Quad Cooling Channel Simulation

by COSY Infinity

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MuCool meeting, Fermilab, 1/24/2003
**Quad Cooling Cell**

**4m Cell**

- **Incoming Muons:** 180 MeV/c to 245 MeV/c
- **Magnetic Quadrupoles:** $k=2.88$
- **35cm Liquid H Absorber:** Energy loss $\approx -12$ MeV. The same design as Study II 2.75m sFOFO cell.
- **RF Cavity:** Energy gain to compensate the loss. 200 MHz, $\phi = 30^\circ$. 
**COSY Simulation**

**Map Method**

- The transfer map $\mathcal{M}$ is the flow of the system ODE.

$$
\vec{z}_f = \mathcal{M}(\vec{z}_i, \vec{\delta}),
$$

where $\vec{z}_i$ is the initial condition, $\vec{z}_f$ is the final condition, $\vec{\delta}$ is system parameters.

- For a repetitive system, only one cell transfer map has to be computed. Thus, it is much faster than tracking codes (i.e. tracing each individual particle through the system).

- The Differential Algebraic (DA) method allows a very efficient computation of Taylor transfer maps.

**Differential Algebra**

- arbitrary order
- very transparent algorithms; effort independent of order
- can keep system parameters in map
- etc. etc.
Field Description in Differential Algebra

There are various DA algorithms to treat the fields of beam optical systems efficiently.

For example, **DA PDE Solver**

- requires to supply only
  - the midplane field for a midplane symmetric element.
  - the on-axis potential for a rotationally symmetric element.
- treats arbitrary fields straightforwardly.
  - Magnet (or, Electrostatic) fringe fields:
    The Enge function fall-off model
    
    \[
    F(s) = \frac{1}{1 + \exp(a_1 + a_2 \cdot (s/D) + \ldots + a_6 \cdot (s/D)^5)}
    \]
    
    where \( D \) is the full aperture.
    Or, any arbitrary model including the measured data representation.
  - Solenoid fields including the fringe
  - Measured fields: E.g. Use Gaussian wavelet representation
  - Etc. etc.
Field Profile of Various Magnets
60cm full aperture, 60cm length Magnets

MQ Enge fall-off (measurement from PEP/SLAC) for 60cm full aperture, 60cm length

MQ Enge fall-off (measurement from MSU/S800 Quad II) for 60cm full aperture, 60cm length
MQ Enge fall-off (measurement from LHC/HGQ lead end) for 60cm full aperture, 60cm length

Compare to: Ideal Solenoidal Magnet
60cm full aperture, 60cm length Magnet
200 MeV/c Particles along the $x$-axis at every 2cm are tracked through 100 Quad cells without cooling, nor acceleration. Pictures: in the $x - p_x/p_0$ plane (frame size: $60\text{cm} \times 0.6 \text{ rad}$).

**Sharp Cut-off** Fringe Field Model
( No Fringe Field Effect Consideration )
**PEP/SLAC** Quad Fringe Field Fall-off Model

⇒ We use this model at the moment.

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**S800 Q II** Quad Fringe Field Fall-off Model

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200MeV/c NoCool FR 3 - 100cells, ini:x-axis every 2cm in x, x-a
Treatment of Dynamics through Material

Deterministic Effects

- The mean energy loss.
  \[ \Rightarrow \text{Compensate it by RF cavities.} \]
- Include the effects in the transfer map \( \mathcal{M} \).

Nondeterministic Random Effects

- Multiple scattering. \( \mathcal{R}_{MS} \): A random kick in \( p_x \) and \( p_y \).
  Gaussian distribution.
- Straggling. \( \mathcal{R}_{St} \): A random kick in energy.
  In the absorbers under consideration, the distribution follows Vavilov’s theory.

A set of particles is tracked via the map for the cell \( \mathcal{M} \);
then the Monte Carlo kicks \( \mathcal{R}_{MS} \) and \( \mathcal{R}_{St} \) are added.

\[
\tilde{z}_f = (\mathcal{R}_{MS} \circ \mathcal{R}_{St} \circ \mathcal{M})(\tilde{z}_i)
\]

This procedure is iterated for the next cell.
Mean Energy Loss through Material

The Bethe-Bloch Formula

\[
-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z z^2}{A \beta^2} \left[ \ln \left( \frac{2m_e \gamma^2 v^2 W_{max}}{I^2} \right) - 2\beta^2 - \delta - 2\frac{C}{Z} \right]
\]

\( r_e \): classical electron radius, \( I \): mean excitation potential,
\( W_{max} \): maximum energy transfer in a single collision,
\( \delta \): density correction, \( C \): shell correction.

Through Typical Muon Beam Absorber

![Average Energy Loss through Materials](image)
Pseudo Invariant Ellipses of Quad Cooling Cells

The Quad Cooling channel is a decoupled damped system. By scaling the transfer map by $\sqrt{D}$, ellipses can be restored. ($D$: the determinant of the linear map.)

220 MeV/c. $x - p_x/p_0$ plane. $x_0=2, 4, ..., 30$ cm. For 100 cells.

No Cooling

With Cooling
No Scattering

Pseudo Invariant Ellipses w Cooling
Energy Straggling
– Vavilov Energy Loss Distribution –

Through 35cm thick Liquid H

Vavilov Energy Loss Distribution : Liquid H 35cm

Distribution of Vavilov Random Numbers : 1e7 muons (200MeV/c) through Liquid H 35cm
Muons through the Quad Cooling Cells

Transversal Emittance starting from the beam acceptance. \( p_0 = 200 \) MeV/c. The initial longitudinal emittance: 0. The simulation includes multiple scattering and straggling.
Transversal Cooling and Equilibrium in the Quad Cooling Channel
Outlook

- Design the matching section.
- Send the realistic beam distribution.
- Simulate through from the capture through the cooling channel.
- ... 
- Treat the detailed absorber shape.
- ... 
- Optimize the system.
- More details on RF cavities.
- More details on magnets.
- ...