Quad Cooling Channel Simulation

by COSY Infinity

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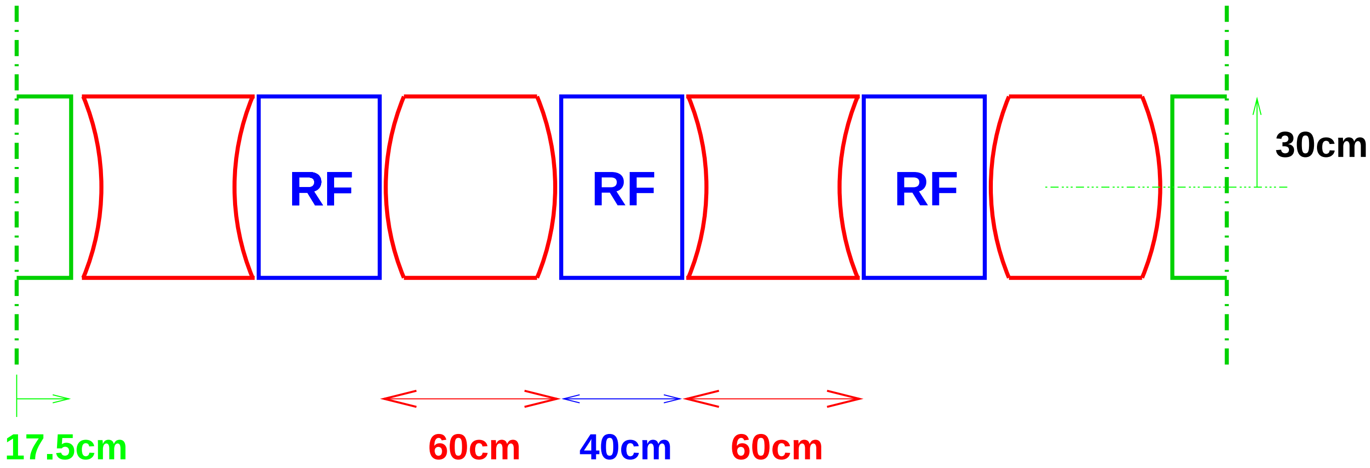
1. Quad Cooling Cell
2. COSY Simulation Technique
3. Some Results
4. Outlook

MuCool meeting, Fermilab, 1/24/2003

Quad Cooling Cell

4m Cell



- Incoming Muons: 180 MeV/c to 245 MeV/c
- Magnetic Quadrupoles: k=2.88
- 35cm Liquid H Absorber: Energy loss ≈ -12 MeV.
  The same design as Study II 2.75m sFOFO cell.
- RF Cavity: Energy gain to compensate the loss.
  200 MHz, $\phi = 30°$.

COSY Simulation

Map Method

- The transfer map $\mathcal{M}$ is the flow of the system ODE.

$$\vec{z}_f = \mathcal{M}(\vec{z}_i, \vec{\delta}),$$

  where $\vec{z}_i$ is the initial condition, $\vec{z}_f$ is the final condition, $\vec{\delta}$ is system parameters.

- For a repetitive system, only one cell transfer map has to be computed. Thus, it is much faster than tracking codes (i.e. tracing each individual particle through the system).
- The Differential Algebraic (DA) method allows a very efficient computation of Taylor transfer maps.

Differential Algebra

- arbitrary order
- very transparent algorithms; effort independent of order
- can keep system parameters in map
- etc. etc.

Field Description in Differential Algebra

There are various DA algorithms to treat the fields of beam optical systems efficiently.

For example, **DA PDE Solver**

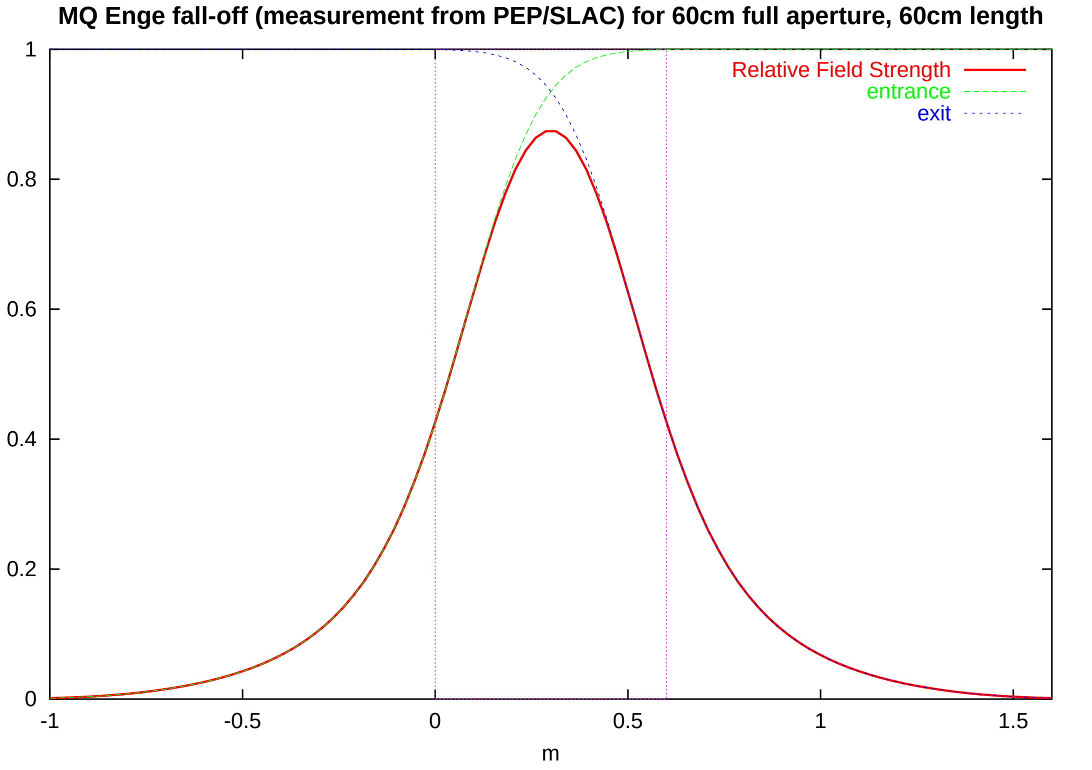
- requires to supply only
  - the midplane field for a midplane symmetric element.
  - the on-axis potential for a rotationally symmetric element.
- treats arbitrary fields straightforwardly.
  - Magnet (or, Electrostatic) fringe fields:
    The Enge function fall-off model

