6D Muon Cooling in a Helical Dipole Channel

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for J-Lab♣/Muons Inc.♠ SBIR/STTR

'Six-Dimensional Beam Cooling in a Gas Absorber'

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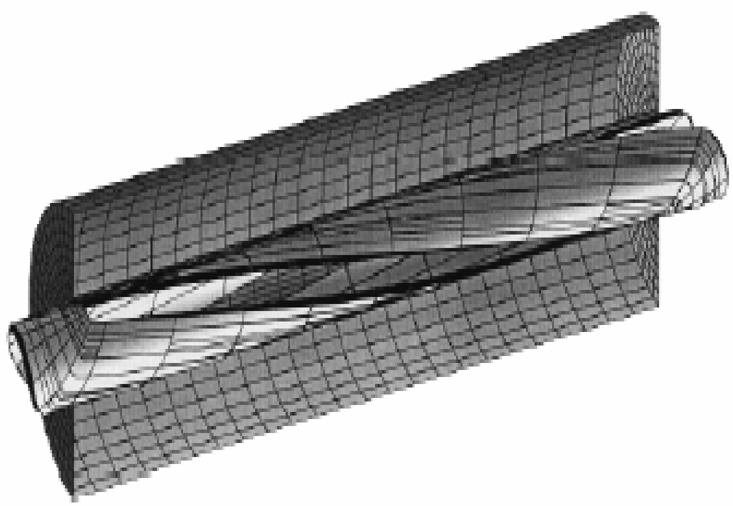
The Magnetic Cooling Channel

- A long solenoid with superimposed transverse fields (dipole, quadrupole, sextupole) arranged to make the reference particle move in a helical orbit as it moves down the solenoid.

- The technology of these helical dipole magnets is well known (Brookhaven – spin control in RHIC). For initial ionization cooling of a muon beam, the helical magnets will require a larger aperture than has been used up to now



Direct Wind Type

Parameter	Unit	Initial	Middle ****)	Final
Beam momentum, p *)	MeV/c	100	100	100
Solenoid field, B	T	3.5	8	15
Cyclotron wave length, $\lambda_c = 2\pi / k_c$ *)	m	0.60	0.26	0.13
Helix period, $\lambda = 2\pi / k$	m	1	0.44	0.22
Helical magnet inner radius	cm	30	12	7
Transverse field at magnet	T	1.2	3	5
Transverse field at beam center	T	0.5	1.25	2.1
Helix quadrupole gradient	T/m	1.2	7.5	20
Helix orbit radius, a *)	cm	15	6	3
Dispersion, D	cm	37	15	7.5
Transverse beta functions, β_+/β_-	cm	16/26	6/10	3.2/5.2
Accelerating RF field amplitude	MV/m	30	30	30
Frequency, $f=\omega/2\pi$	GHz	0.2	0.8	1.6
Energy loss rate in absorber	MeV/m	20	20	20
6D cooling decrement length, Λ^{-1}	m	4	4	4
Relative momentum spread	%	7.5	3	2
Bunch length	cm	30	7.5	1.1
Transverse emittances, $\varepsilon_+/\varepsilon_-$	cm x rad	1.7/1.	0.2/0.2	$(1/3)10^{-2}$
Beam widths, σ_1/σ_2	cm	8/5	1.8/1.1	0.45/0.28

Summary

- Cooling in a helical dipole channel – single particle dynamics
  - momentum offset dependant path length – momentum cooling
  - transversely correlated beam – emittance exchange
- Fields in a helical cooling channel – analytic solution
- Cooling rates – analytic predictions vs GEANT4 simulation
  - 6D pase-space compression – promise of 10^{5}, over 60 m channel
  - Multi-particle tracking studies with g4bl (work in progress)
- Simplicity of the cooling scheme ....

Scaling Gas-filled Muon Ring Coolers

Al Garren, UCLA

Ringcooler Mini-workshop

Tucson, December 15-16, 2003

Ring types considered

- Weak Focusing Scaling Rings:
  Radial sectors with uniform magnetic fields *ie* zero gradients.
- Strong Focusing FFAG Scaling Rings:
  Radial sectors with alternating field direction and magnetic field strength determined by the field exponent *k*.

Weak Focusing Scaling Rings Parameters

- Horizontal and vertical focusing is given by the combination of body and edge focusing of the sector magnets. The essential parameters are:
  - Muon momentum of reference orbit, *ie* momentum of cooled beam
  - Number of magnet sectors
  - Magnetic field
  - Relative angular width of magnets and drifts

Strong Focusing Scaling Rings Parameters

- Assumptions:
  - Inward and outward bending magnets have identical field profiles except the signs are opposite. Outward bends are horizontally focusing, inward bends are vertically focusing.
- Horizontal and vertical focusing is given by body and edge focusing of the sector magnets and by their gradients.
- The essential parameters are:
  - Muon momentum
  - Number of magnet sectors
  - Magnetic fields on reference orbit
  - Relative angular widths of F and D magnets and drifts (if any).
- The geometry including edge angles can be derived from the above.

