Introduction

1) Initial Assumptions
2) Optimize Shape Parameter
3) Pure Sine
4a) Add Field Gradient
4b) Optimize Sine Amplitude

Conclusion
A "Scaling" FFAG, almost a scaling FFAG

Pole angles ≡ D focus

H₂ gas filled

H2 gas filled

Fixed Field Magnet

Pole angles

A "Scaling" FFAG, almost a scaling FFAG

Pole angles ≡ D focus

H₂ gas filled

Fixed Field Magnet

Pole angles
Magnets are assumed to have hyperbolic tangent fall offs

\[ \frac{\tau_0}{B} = \int \frac{z}{\sqrt{z^2 + \tau^2}} \]

Fields off this reference are calculated from Maxwell’s Equations

Above fields are those on a true circular reference "orbit"

Maximum smoothing = pure sine z variation

By limiting the number of components the fields can be "smoothed"

These are then Fourier transformed, using 50 coefficients

\[
\frac{\left( \frac{\tau_0}{B} + \frac{\tau_0}{z^2} \right)}{\left( \frac{\tau_0}{z} - \frac{\tau_0}{z^2} \right)} - \frac{\left( \frac{\tau_0}{z} + \frac{\tau_0}{z^2} \right)}{\left( \frac{\tau_0}{z} - \frac{\tau_0}{z^2} \right)} \right] \frac{z}{\tau_0} = B
\]

ICOOL Method
2 examples from a PRISM Study: Sensitivity to Order of ICOOL Field Calculation

Probably less than 2% error for 5th order calculation

turns=10
order=5

Use:

<table>
<thead>
<tr>
<th>Order</th>
<th>Py max (MeV/e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>20</td>
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<tr>
<td>3</td>
<td>25</td>
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<td>4</td>
<td>30</td>
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Linear 14 cm
Scaling fitted
Sensitivity to Number of Fourier Terms

Acceptance $\pi$ mm

Number of Fourier terms

- 5
- 20
- 30
- 40
- 50

$10 \pm 11$ $12 \pm 13$ $14 \pm 15$

50 terms seems more than adequate

Parameters Used for All Calculations

- $n$ cells = 4
- Central Momentum = 172 MeV/c
- Cell Length = 1.000655 (m)
- Magnet Length = 1.000655/2 (m)
- Nominal radius = 0.637037 (m)

in cells = 4

Sensitivity to Number of Fourier Terms
Tunes are flat, so it is "Scaling"
Vary Initial x amplitude

Very large acceptance in x

Y tune depends little on amplitude
Resonance as $y$ tune $=0.25$, but no loss.

$y$ tune depends on amplitude.

Significant x-y mixing and less $y$ acceptance than in $x$.

Vary Initial $y$ amplitude.
Acceptances significantly less than "hard edged" calculations

Very large x acceptance only for very small y amplitudes

\[ x \] acceptance does not fill vertical aperture

\[ y \] acceptance is for maximum equal acceptance in \( x \) and \( y \)

Ellipse is for maximum equal acceptance in \( x \) and \( y \)

Aperture in \( x \) and \( y \) simultaneously
Acceptance improved 37%.

8 cm better than 10 cm, but how?

Optimize Shape Parameter Gamma

Acceptance = 37%

ngood = 32

Acceptance = 48 (mm)

ngood = 43

Gamma = 8 cm

B (T)

z (m)

Gamma = 8 cm

Gamma = 10 cm

Initial px (MeV/c)

Initial py (MeV/c)

nx = 4

cell = 0.00625 (m)

radius = 0.637037 (m)

Cells = 4

Accept = 48 (mm)

Gamma = 8 cm

Initial px (MeV/c)

Initial py (MeV/c)

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Accept = 32 (mm)
3) Try pure sine variation with azimuth

Acceptance improved by 52% •

Less acceptance in x •

Acceptance much better in y

Acceptance improved by 52% •

Acceptance much better in y

Acceptance much better in y
Acceptance improved by > 40%

Now good in x and y

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\text{Acceptance improved by } > 40\%
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\]
Vary amplitude of sine variation
But keep gradient=0
Relative $B_{\text{max}} - B_{\text{min}}$

$0.25$ $0.50$ $0.75$ $1.00$ $1.25$

$0$ $25$ $50$ $75$ $100$

$\pi$ mm

$B(T)$
$z(\mu m)$

2 solutions, but lower has low $\nu$ and high $\beta$ (bad for cooling)

Acceptance improved by further 20%

But keep gradient=0

Vary amplitude of sine variation
Acceptance improved by factor 3.4

Compare Initial (# I) with Best (# 4b)
<table>
<thead>
<tr>
<th></th>
<th>11</th>
<th>4</th>
<th>7 6 Pole</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>120</td>
<td>0.279</td>
<td>1.15</td>
</tr>
<tr>
<td>2.9</td>
<td>101</td>
<td>0.272</td>
<td>1.15</td>
</tr>
<tr>
<td>73</td>
<td>73</td>
<td>0.272</td>
<td>0</td>
</tr>
<tr>
<td>48</td>
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<tr>
<td>35</td>
<td>35</td>
<td>0.272</td>
<td>0</td>
</tr>
</tbody>
</table>

Summary

Much Worse Needs study
- Gradient instead of Pole angles
- W

Also Tried
Yet included

But inside and outside fall off of B with $\theta$ and $\pi$

Physical aperture rather than dynamics

For two cases (4a and 4b), acceptance limited by

No gain with more poles

Even greater gain if sine fluctuation enhanced (How?)

Significant gain with some positive field gradient

Pure sine field variation with $z$

Large gain in acceptance for

Conclusion