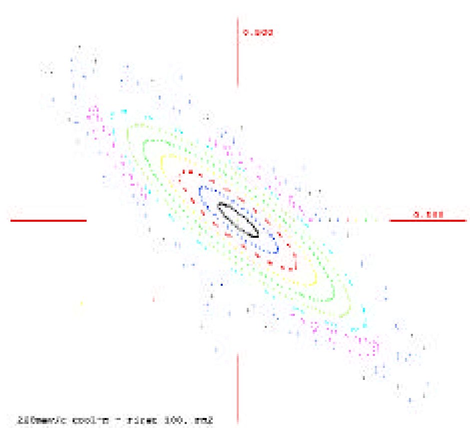
The Normal Form method can be used for analysis of nonlinear behavior.

Tracking a Decoupled Damped System
- Quadrupole Cooling Cells -

Tracking via the damped transfer map



Tracking via the determinant 1 restored transfer map of the damped system



Normal Form Theory

Goal: perform a nonlinear change of variables such that the motion in the new variable pairs is rotationally invariant:

$$\mathbf{M} \circ \mathbf{R} = \mathbf{R} \circ \mathbf{M}$$

If the map is symplectic, this means circles. If the map is damped, we obtain logarithmic spirals.

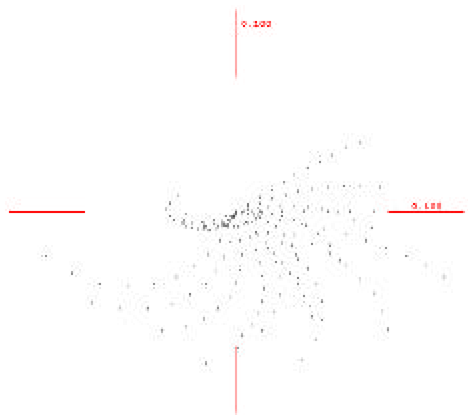
Advantage: Tune with amplitude is trivial to compute, since each iteration of the map corresponds to the same angle advance.

Other Advantages: - Provides pseudo invariants the quality of which allows conclusions about the map; - sensitive to resonances, allows efficient study of resonances.

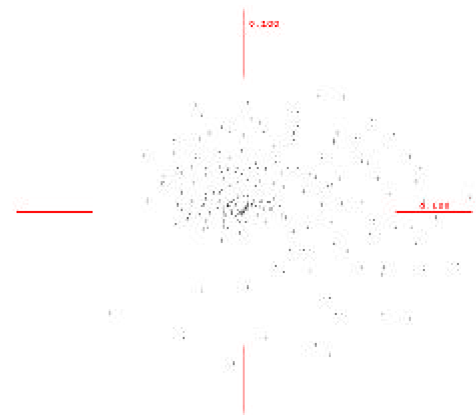
Tracking a Coupled Non-Damped System
- sFOFO 2.75m Cells from Study II -

Tracking in Regular (x-x') Coordinates

1-20 Cells

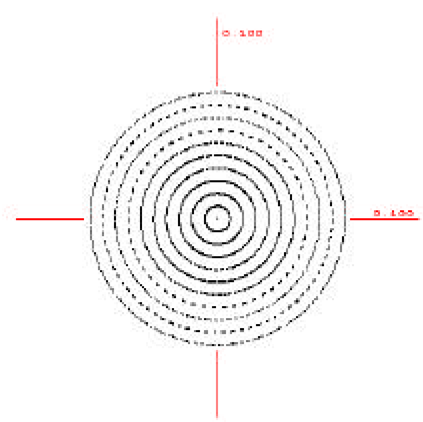


101-120 Cells



Tracking in Normal Form Coordinates

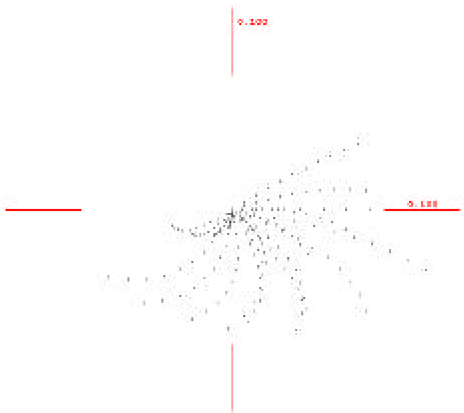
1-300 Cells



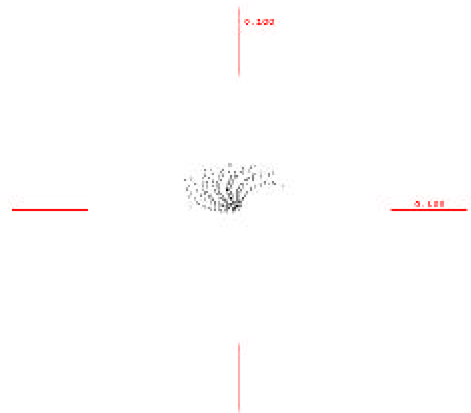
Tracking a Coupled Damped System
- sFOFO 2.75m Cooling Cells from Study II -

Tracking in Regular (x-x’) Coordinates

1-20 Cells

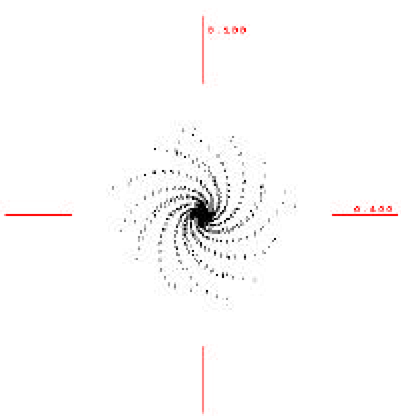


101-120 Cells



Tracking in Normal Form Coordinates

1-300 Cells



41-60 Cells