6D Muon Cooling in a Helical Dipole Channel

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for J-Lab* / Muons Inc. * SBIR/STTR
‘Six-Dimensional Beam Cooling in a Gas Absorber’

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Rolland. Johnson*
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The Magnetic Cooling Channel

- A long solenoid with superimposed transverse fields (dipole, quadrupole, sextupole) arranged to make the reference particle move in a helical orbit as it moves down the solenoid.

- The technology of these helical dipole magnets is well known (Brookhaven - spin control in RHIC). For initial ionization cooling of a muon beam, the helical magnets will require a larger aperture than has been used up to now.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Initial</th>
<th>Middle</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam momentum, $p$ *)</td>
<td>MeV/c</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Solenoid field, $B$</td>
<td>T</td>
<td>3.5</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>Cyclotron wave length, $\lambda_c = 2\pi / k_c$ *)</td>
<td>m</td>
<td>0.60</td>
<td>0.26</td>
<td>0.13</td>
</tr>
<tr>
<td>Helix period, $\lambda = 2\pi / k$</td>
<td>m</td>
<td>1</td>
<td>0.44</td>
<td>0.22</td>
</tr>
<tr>
<td>Helical magnet inner radius</td>
<td>cm</td>
<td>30</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>Transverse field at magnet</td>
<td>T</td>
<td>1.2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Transverse field at beam center</td>
<td>T</td>
<td>0.5</td>
<td>1.25</td>
<td>2.1</td>
</tr>
<tr>
<td>Helix quadrupole gradient</td>
<td>T/m</td>
<td>1.2</td>
<td>7.5</td>
<td>20</td>
</tr>
<tr>
<td>Helix orbit radius, $a$ *)</td>
<td>cm</td>
<td>15</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Dispersion, $D$</td>
<td>cm</td>
<td>37</td>
<td>15</td>
<td>7.5</td>
</tr>
<tr>
<td>Transverse beta functions, $\beta_+ / \beta_-$</td>
<td>cm</td>
<td>16/26</td>
<td>6/10</td>
<td>3.2/5.2</td>
</tr>
<tr>
<td>Accelerating RF field amplitude</td>
<td>MV/m</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Frequency, $\tilde{\omega} = \omega / 2\pi$</td>
<td>GHz</td>
<td>0.2</td>
<td>0.8</td>
<td>1.6</td>
</tr>
<tr>
<td>Energy loss rate in absorber</td>
<td>MeV/m</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>6D cooling decrement length, $\Lambda^{-1}$</td>
<td>m</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Relative momentum spread</td>
<td>%</td>
<td>7.5</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Bunch length</td>
<td>cm</td>
<td>30</td>
<td>7.5</td>
<td>1.1</td>
</tr>
<tr>
<td>Transverse emittances, $\varepsilon_+ / \varepsilon_-$</td>
<td>cm x rad</td>
<td>1.7/1.</td>
<td>0.2/0.2</td>
<td>(1/3)10^-2</td>
</tr>
<tr>
<td>Beam widths, $\sigma_+ / \sigma_-$</td>
<td>cm</td>
<td>8/5</td>
<td>1.8/1.1</td>
<td>0.45/0.28</td>
</tr>
</tbody>
</table>
Summary

- Cooling in a helical dipole channel – single particle dynamics
  - momentum offset dependant path length – momentum cooling
  - transversely correlated beam – emittance exchange

- Fields in a helical cooling channel – analytic solution

- Cooling rates – analytic predictions vs GEANT4 simulation
  - 6D pase-space compression – promise of $10^5$, over 60 m channel
  - Multi-particle tracking studies with g4bl (work in progress)

- Simplicity of the cooling scheme ....
Scaling Gas-filled Muon Ring Coolers

Al Garren, UCLA
Ringcooler Mini-workshop
Tucson, December 15-16, 2003
Ring types considered

- Weak Focusing Scaling Rings: Radial sectors with uniform magnetic fields \textit{ie} zero gradients.
- Strong Focusing FFAG Scaling Rings: Radial sectors with alternating field direction and magnetic field strength determined by the field exponent $k$. 
Weak Focusing Scaling Rings
Parameters