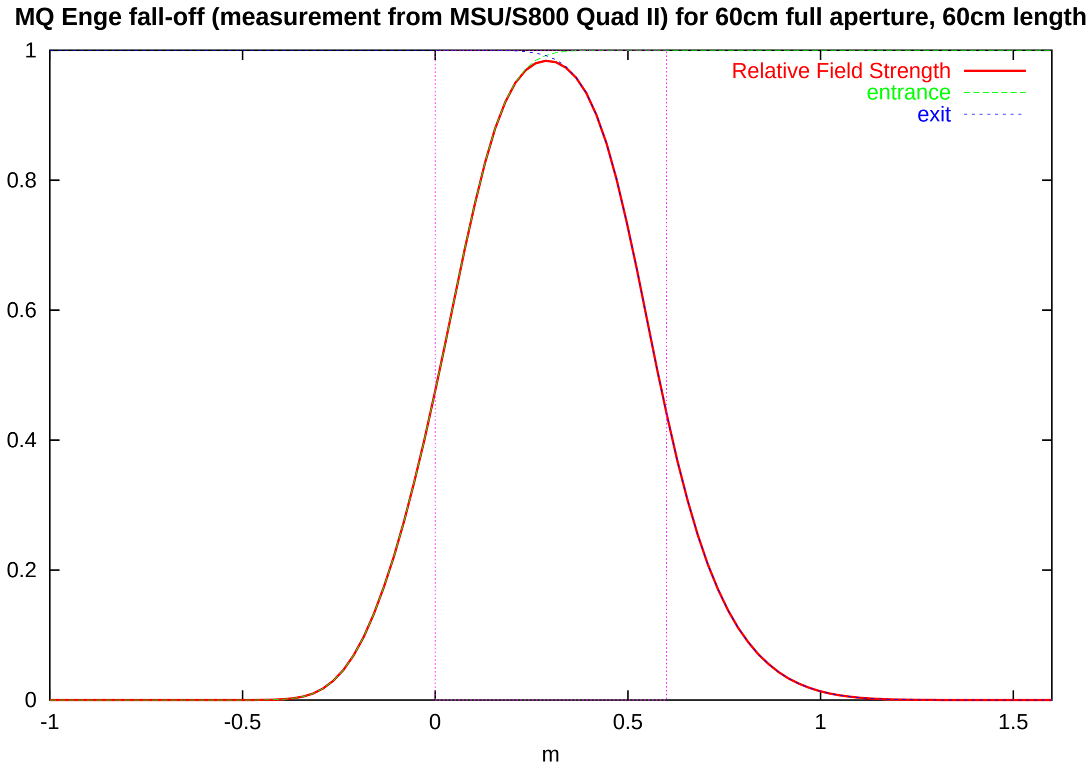
    $$F(s) = \frac{1}{1 + \exp(a_1 + a_2 \cdot (s/D) + ... + a_6 \cdot (s/D)^5)}$$

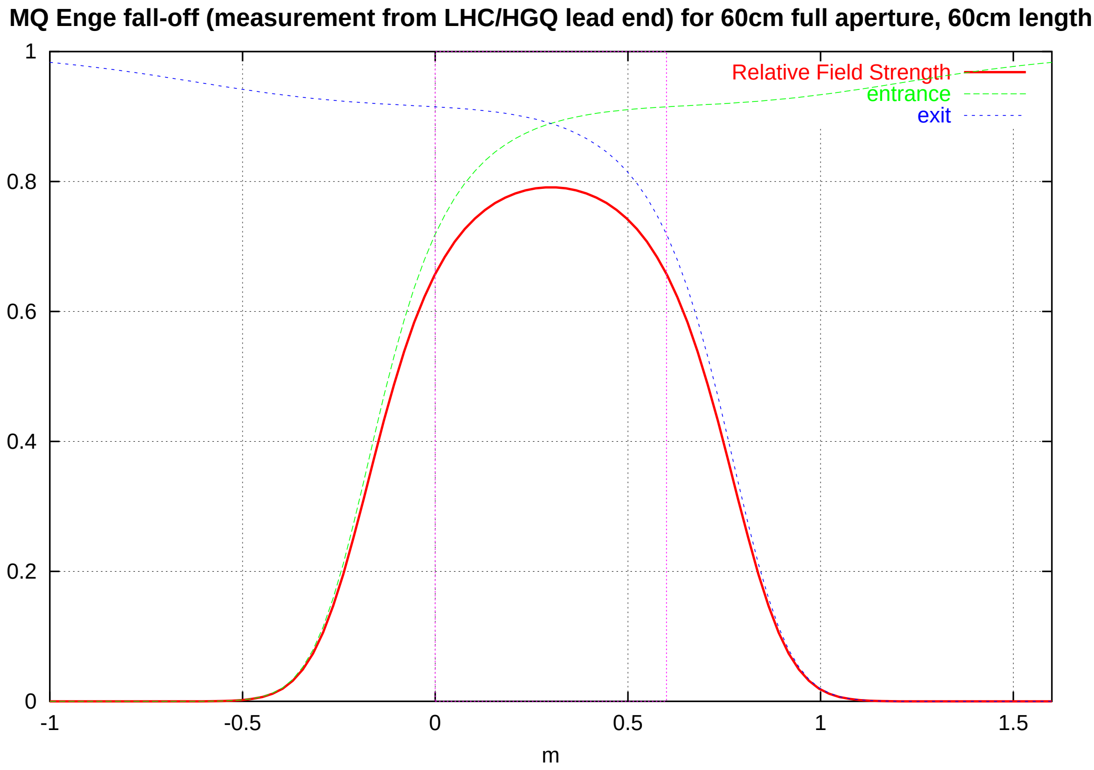
    where D is the full aperture.
    Or, any arbitrary model including the measured data representation.
  - Solenoid fields including the fringe
  - Measured fields: E.g. Use Gaussian wevelet representation
  - Etc. etc.

