Neutrino Radiation Hazards at a High-Energy Muon Collider

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Introduction

- High-energy muons decaying in flight create a narrow beam of high-energy neutrinos
- Neutrinos will occasionally interact with matter in the Earth until it exits the surface
- At the surface, secondaries produced from neutrino interactions can be numerous and energetic enough to cause a hazard

Dose Equivalent Conversion

- **Dose Equivalent Conversion Coefficients:**

$$C_i(E_\nu) = C_0 E_\nu^2 \qquad C_0 = 10^{-15}\ \mathrm{Sv\,cm^2\,GeV^{-2}}$$

1 Sv = 100 rem

$$D_i = \int dE_\nu \frac{d\Phi_i}{dE_\nu} C_i(E_\nu)$$

- **Assumes (Cossairt, *et al*):**

  Equilibrium & Heavy shielding (concrete)

  500 MeV < E_ν < 10 TeV

  Good for both electron- and muon-type neutrinos

  For a given type ***i*** of neutrino

Neutrino Production

- **Number of Neutrinos Produced along an arc *ds* and into the phase space volume *dx* $d\Omega_\nu$:**

$$dN_i = \left(\frac{N_\mu(s)}{\beta\gamma c\tau_\mu}\right) \frac{x^2(a_i - b_i x)}{4\pi\gamma^2(1-\beta\cos\theta_\nu)^2}\, ds\, dx\, d\Omega_\nu$$

- **Where:**

$$(a_i, b_i) = \begin{cases} (6,4) & \text{if } \nu_\mu \text{ or } \bar{\nu}_\mu \\ (12,12) & \text{if } \nu_e \text{ or } \bar{\nu}_e \end{cases}$$

$$dE_\nu = \frac{m_\mu c^2}{2\gamma(1-\beta\cos\theta_\nu)} dx$$

Dose Equivalent

- **Integrating over phase space, *x* and Ω_ν:**

$$D \approx \frac{m_\mu^2 c^4 C_0}{16\pi\beta\gamma^5 L^2} \int \frac{ds}{c\tau_\mu} \frac{N_\mu(s)}{(1-\beta\cos\theta_\nu)^4}$$