- Horizontal and vertical focusing is given by the combination of body and edge focusing of the sector magnets. The essential parameters are:
  - Muon momentum of reference orbit, \textit{i.e.} momentum of cooled beam
  - Number of magnet sectors
  - Magnetic field
  - Relative angular width of magnets and drifts
Strong Focusing Scaling Rings
Parameters

- Assumptions:
  - Inward and outward bending magnets have identical field profiles except the signs are opposite. Outward bends are horizontally focusing, inward bends are vertically focusing.
  - Horizontal and vertical focusing is given by body and edge focusing of the sector magnets and by their gradients.
- The essential parameters are:
  - Muon momentum
  - Number of magnet sectors
  - Magnetic fields on reference orbit
  - Relative angular widths of F and D magnets and drifts (if any).
- The geometry including edge angles can be derived from the above.
Dipole only 6D cooling Rings

Ring Cooler Meeting

Tucson

December 15-16, 2003
Recalculation with ICOOL V2.66

ICOOL V2.59

ICOOL V.2.66

Invariant Emittance

Full Turns

X : mm-rad  
Y : mm-rad  
Z : mm

Transmission/Merit

Full Turns

Transmission, %  
Merit with Decay

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Reduced Radius Performance

\[ B = 2.6\, \text{T} \quad P_0 = 125\, \text{MeV/c} \]

\[ B = 5.2\, \text{T} \quad P_0 = 250\, \text{MeV/c} \]

\[ \text{Invariant Emittance} \]

\[ \text{Full Turns} \]

\[ \text{Transmission/Merit} \]

\[ \text{Transmission, \%} \]

\[ \text{Merit with Decay} \]

\[ \text{X : mm-rad} \]

\[ \text{Y : mm-rad} \]

\[ \text{Z : mm} \]

\[ \text{Radius = 30 cm} \]

\[ \text{RF at 30 MV/m} \]

\[ \text{RF at 45 MV/m} \]

Harold G. Kirk
Merit Factor Comparison

RF at 25 MV/m

- R=60cm
- R=30cm--250 MeV
- R=30cm--125 MeV
LEAR ANTI-CYCLOTRON

Don Summers and Romulus Godang
(University of Mississippi-Oxford)

Muon Ring Cooler Mini-Workshop
Tucson, Arizona
15-16 December 2003
B-Field

- Magnet 1.6 Tesla
  4 concentric coils
  Weak focusing
  Azimuthally symmetric field

- Injection radius = 110 mm,
  p = 202 MeV/c, 0.3 mbar hydrogen

- Anti-protons adiabatically spiral to the center

- dE/dx cannot be too high
Examining the Garren-Kirk Dipole Cooling Ring with Realistic Fields

Steve Kahn
Alper Garren
Harold Kirk
Half Cell Geometry Description

Based on a Sketch from A. Garren
Closed Orbit

Closed orbit trajectory for 250 MeV/c $\mu$ started at $x=55.02994$ cm.

Note that there is curvature in region between magnets since there is still a significant field.
Figure shows $B_y$ along the 250 MeV/c reference path.

- The blue curve indicates the field from the Tosca field map.
- The red curve is the hard edge field.

Note the $-0.5$ T field in the gap mid-way between the magnets.
90° Transfer Matrix

This is the transfer matrix for transversing a quarter turn:

\[
\begin{bmatrix}
\delta x \\
\delta x' \\
\delta y \\
\delta y'
\end{bmatrix} =
\begin{bmatrix}
-0.29145 & 31.965 & 0 & 0 \\
-0.0287 & -0.289 & 0 & 0 \\
0 & 0 & -0.18336 & 52.9949 \\
0 & 0 & -0.01823 & -0.1853
\end{bmatrix}
\begin{bmatrix}
\delta x_0 \\
\delta x'_0 \\
\delta y_0 \\
\delta y'_0
\end{bmatrix}
\]

This should be compared to the 2×2 matrix to obtain the twiss variables:

\[
\begin{bmatrix}
\cos \mu + \alpha \sin \mu & \beta \sin \mu \\
\gamma \sin \mu & \cos \mu - \alpha \sin \mu
\end{bmatrix}
\]
**Twiss Variables Half Way Between Magnets**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Tosca</th>
<th>A. Garren Synch</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_x$</td>
<td>98.38°</td>
<td>99.8784°</td>
</tr>
<tr>
<td>$\beta_x$</td>
<td>32.3099 cm</td>
<td>37.854 cm</td>
</tr>
<tr>
<td>$\alpha_x$</td>
<td>-0.00124</td>
<td>0</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>100.62°</td>
<td>92.628°</td>
</tr>
<tr>
<td>$\beta_y$</td>
<td>53.9188 cm</td>
<td>56.891 cm</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>0.0009894</td>
<td>0</td>
</tr>
</tbody>
</table>
Using the Field Map