Field Profile of Various Magnets

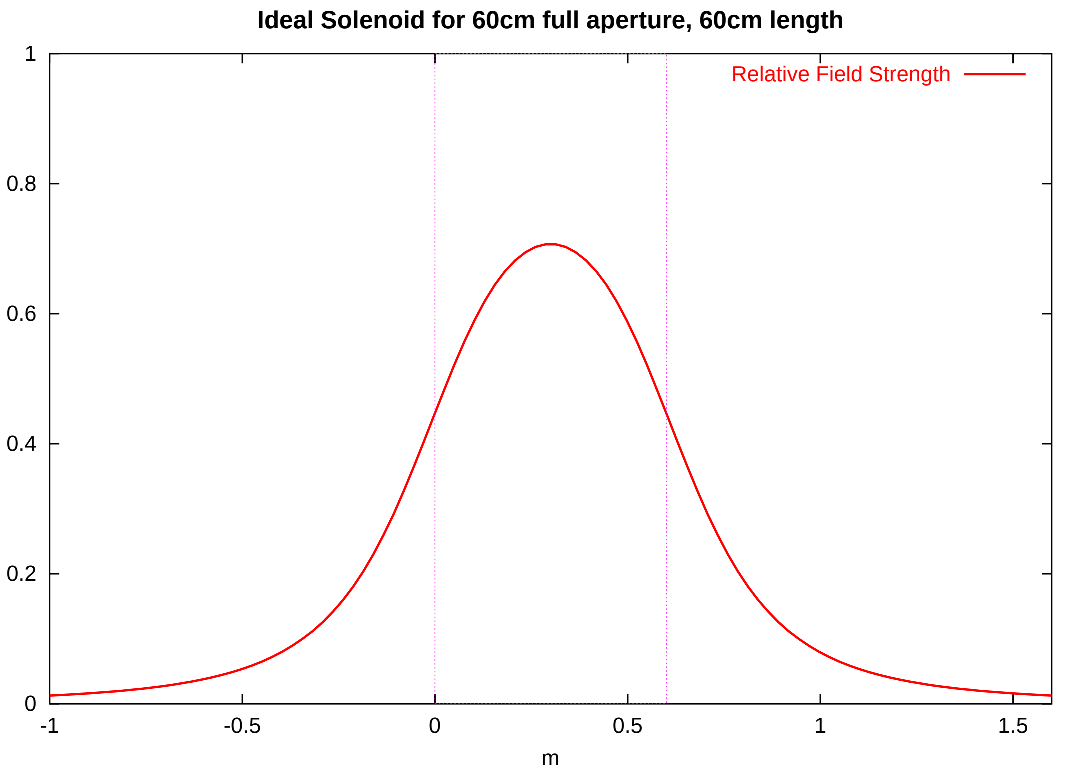
60cm full aperture, 60cm length Magnets







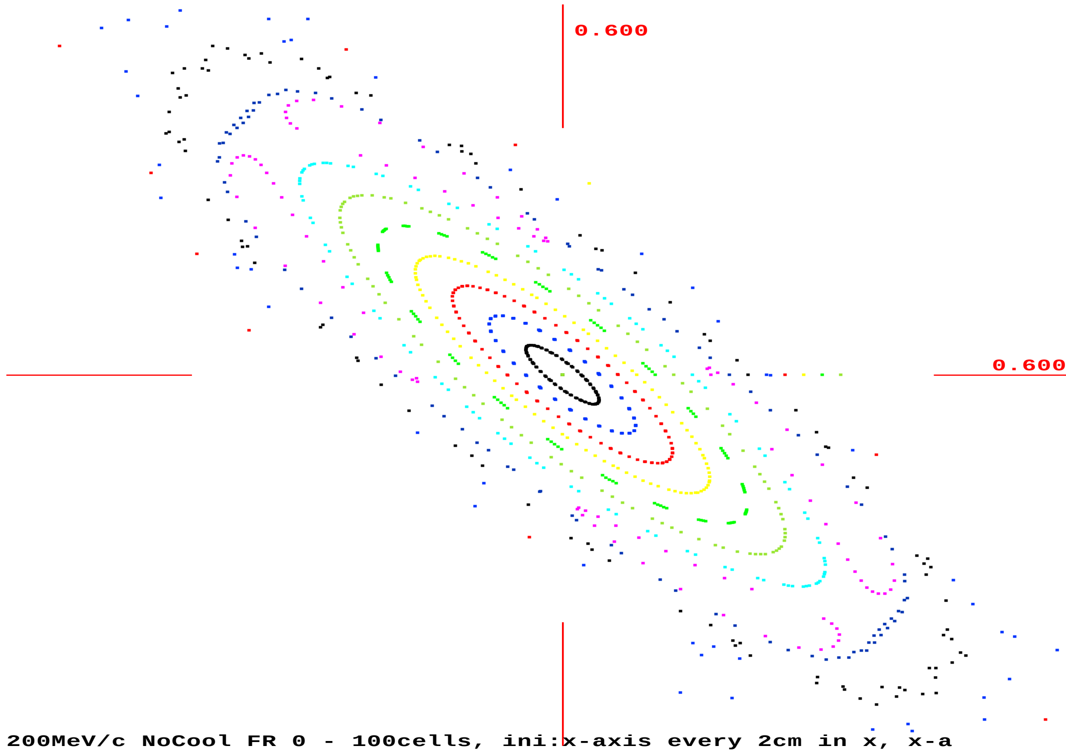
Compare to: Ideal Solenoidal Magnet
60cm full aperture, 60cm length Magnet



