ICOOL Acceptance Studies of PRISM

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ICOOL Method

Represent Scaling by sum of multipoles

Fields defined on a true circle that approximates the reference orbit

\[
B_y(x) = B_o(x + R)^k \approx B_o \sum_{n=1}^{5} M_n r^n
\]

\[
M_n = \frac{\Pi_{i=0}^{i=n-1} k_i}{n!}
\]

Hyperbolic tangent approx to ends

\[
dz = \frac{\text{Distance from end}}{\Gamma}
\]

\[
B = B_o \frac{e^{dz} - e^{-dz}}{e^{dz} + e^{-dz}}
\]

ICOOL Field Calculations

Using Maxwell’s equations fields off axis derived from multipoles on axis

\[
\text{Scaling fitted}
\]

\[
\text{Linear 14 cm}
\]

\[
5\text{th order calculations used}
\]
End Shapes Method 1

- magnet lengths from figure in Arimoto at Nufac04
- Shape parameter $\Gamma = 15$ cm (typical for normal magnet)
- Adjust Fields to get tunes from Sato’s talk at Nufac04

<table>
<thead>
<tr>
<th>Table 1: Present parameters of PRISM-FFAG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of sectors</td>
</tr>
<tr>
<td>Magnet type</td>
</tr>
<tr>
<td>Field index ($k$-value)</td>
</tr>
<tr>
<td>F/D ratio</td>
</tr>
<tr>
<td>Opening angle of magnets</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Half gap of magnets</td>
</tr>
<tr>
<td>Maximum field</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Average radius</td>
</tr>
<tr>
<td>Tune</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Momentum</th>
<th>MeV/c</th>
<th>68</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field Index k</td>
<td>4.6</td>
<td></td>
</tr>
<tr>
<td>beta x</td>
<td>m</td>
<td>1.75</td>
</tr>
<tr>
<td>beta y</td>
<td>m</td>
<td>3.8</td>
</tr>
<tr>
<td>momentum range for $\pm 40$ cm</td>
<td>$\times 2.11$</td>
<td></td>
</tr>
<tr>
<td>x tune at 68 MeV/c</td>
<td>.272 x 10 = 2.72</td>
<td></td>
</tr>
<tr>
<td>y tune at 68 MeV/c</td>
<td>0.160 x 10 = 1.60</td>
<td></td>
</tr>
</tbody>
</table>
ncells = 9.999959

cell = 4.085 (m)

B spread = 15 cm (cf half gap 17 cm)

disp x0 (cm)

momentum (GeV/c)

0.04 0.06 0.08 0.10

0.0 0.1 0.2 0.3

0.04 0.05 0.06 0.07 0.08 0.09 0.10

0.27 0.27 0.27 0.27 0.27 0.27 0.27

0.27 0.27 0.27 0.27 0.27 0.27 0.27

momentum (GeV/c)

0.04 0.06 0.08 0.10

0.0 0.1 0.2 0.3

0.04 0.05 0.06 0.07 0.08 0.09 0.10

0.16 0.16 0.16 0.16 0.16 0.16 0.16

0.15 0.15 0.15 0.15 0.15 0.15 0.15

cell x tune

0.04 0.05 0.06 0.07 0.08 0.09 0.10

0.27 0.27 0.27 0.27 0.27 0.27 0.27

cf 0.27

cell y tune

0.04 0.05 0.06 0.07 0.08 0.09 0.10

0.16 0.16 0.16 0.16 0.16 0.16 0.16

cf 0.16
Dynamic aperture

x motion with no initial y

\[ x_{\text{motion}} \text{ with no initial } y \]

\[ \theta_x \ (\text{rad}) \]

\[ y_{\text{(cm)}} \]

\[ \begin{array}{c}
-100 \\
-75 \\
-50 \\
-25 \\
0 \\
25 \\
\end{array} \]

\[ \begin{array}{c}
0.0 \\
0.1 \\
0.2 \\
0.3 \\
0.4 \\
0.5 \\
\end{array} \]

\[ \begin{array}{c}
\text{cell x tune} \\
\text{theta y} \\
\text{theta 0 (rad)} \\
\end{array} \]

x motion with epsilon initial y

\[ x_{\text{motion}} \text{ with epsilon initial } y \]

\[ \theta_x \ (\text{rad}) \]

\[ y_{\text{(cm)}} \]

\[ \begin{array}{c}
-100 \\
-75 \\
-50 \\
-25 \\
0 \\
25 \\
\end{array} \]

\[ \begin{array}{c}
0.0 \\
0.1 \\
0.2 \\
0.3 \\
0.4 \\
0.5 \\
\end{array} \]
y motion with epsilon initial x

Theta 0 (rad)

cell y tune
Acceptance with both x and y amplitudes

Values on axes give optimistic estimate of acceptance
End Fields  Method 2

$F_{len} = 0.660 \text{ m}$
$D_{len} = 0.247 \text{ m}$
$D/F \text{ fields } = -0.200$
$F_{shape} = 0.087 \text{ m}$
$D_{shape1} = 0.087 \text{ m}$
$D_{shape2} = 0.240 \text{ m}$
$F-D \text{ gap } = 0.153 \text{ m}$
Compare Acceptances

Acceptance in y worse probably because of steeper dB/ds
Linear Magnets i.e. Non-Scaling

Move momentum range up to avoid 3rd order resonance at low end
Range remains approximately the same: factor of 2
After recalcing to 68 MeV/c Fields are lower

- Focus Magnet
- Non-linear Scaling
- Linear Non-Scaling

By (Gauss)

- $10^3$
- $10^2$
- $10^1$

x (m)

-5
-2.5

0

2.5

5
Linear lattices have larger acceptance
Best solution gain is more than 3 times
Summary and Conclusions

<table>
<thead>
<tr>
<th>End shapes</th>
<th>B vs x</th>
<th>x</th>
<th>y</th>
<th>xy</th>
</tr>
</thead>
<tbody>
<tr>
<td>fitted</td>
<td>scaling</td>
<td>12.58</td>
<td>2.04</td>
<td>25.6</td>
</tr>
<tr>
<td>14 cm</td>
<td>scaling</td>
<td>12.92</td>
<td>2.73</td>
<td>35.3</td>
</tr>
<tr>
<td>fitted</td>
<td>linear</td>
<td>21.66</td>
<td>1.95</td>
<td>42.3</td>
</tr>
<tr>
<td>14 cm</td>
<td>linear</td>
<td>23.71</td>
<td>2.47</td>
<td>58.6</td>
</tr>
</tbody>
</table>

- The ICOOL simulation using 5 multiploles to represent the exponent k, gives a good representation of a scaling lattice. Tracking a single particle through 100 cells takes approximately 6 seconds on a 2.4 GHz Pentium laptop.

- The observed x acceptances for zero perpendicular amplitudes agree qualitatively with Sato’s report at NUFAC04, but other acceptances appear somewhat lower.

- Using a fit to the s field dependence in Arimoto’s NUFACT04 talk gave somewhat less acceptances than more gentle field shapes.

- Removing all higher moments, thus making the magnets simple combined function (dipole + quadrupole), gave almost the same momentum acceptance, and 2 times larger dynamic aperture at the chosen central momentum.

- But we have not studied the acceptance as a function of momentum, as is now required since different momenta have quite different tunes.