- We can produce a 3D field map from TOSCA.
  - We could build a GEANT model around this field map however this has not yet been done.
  - We have decided that we can provide a field to be used by ICOOL.
    - ICOOL works in a beam coordinate system.
      - We know the trajectory of the reference path in the global coordinate system.
        - We can calculate the field and its derivatives along this path.
$b_n$ along the path
Fourier Expansion of $b_n(s)$

- The $b_n(s)$ can be expanded with a Fourier series:

$$b_n = \Re \sum_{k=0}^{N-1} c_{k,n} e^{-\frac{ik^s}{T}}$$

where

$$c_{k,n} = \frac{1}{T} \int_0^T b_n(s) e^{\frac{ik^s}{T}}$$

- These Fourier coefficients can be fed to ICOOL to describe the field with the BSOL 4 option.
- We use the $b_n$ for $n=0$ to 5.
Storage Ring Mode

- Modify Harold Kirk’s ICOOL deck to accept the Fourier description of the field.
  - Scale the field to 250 MeV/c on the reference orbit.
    - This is a few percent correction.
- Verify the configuration in storage ring mode.
  - RF gradient set to zero.
  - Material density set to zero.
- Use a sample of tracks with:
  - \( \delta x = \pm 1 \text{ mm}; \delta y = \pm 1 \text{ mm}; \delta z = \pm 1 \text{ mm}; \)
  - \( \delta p_x = \pm 10 \text{ MeV/c}; \delta p_y = \pm 10 \text{ MeV/c}; \delta p_z = \pm 10 \text{ MeV/c}; \)
  - Also the reference track.
The figures show $\delta x$ and $\delta y$ for the 13 sample tracks.

Two tracks are lost. The others stay in a range of $\delta x$ and $\delta y = \pm 2$ cm.

Most of the track survive $>100$ m (25 turns).

- The lost tracks are the two with $\delta p_z = \pm 10$ MeV/c.
Transverse Momentum Deviation in Storage Ring Mode

- The figures show $\delta p_x$ and $\delta p_y$ deviations.
- They stay in the range $\pm 10$ MeV/c.
Now With RF

- We now set the material of the absorber to gaseous H$_2$ with 100 atm pressure.
- We need to optimize the RF phase relative to the reference particle.
- We must optimize the RF gradient such that the muon momentum is stable from period to period.
- Figures show plots of $E_z$ and $p_z$ vs. $s$ for gradient of 20 mV/m and phase of 30°
A Strong Focusing AG Lattice

Lattice consists of alternating Horz. Defocusing and Horz. Focusing with $L_{HD} = \frac{1}{2} L_{HF}$.
No drift cells between dipole elements.

Parameters
12 cells
Bend angles $30^\circ$ and $-15^\circ$
Circumference = 6m
$B_0 = 2.6T$ and $P_0 = 250$ MeV/c
Dispersion = 25 cm

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Strong Focusing Lattice Performance

12 Sector Oct. 21, 03 Lattice: 250 MeV/c

- X : mm-rad
- Y : mm-rad
- Z : mm
- FFAG-like Lattice

RF at 25 MV/m over 60% of circumference

Horizontal Emittance Reduction Factor 10
Vertical Emittance Reduction Factor 11
Longitudinal Emittance Reduction Factor 2

Harold G. Kirk
Dec. 16, 2003

Preliminary Conclusions of the Tucson Workshop

1) Magnets are feasible for ring coolers at 2.6 T, 30 cm aperture, 60 cm radius. May be scaled to higher field magnets.

2) Weak Focusing / Strong Focusing Ring. Continue studying on both options.

3) Insertion of rf cells into dipole elements maybe feasible. Quarter wavelength coaxial rf cavities could do the job.


5) We should consider designing and constructing a test muon ring cooler based on the gas filled model.

5) 6D cooling continues to look promising.