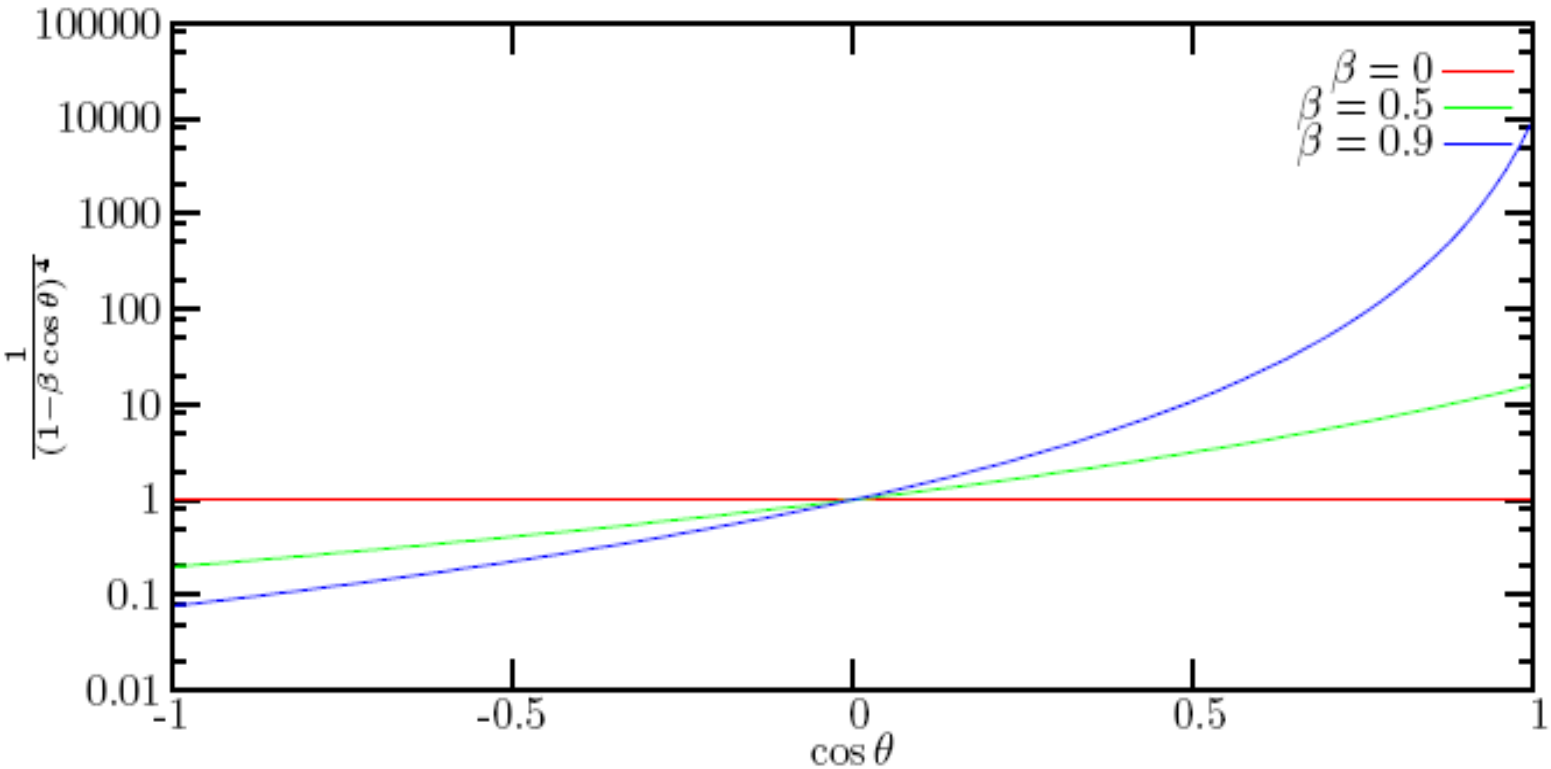
- **Assuming:**

$$\frac{d\Phi_i}{dx} = \frac{1}{L^2} \frac{d^2 N_i}{dx\, d\Omega_\nu}$$

$$\cos\theta_\nu = \hat{n} \cdot \hat{e}_\mu(s) = \hat{n} \cdot \frac{d\vec{r}}{ds}$$

Effect of the Boost

- **The integrand depends strongly on β:**



Single-Turn Dose from Bends

- **Integrating 1 turn around a ring of radius R:**

$$D_b^* \approx \frac{5}{16}\gamma^3 N_\mu \left(\frac{m_\mu}{e\tau_\mu B}\right)\left(\frac{m_\mu^2 c^4 C_0}{L^2}\right)$$

- **Assuming:**

$$R = \beta\gamma\left(\frac{m_\mu c}{eB}\right)$$

$$N_\mu(s) = N_{\mu 0}$$

Single-Pass Dose from Straights

- **Integrating over a straight of length ℓ_s:**

$$D_s^* \approx \frac{1}{\pi}\gamma^3 N_\mu \left(\frac{\ell_s}{c\tau_\mu}\right)\left(\frac{m_\mu^2 c^4 C_0}{L^2}\right)$$

- **Where:**

$$N_\mu(s) = N_{\mu 0}$$

$$\cos\theta_\nu = 1$$ (i.e., forward contribution)

Total Dose Equivalent

- N_μ **changes with each turn/pass:**

$$D = N_\mu D_1$$

- **And over the course of 1 year:**

$$N_\mu = 2\, f_p\, T\, n_b \sum_{n=1}^{n_T} N_n$$

$$= 2\, f_p\, T\, n_b\, n_T \langle N_b \rangle$$

Luminosity

- **The desired luminosity in the collider:**

$$\mathcal{L} \approx \beta\gamma\, f_p\, n_b\, n_T \left(\frac{\langle N_b \rangle^2}{4\, \epsilon_{NT}\, \beta_T^*} \right)$$

