FFAG Lattice Costs

J. Scott Berg
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Modified Palmer Cost Model

\[ B_{i\pm} = B_0 \pm B_1 f_R R \]
\[ R_{o\pm} = f_R R + t_C |B_{i\pm}| \]
\[ B_{o\pm} = B_0 \pm B_1 R_{o\pm} \]

\[ B_{e\pm} = \begin{cases} 
B_{i\pm} & |B_{i\pm}| \geq |B_{o\pm}| \\
B_{o\pm} & |B_{i\pm}| < |B_{o\pm}| 
\end{cases} \]

\[ C_{\pm} = C_m 0 B_{e\pm}^{1.5} R_{o\pm} (L + f_E f_R R) \]

\[ f_Q = \frac{|B_{e+} + B_{e-}|}{|B_{e+}| + |B_{e-}|} k_D + \frac{|B_{e+} - B_{e-}|}{|B_{e+}| + |B_{e-}|} k_Q \]

\[ C_{\text{mag}} = \frac{1}{2} (C_+ + C_-) \left( \frac{n_0}{n} \right)^{1/3} f_A f_Q \]

\[ C_{\text{rf}} = \frac{k_C V G_0}{G} + \frac{k_P V G}{G_0} \]

\[ C_{\text{lin}} = C_L L_R \]

- Costs \( C_{\text{mag}} \) (magnets), \( C_{\text{rf}} \) (RF), and \( C_{\text{lin}} \) (linear)
- Central field is \( B_0 \), gradient is \( B_1 \)
- \( n \) magnets, magnet radius is \( f_R R \), magnet length is \( L \)
- Total installed voltage is \( V \), RF gradient is \( G \)
- Ring length is \( L_R \)
Modified Palmer Cost Model (cont.)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_R$</td>
<td>1.3</td>
</tr>
<tr>
<td>$C_{m0}$</td>
<td>22.5 mPB/T$^{1.5}$/m²</td>
</tr>
<tr>
<td>$k_D$</td>
<td>1</td>
</tr>
<tr>
<td>$n_0$</td>
<td>300</td>
</tr>
<tr>
<td>$k_C$</td>
<td>30 PB/GV</td>
</tr>
<tr>
<td>$G_0$</td>
<td>16 MV/m</td>
</tr>
<tr>
<td>$t_C$</td>
<td>2 mm</td>
</tr>
<tr>
<td>$f_E$</td>
<td>20</td>
</tr>
<tr>
<td>$k_Q$</td>
<td>1.5</td>
</tr>
<tr>
<td>$f_A$</td>
<td>1.5</td>
</tr>
<tr>
<td>$k_P$</td>
<td>26.8 PB/GV</td>
</tr>
<tr>
<td>$C_L$</td>
<td>25 mPB/m</td>
</tr>
</tbody>
</table>

- PB is our cost unit the “Palmer Buck”
• Non-Scaling FFAG lattices optimized to same design parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum energy</td>
<td>10 GeV</td>
</tr>
<tr>
<td>Maximum energy</td>
<td>20 GeV</td>
</tr>
<tr>
<td>RF frequency</td>
<td>201.25 MHz</td>
</tr>
<tr>
<td>Voltage per cavity</td>
<td>7.5 MV</td>
</tr>
<tr>
<td>RF drift length</td>
<td>2 m</td>
</tr>
<tr>
<td>Short drift length</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Minimum energy tunes</td>
<td>0.35</td>
</tr>
<tr>
<td>$\Delta T$ per cell</td>
<td>7.12 ps</td>
</tr>
<tr>
<td>Normalized acceptance</td>
<td>30 mm</td>
</tr>
<tr>
<td>Pole tip fields</td>
<td>7 T</td>
</tr>
</tbody>
</table>

• Compare different lattice types: triplet, doublet, FODO

• Not optimized for cost
<table>
<thead>
<tr>
<th>Type</th>
<th>FDF</th>
<th>FD</th>
<th>FODO 1 RF</th>
<th>FODO 2 RF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cells</td>
<td>93</td>
<td>101</td>
<td>113</td>
<td>82</td>
</tr>
<tr>
<td>D Length (cm)</td>
<td>128</td>
<td>101</td>
<td>81</td>
<td>99</td>
</tr>
<tr>
<td>D Radius (cm)</td>
<td>8.4</td>
<td>6.9</td>
<td>8.0</td>
<td>8.3</td>
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<tr>
<td>F Length (cm)</td>
<td>45</td>
<td>81</td>
<td>60</td>
<td>74</td>
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<tr>
<td>F Radius (cm)</td>
<td>9.5</td>
<td>12.4</td>
<td>14.2</td>
<td>17.8</td>
</tr>
<tr>
<td>RF Voltage (MV)</td>
<td>698</td>
<td>758</td>
<td>848</td>
<td>1230</td>
</tr>
<tr>
<td>Circumference (m)</td>
<td>481</td>
<td>436</td>
<td>612</td>
<td>470</td>
</tr>
<tr>
<td>Magnet cost (PB)</td>
<td>76</td>
<td>78</td>
<td>100</td>
<td>112</td>
</tr>
<tr>
<td>RF cost (PB)</td>
<td>45</td>
<td>49</td>
<td>55</td>
<td>80</td>
</tr>
<tr>
<td>Linear cost (PB)</td>
<td>12</td>
<td>11</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>Total cost (PB)</td>
<td>134</td>
<td>138</td>
<td>170</td>
<td>203</td>
</tr>
</tbody>
</table>

- Magnet costs generally high: re-optimize for cost, probably lower fields
**Cost Optimized FFAG Lattices**

- Allow magnet lengths to vary, find minimum cost lattice

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<thead>
<tr>
<th>Type</th>
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<th>FD</th>
<th>FODO 1 RF</th>
<th>FODO 2 RF</th>
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<tbody>
<tr>
<td>Cells</td>
<td>113</td>
<td>127</td>
<td>91</td>
<td></td>
</tr>
<tr>
<td>D Length (cm)</td>
<td>135</td>
<td>130</td>
<td>139</td>
<td></td>
</tr>
<tr>
<td>D Radius (cm)</td>
<td>8.7</td>
<td>9.7</td>
<td>10.1</td>
<td></td>
</tr>
<tr>
<td>D Pole Tip (T)</td>
<td>4.7</td>
<td>4.0</td>
<td>4.8</td>
<td></td>
</tr>
<tr>
<td>F Length (cm)</td>
<td>222</td>
<td>213</td>
<td>256</td>
<td></td>
</tr>
<tr>
<td>F Radius (cm)</td>
<td>13.6</td>
<td>15.5</td>
<td>19.3</td>
<td></td>
</tr>
<tr>
<td>F Pole Tip (T)</td>
<td>2.3</td>
<td>1.9</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>RF Voltage (MV)</td>
<td>847</td>
<td>950</td>
<td>1362</td>
<td></td>
</tr>
<tr>
<td>Circumference (m)</td>
<td>855</td>
<td>941</td>
<td>722</td>
<td></td>
</tr>
<tr>
<td>Magnet cost (PB)</td>
<td>34</td>
<td>33</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>RF cost (PB)</td>
<td>55</td>
<td>62</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>Linear cost (PB)</td>
<td>17</td>
<td>24</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Total cost (PB)</td>
<td>106</td>
<td>118</td>
<td>144</td>
<td></td>
</tr>
</tbody>
</table>
- Optimum cost substantially reduced from 7 T value
- Optimum seems to favor RF costs higher than magnet+linear.
- Optimum wants lower pole tips on FODO F quads
  - Larger aperture in F quads
- Pole tips relatively low!
  - May be able to shorten RF drift a bit