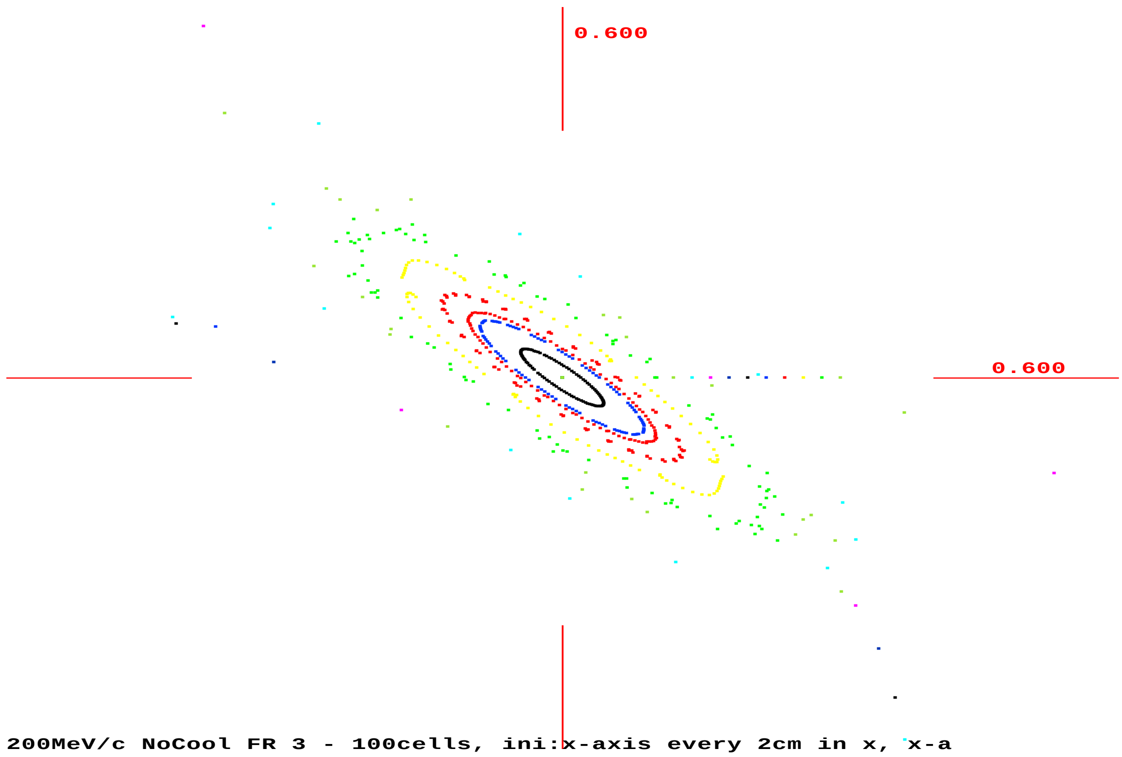
Fringe Field Effects

200 MeV/c Particles along the x-axis at every 2cm are tracked through 100 Quad cells without cooling, nor acceleration. Pictures: in the x - p_x/p_0 plane (frame size: 60cm $\times$ 0.6 rad).

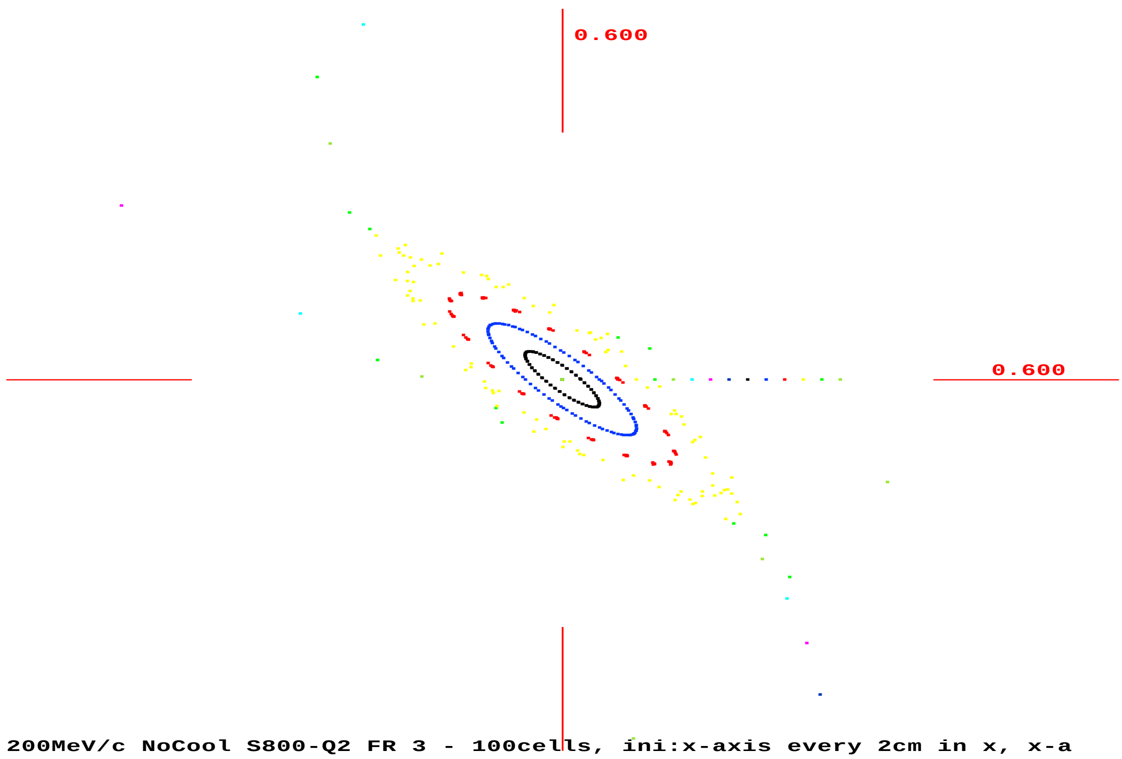
Sharp Cut-off Fringe Field Model
(No Fringe Field Effect Consideration)