- **Where:** $\delta = 1 - \exp\left(-\frac{n_T \mathcal{C}}{\beta\gamma c \tau_\mu} \right)$

$$\epsilon_{NT} = \sqrt{\epsilon_{Nx}\, \epsilon_{Ny}}$$

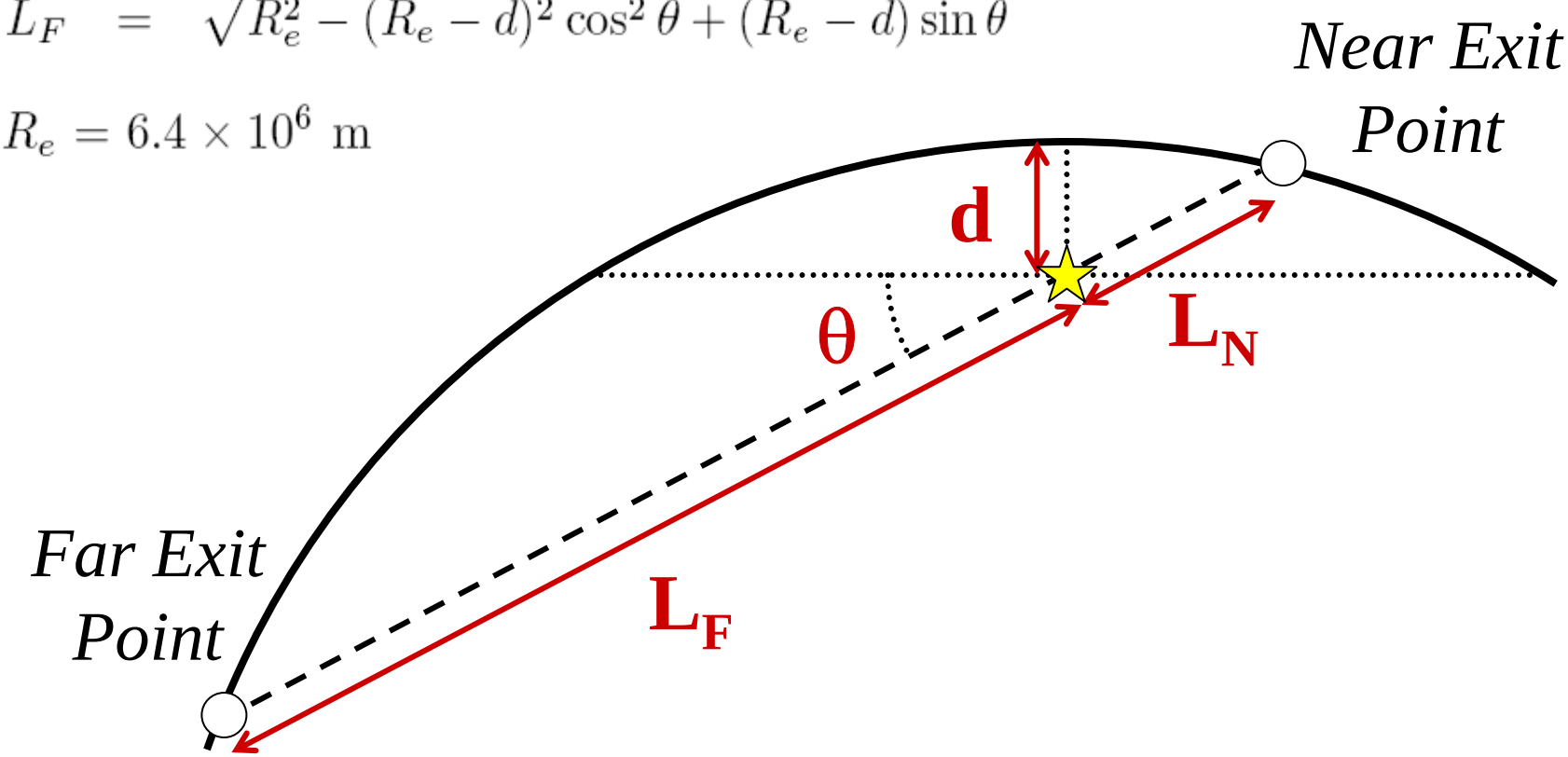
$$\beta_T^* = \sqrt{\beta_x^*\, \beta_y^*} \; .$$

