10.4 to 20 MeV, Electron Model for an Isochronous FFAG with Insertions

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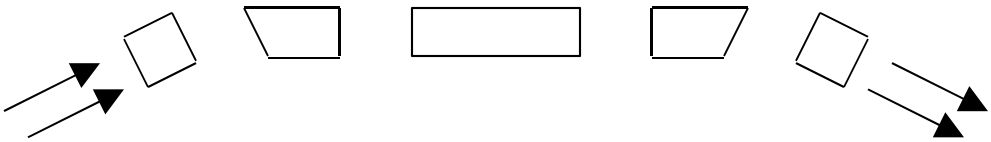
1. Introduction

FFAGs: 1-3 are for proton drivers or medical rings, and 4-6 are for rapid acceleration:

1.	S-FFAG	scaling, non-linear
2.	N-FFAG	non-scaling, non-linear, non-isochronous
3.	N-FFAG-I	N-FFAG with insertions
4.	L-FFAG	non-scaling, linear
5.	I-FFAG	non-scaling, non-linear, isochronous
6.	I-FFAG-I	I-FFAG with insertions

A 10.4-20 MeV electron model for an I-FFAG-I is outlined here. The ring circumference is 27.0 m and there are three superperiods, each of nine normal and four insertion cells. The normal and insertion cell lengths are 0.6 and 0.9 m, respectively, and both cells are of the pumplet type, as shown below, and as used previously for the I-FFAG designs.

O bd(-) o F(±) o BD(+) o F(±) o bd(-) O



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.05	Normal Cell (0.6 m)	.05
.20	Insertion Cell (0.9 m)	.20

Pumplet Cell, Orbit Lengths for the 20 MeV Electrons.

Parameters of the insertion cells are shown in Tables 1a and 1b, and of the normal cells in Tables 2a and 2b. Field gradients, K_v, are normalised and given in units of m^{-2}. The small amplitude, betatron and dispersion functions of the cells are tabulated for the centre of the O straight section. On adding insertions at this point, a small ripple develops in the radial, beta-function maxima, but vertical betatron and dispersion functions are unchanged.

Acceleration is over 16 turns at 0.6 MeV energy gain per turn, using three, 3-cell cavities, with one in each superperiod. The harmonic number of 270 corresponds to 2996.95 MHz. During acceleration, there are no vertical, integer or half-integer, resonances crossed, and only five horizontal, integer resonances, with none of the latter over the first five turns. Similar features are to be introduced for the muon rings.

The orbit separations are given for three different electron energy ranges in Table 3.

Tables 1a and 1b: *Insertion Cell Parameters*

T(MeV)	$\gamma = \gamma$-t	$\mu_h/2\pi$	$\mu_v/2\pi$	β_h	β_v	α_p	bd(T)	F(T)	BD(T)
20.0	40.1385	.40189	.140	.22709	.85349	.02974	–.124633	.086657	.091195
19.4	38.9643	.38392	.140	.25773	.85985	.03152	–.124200	.082263	.092685
18.8	37.7902	.36914	.140	.28060	.86776	.03329	–.123667	.077040	.094269
18.2	36.6160	.35641	.140	.29881	.87699	.03509	–.122985	.072936	.095948
17.6	35.4419	.34400	.140	.31664	.88663	.03705	–.122120	.067930	.097727
17.0	34.2677	.33187	.140	.33436	.89679	.03920	–.121050	.062661	.099611
16.4	33.0936	.32013	.140	.35176	.90726	.04153	–.119754	.057078	.101601
15.8	31.9194	.30872	.140	.36913	.91832	.04409	–.118204	.051254	.103703
15.2	30.7452	.29772	.140	.38632	.92992	.04689	–.116370	.045062	.105915
14.6	29.5711	.28722	.140	.40313	.94213	.04994	–.114218	.038511	.108235
14.0	28.3969	.27733	.140	.41923	.95499	.05327	–.111712	.031575	.110654
13.4	27.2228	.26821	.140	.43416	.96859	.05687	–.108830	.024240	.113178
12.8	26.0486	.26013	.140	.44703	.98303	.06070	–.105467	.016477	.115711
12.2	24.8745	.25341	.140	.45666	.99845	.06472	–.101642	.008309	.118273
11.6	23.7003	.24859	.140	.46130	1.01510	.06878	–.097296	–.000240	.120769
11.0	22.5262	.24640	.140	.45852	1.03334	.07266	–.092398	–.009100	.123095
10.7	21.9391	.24659	.140	.45338	1.04320	.07447	–.089733	–.013738	.124262
10.4	21.3520	.24798	.140	.44507	1.05372	.07605	–.086935	–.018261	.125208

T(GeV)	$K_v(m^{-2})$ - bd - X(mm)		$K_v(m^{-2})$ - F - X(mm)		$K_v(m^{-2})$ - BD - X(mm)	
20.0	6.70	16.50648	–95.59182	13.90471	62.85972	8.11824
19.4	7.50	15.60108	–91.97009	13.20960	60.77155.	7.76064
18.8	9.00	14.61247	–89.33443	12.43986	58.84210	7.35581
18.2	11.00	13.53473	–87.37943	11.58785	56.99061	6.89852
17.6	13.00	12.35839	–85.45909	10.64332	55.06240	6.38144
17.0	15.00	11.06990	–83.50392	9.59236	53.03618	5.79450
16.4	17.00	9.65691	–81.69720	8.42175	50.92631	5.12772
15.8	19.00	8.10022	–79.83924	7.11138	48.68579	4.36611
15.2	21.00	6.38072	–77.98307	5.64074	46.30319	3.49384
14.6	23.00	4.47555	–76.11612	3.98511	43.75160	2.49149
14.0	25.00	2.35860	–74.22266	2.11591	40.99781	1.33602
13.4	27.00	0.00000	–72.27432	0.00000	37.99446	0.00000
12.8	29.00	–2.63280	–70.24573	–2.39928	34.68828	–1.54803
12.2	31.00	–5.57489	–68.09147	–5.12214	31.00328	–3.34401