PEP/SLAC Quad Fringe Field Fall-off Model
⇒ We use this model at the moment.



S800 Q II Quad Fringe Field Fall-off Model



Treatment of Dynamics through Material

Deterministic Effects

- The mean energy loss.
  $\Rightarrow$ Compensate it by RF cavities.
- Include the effects in the transfer map $\mathcal{M}$.

Nondeterministic Random Effects

- Multiple scattering. $\mathcal{R}_{\mathrm{MS}}$: A random kick in p_x and p_y.
  Gaussian distribution.
- Straggling. $\mathcal{R}_{\mathrm{St}}$: A random kick in energy.
  In the absorbers under consideration, the distribution follows Vavilov's theory.

A set of particles is tracked via the map for the cell $\mathcal{M}$; then the Monte Carlo kicks $\mathcal{R}_{\mathrm{MS}}$ and $\mathcal{R}_{\mathrm{St}}$ are added.

$$\vec{z}_f = (\mathcal{R}_{\mathrm{MS}} \circ \mathcal{R}_{\mathrm{St}} \circ \mathcal{M})(\vec{z}_i)$$

This procedure is iterated for the next cell.

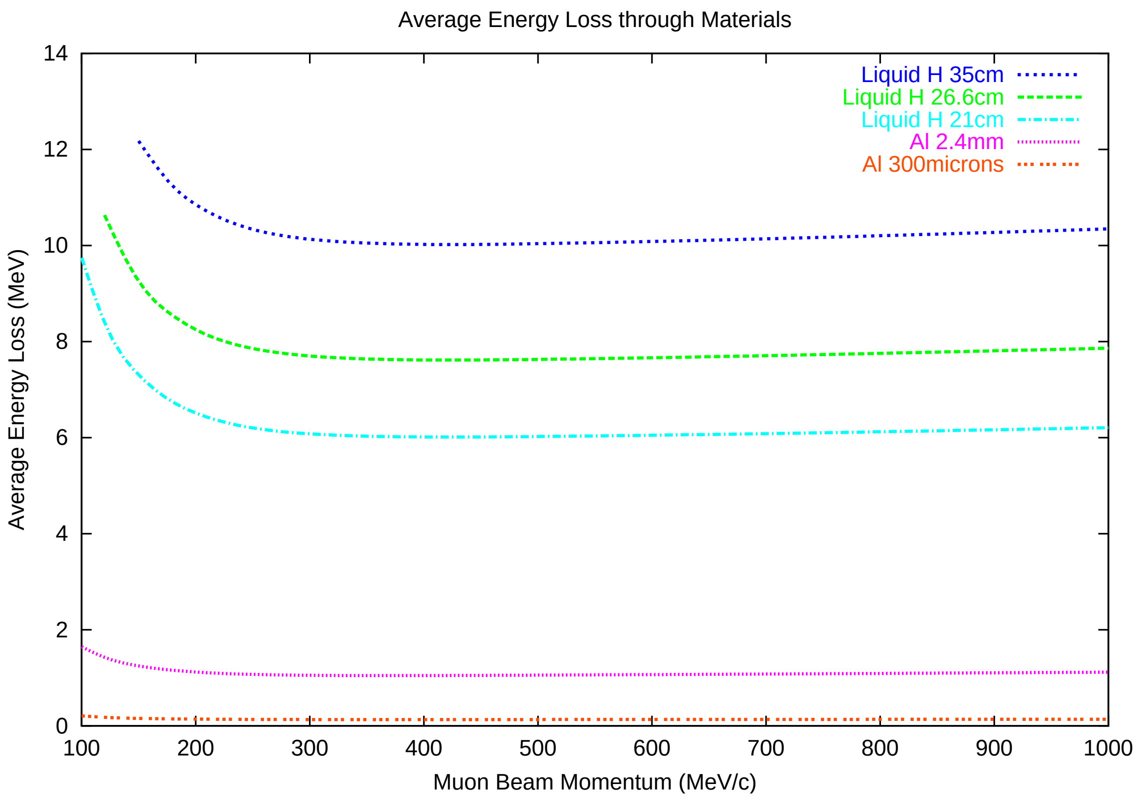
Mean Energy Loss through Material

The Bethe-Bloch Formula

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_e\gamma^2 v^2 W_{max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$

r_e : classical electron radius, I : mean excitation potential,
W_{max} : maximum energy transfer in a single collision,
δ : density correction, C : shell correction.