Digging Deep

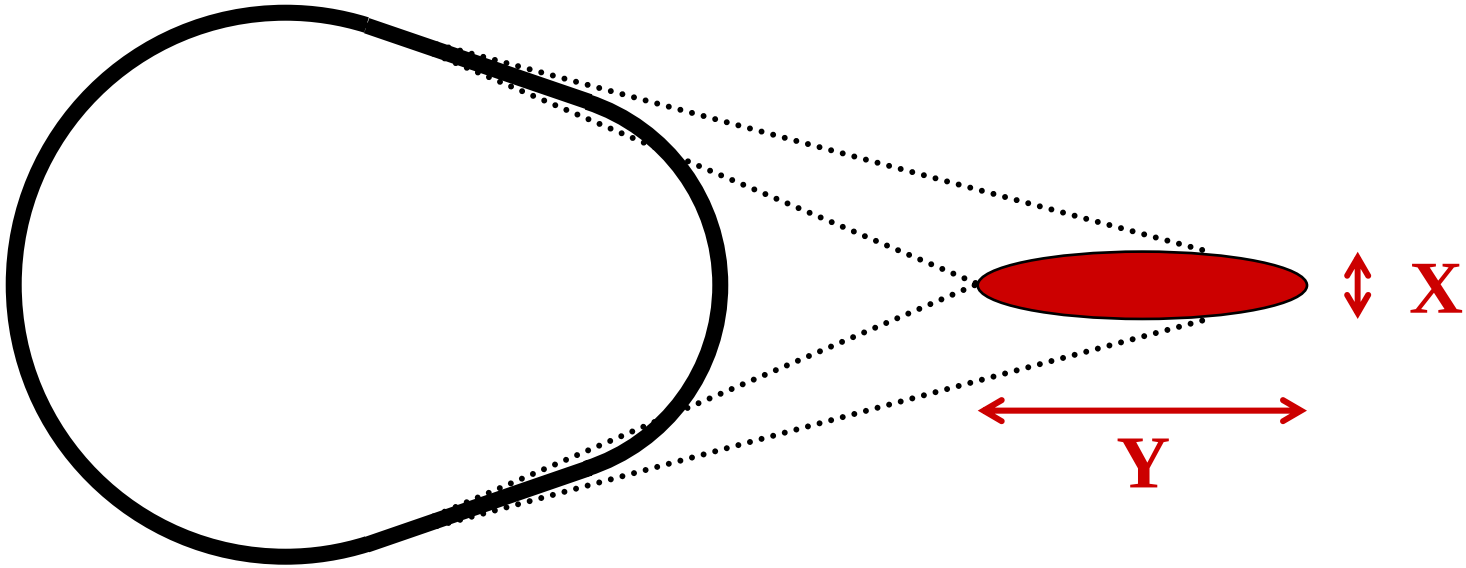
$$L_N = \sqrt{R_e^2 - (R_e - d)^2 \cos^2\theta} - (R_e - d)\sin\theta$$

$$L_F = \sqrt{R_e^2 - (R_e - d)^2 \cos^2\theta} + (R_e - d)\sin\theta$$

$$R_e = 6.4 \times 10^6 \text{ m}$$



The Near Hot Spot



$$X = 2L \tan\frac{1}{\gamma}$$

$$Y = \frac{1}{2}X\left(\frac{1}{\sin(\theta_\chi + \frac{1}{\gamma})} + \frac{1}{\sin(\theta_\chi - \frac{1}{\gamma})}\right)\cos\frac{1}{\gamma}$$

$$\sin\theta_\chi = \sqrt{1 - (1 - \frac{d}{R})^2 \cos^2\theta}$$

Annual Dose Equivalent

- **At "Hot Spots" from Straights:**

$$D_s^* = \frac{4}{\pi}\gamma^3\sqrt{-n_b \ln(1-\delta)} \times \sqrt{\frac{\mathcal{C}}{c\tau_\mu}} \times T\sqrt{\epsilon_{NT}\,\beta_T^*\,\mathcal{L}\,f_p} \times \left(\frac{\ell_s}{c\tau_\mu}\right) \times \left(\frac{m_\mu^2 c^4 C_0}{L^2}\right)$$