11.6	33.00	–8.86107	–65.74973	–8.20950	26.83991	–5.42687
11.0	35.00	–12.52397	–63.12911	–11.70110	22.06250	–7.83777
10.7	36.00	–14.55626	–61.65303	–13.65805	19.32717	–9.21238
10.4	37.00	–16.63658	–60.04173	–15.67483	16.41819	–10.64661

Tables 2a and 2b: *Normal Cell Parameters*

T(MeV)	$\gamma = \gamma$-t	$\mu_h/2\pi$	$\mu_v/2\pi$	β_h	β_v	α_p	bd(T)	F(T)	BD(T)
20.0	40.1385	.31958	.088942	.37203	.85349	.02974	–.124633	.086657	.091195
19.4	38.9643	.30611	.089325	.37902	.85985	.03158	–.124606	.082666	.092575
18.8	37.7902	.29514	.089867	.38254	.86776	.03307	–.124489	.078524	.094044
18.2	36.6160	.28478	.090369	.38632	.87699	.03514	–.124230	.074183	.095605
17.6	35.4419	.27502	.090933	.39015	.88663	.03708	–.123802	.069620	.097259
17.0	34.2677	.26501	.091413	.39610	.89679	.03928	–.123188	.064817	.099013
16.4	33.0936	.25618	.092084	.40017	.90726	.04151	–.122360	.059749	.100867
15.8	31.9194	.24645	.092523	.40821	.91832	.04418	–.121299	.054398	.102825
15.2	30.7452	.23794	.093154	.41397	.92992	.04691	–.119987	.048748	.104885
14.6	29.5711	.22922	.093653	.42179	.94213	.05002	–.118386	.042773	.107044
14.0	28.3969	.22131	.094243	.42832	.95499	.05332	–.116468	.036457	.109286
13.4	27.2228	.21373	.094761	.43524	.96859	.05695	–.114212	.029790	.111613
12.8	26.0486	.20710	.095333	.44029	.98303	.06077	–.111569	.022764	.113978
12.2	24.8745	.20133	.095870	.44374	.99845	.06482	–.108505	.015388	.116344
11.6	23.7003	.19700	.096448	.44342	1.0151	.06888	–.104989	.007695	.118645
11.0	22.5262	.19432	.096990	.43876	1.0333	.07286	–.100997	–.000249	.120787
10.7	21.9391	.19144	.096319	.44388	1.0432	.07587	–.098830	–.004315	.121799
10.4	21.3520	.19535	.098082	.42202	1.0537	.07582	–.096569	–.008328	.122674

T(GeV)	$K_v(m^{-2})$ - bd - X(mm)		$K_v(m^{-2})$ - F - X(mm)		$K_v(m^{-2})$ - BD - X(mm)	
20.0	0.03000	16.46015	–89.72807	13.49458	61.07048	7.81610
19.4	0.87480	15.55711	–86.01770	12.82084	58.89790	7.47474
18.8	2.77800	14.57105	–83.85099	12.07417	57.05309	7.08733
18.2	4.84650	13.49610	–81.96025	11.24740	55.20101	6.64891
17.6	7.08500	12.32282	–80.33223	10.33046	53.33310	6.15234
17.0	9.03460	11.03778	–78.37648	9.31030	51.29269	5.58798
16.4	11.48630	9.62853	–77.06130	8.17308	49.30172	4.94584
15.8	13.23130	8.07611	–74.92592	6.90066	47.00421	4.21192
15.2	15.48273	6.36142	–73.38966	5.47254	44.70412	3.37069
14.6	17.34380	4.46177	–71.41603	3.86544	42.15576	2.40369
14.0	19.40060	2.35118	–69.64145	2.05168	39.46220	1.28876

13.4	21.25223	0.00000	–67.61805	0.00000	36.50471	0.00000
12.8	23.19971	–2.62413	–65.63194	–2.32470	33.28799	–1.49246
12.2	25.03453	–5.55598	–63.43747	–4.96026	29.71647	–3.22222
11.6	26.92935	–8.83008	–61.14873	–7.94537	25.72655	–5.22546
11.0	28.71700	–12.47863	–58.53678	–11.31641	21.19202	–7.53914
10.0	27.84423	–14.50187	–55.56818	–13.19809	18.60165	–8.88083
10.4	31.43630	–16.57368	–56.34551	–15.14426	15.93918	–10.25446

Table 3: *Orbit Separations (mm) for Different Electron Energies*

Energy (MeV)	11.6 to 20.0	11.0 to 20.0	10.4 to 20.0
Long straight sections	29.309	31.344	33.424
Insertion cell bd unit	29.030	31.063	33.143
Normal cell bd unit	28.939	30.962	33.034
Insertion cell F quad	25.606	27.563	29.580
Normal cell F quad	24.811	26.693	28.639
Insertion cell BD unit	15.956	17.331	18.765
Normal cell BD unit	15.355	16.697	18.071

In the Tables 1a, 1b, 2a and 2b, the orbit separations, X, are given relative to the orbit at 13.4 MeV. In Table 3, the orbit separations for the specified energies are given relative to the 20.0 MeV orbit.