Through Typical Muon Beam Absorber

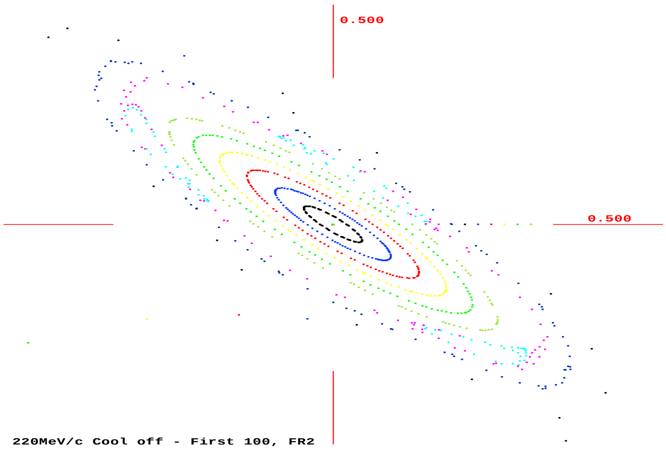


Pseudo Invariant Ellipses of Quad Cooling Cells

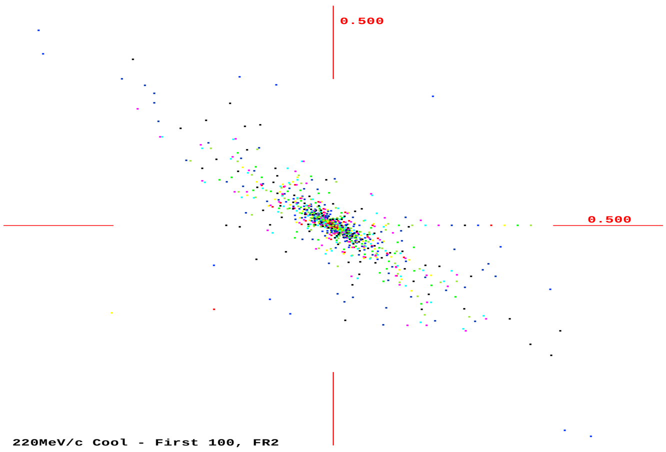
The Quad Cooling channel is a decoupled damped system.
By scaling the transfer map by $\sqrt{D}$, ellipses can be restored.
(D: the determinant of the linear map.)