- **In Plane of Ring from Bends:**

$$D_b^* = \frac{5}{4}\gamma^3\sqrt{-n_b \ln(1-\delta)} \times \sqrt{\frac{\mathcal{C}}{c\tau_\mu}} \times T\sqrt{\epsilon_{NT}\,\beta_T^*\,\mathcal{L}\,f_p} \times \left(\frac{m_\mu}{e\tau_\mu B}\right) \times \left(\frac{m_\mu^2 c^4 C_0}{L^2}\right)$$

			Tevatron	μ-Tevatron I	μ-Tevatron II	μ-Tevatron III
L	Average Luminosity	$cm^{-2}\ s^{-1}$	1.E+34	1.E+35	1.E+35	1.E+35
E_μ	Beam Energy	*TeV*	0.250	1.000	2.500	2.500
T	Operational Year	*s*	1.E+07	1.E+07	1.E+07	1.E+07
f_{REP}	Pulse Repetition Rate	*Hz*	1.00	1.00	1.00	1.00
C	Circumference of Ring	*km*	6.28	6.28	6.28	3.14
ℓ_s	Length of Straight	*m*	10.00	5.00	1.00	100.00
B	Dipole Field Strength	*T*	8.00	8.00	8.00	12.00
ε_{NT}	Norm. Transv. Emit.	*mm mr*	3.00	1.00	1.00	1.00
β*	Amplitude Function at IP	*cm*	5.00	5.00	2.00	2.00
d	Depth of Ring	*m*	10.00	300.00	300.00	200.00
θ	Ring Tilt Angle	*deg*	0.00	0.00	0.00	0.80
n_b	Number of Bunches		1.00	1.00	1.00	1.00
δ	Fraction of Muon Decays		10%	4%	4%	1%

$\langle D_b \rangle_N$	Avg Dose per yr from Bends (Near)	*Sv*	**3.75E-07**	**9.09E-07**	**8.98E-06**	**4.54E-05**
$\langle D_b \rangle_F$	Avg Dose per yr from Bends (Far)	*Sv*	**3.75E-07**	**9.09E-07**	**8.98E-06**	**2.19E-07**
$\langle D_s \rangle_N$	Avg Dose per yr from Straight (Near)	*Sv*	**8.67E-05**	**1.05E-04**	**2.08E-04**	**1.57E-01**
$\langle D_s \rangle_F$	Avg Dose per yr from Straight (Far)	*Sv*	**8.67E-05**	**1.05E-04**	**2.08E-04**	**7.59E-04**

			Tevatron	μ-Tevatron I	μ-Tevatron II	μ-Tevatron III
n_T	Number of Turns in Ring		26	41	101	50
$n_{T,MAX}$	Number of Turns in Ring / Cycle		47738	47738	47738	95475
$\langle N \rangle$	Avg Number of Muons per Bunch		3.114E+13	2.284E+13	5.777E+12	8.233E+12
N_0	Initial Number of Muons per Bunch		3.281E+13	2.331E+13	5.896E+12	8.274E+12
N_p	Number of Protons per Pulse		7.802E+14	7.802E+14	7.802E+14	7.802E+14
Y_μ	Number of Mu per POT at Injection		0.04206	0.02987	0.00756	0.01061
$\langle I_0 \rangle$	Avg Muon Beam Current per Bunch	*mA*	238.19	174.66	44.19	125.94
I_0	Initial Muon Beam Current per Bunch	*mA*	250.96	178.25	45.09	126.57
$\int dL$	Annual Integrated Luminosity	ab^{-1}	0.100	1.000	1.000	1.000
Z_N	Distance from Beam at Near Exit	*km*	11.314	61.967	61.967	13.330
Z_F	Distance from Beam at Far Exit	*km*	11.314	61.967	61.967	192.041
Z_0	Average Distance from Beam at Exit	*km*	11.314	61.967	61.967	102.685
X_N	Beam Width at Near Exit	*m*	9.563	13.095	5.238	1.127
X_F	Beam Width at Far Exit	*m*	9.563	13.095	5.238	16.233
θ_x	Upward Angle of Neutrinos at Exit	*deg*	1.013E-01	5.548E-01	5.548E-01	9.193E-01
Y_N	Length of Hot Spot at Near Exit	*m*	5737.662	1352.588	540.981	70.228
Y_F	Length of Hot Spot at Far Exit	*m*	5737.662	1352.588	540.981	1011.722