220 MeV/c. x - p_x/p_0 plane. x_0=2, 4, ..., 30cm. For 100 cells.

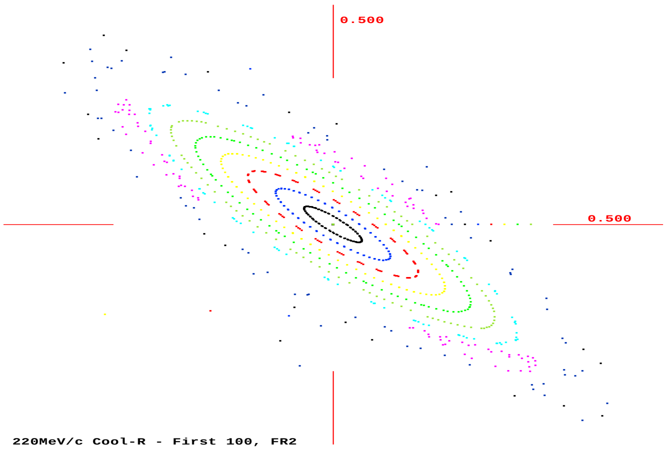
No Cooling



With Cooling
No Scattering

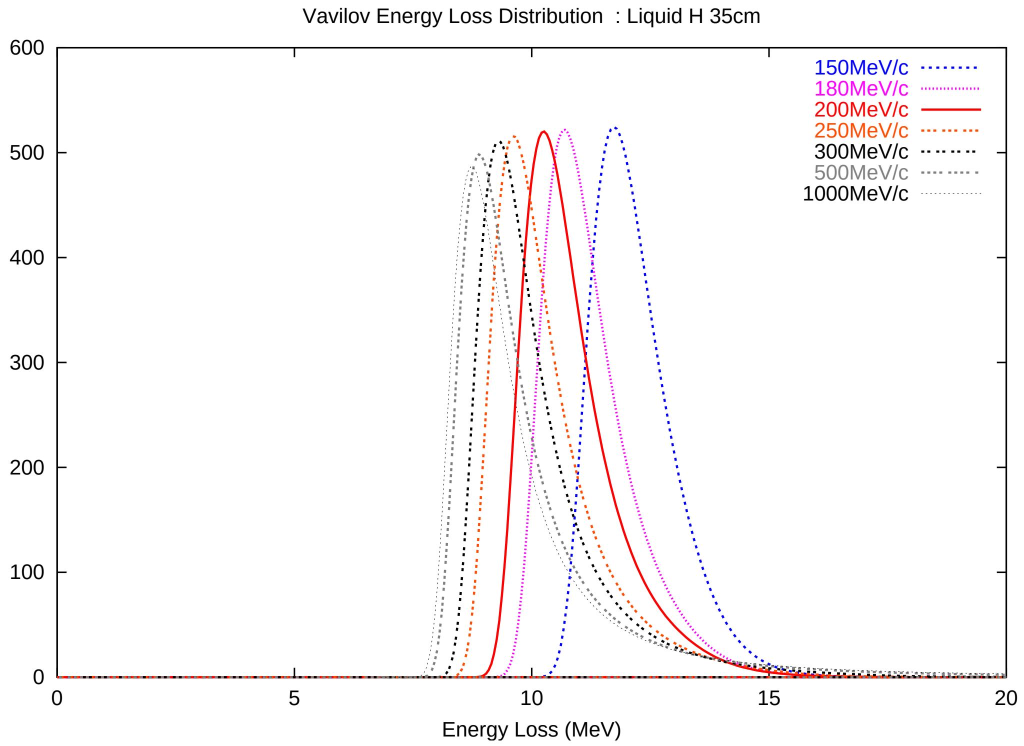


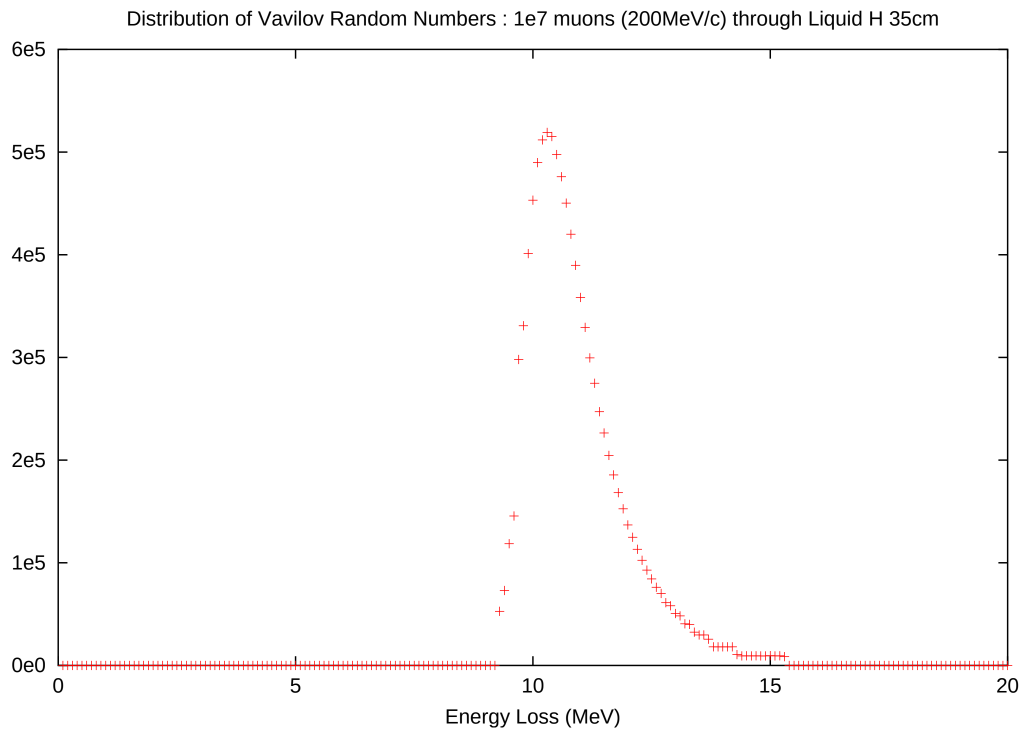
Pseudo Invariant
Ellipses w Cooling



Energy Straggling
– Vavilov Energy Loss Distribution –

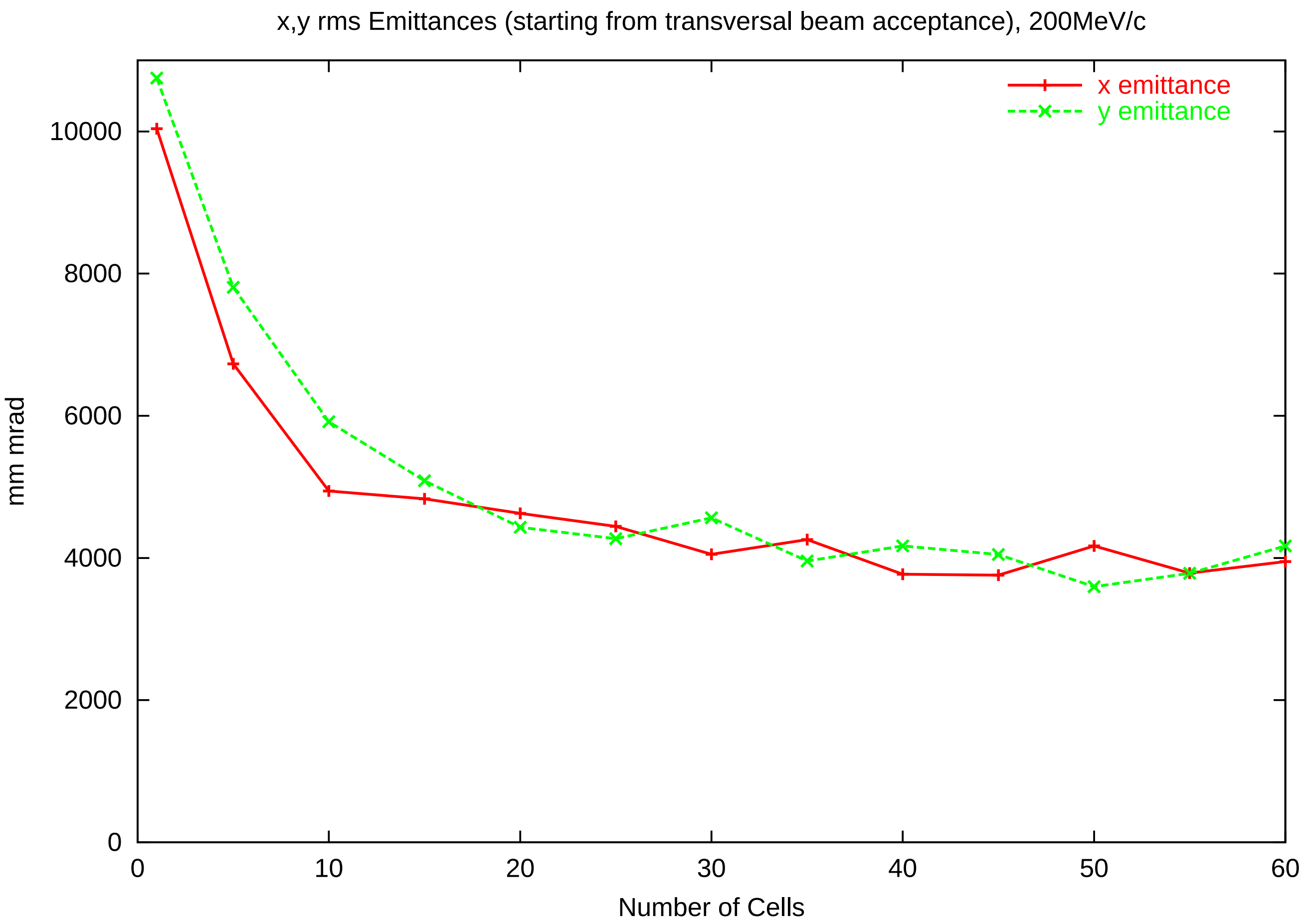
Through 35cm thick Liquid H



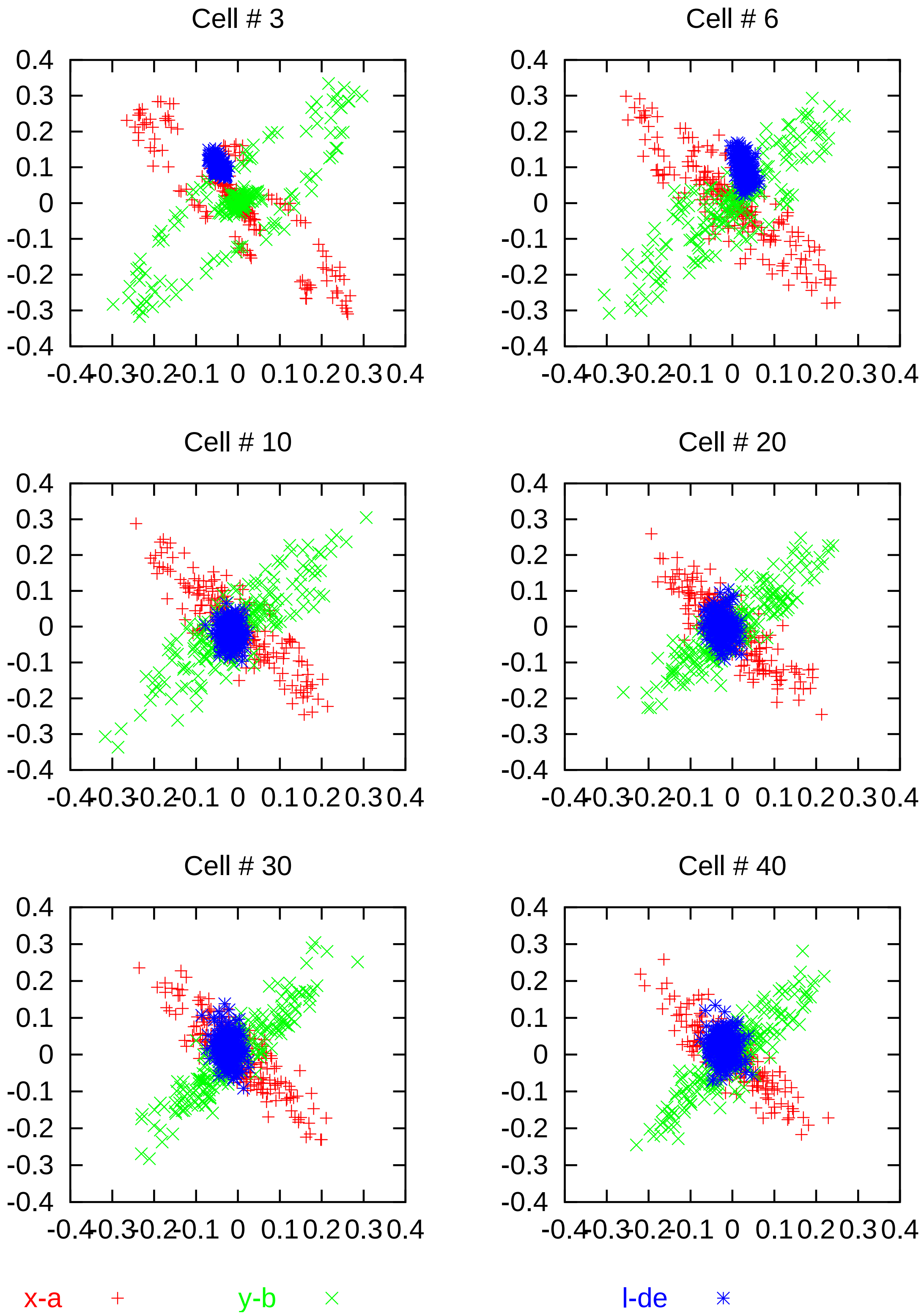


Muons through the Quad Cooling Cells

Transversal Emittance starting from the beam acceptance.
p_0 =200 MeV/c. The initial longitudinal emittance: 0.
The simulation includes multiple scattering and straggling.



Transversal Cooling and Equilibrium in the Quad Cooling Channel



Outlook

- Design the matching section.
- Send the realistic beam distribution.
- Simulate through from the capture through the cooling channel.
- ...
- Treat the detailed absorber shape.
- ...
- Optimize the system.
- More details on RF cavities.
- More details on magnets.
- ...