Dipole only 6D cooling Rings

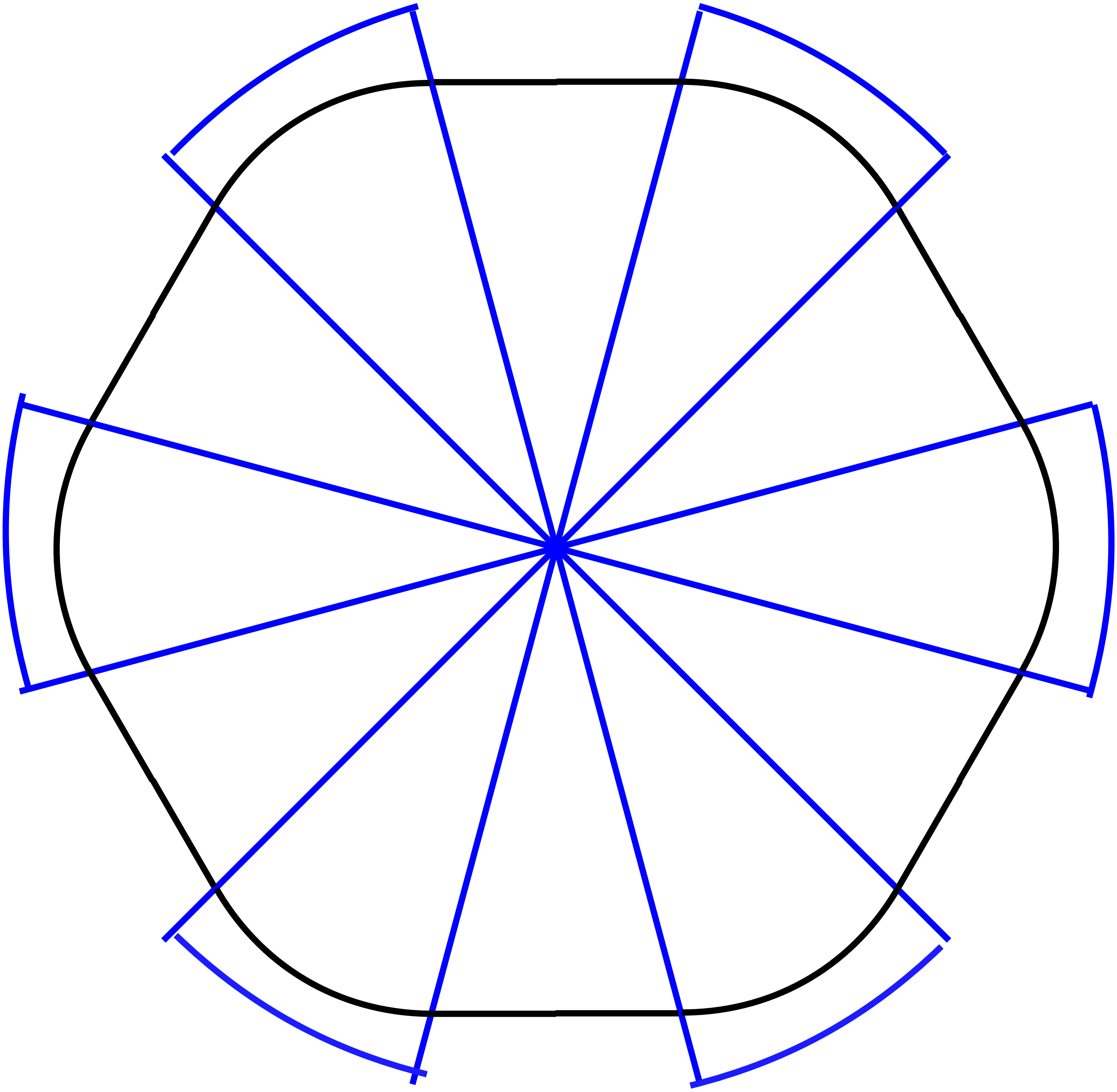
Ring Cooler Meeting

Tucson

December 15-16, 2003

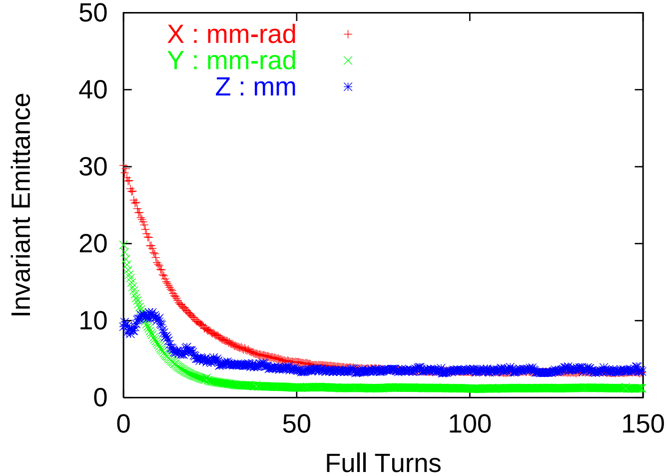
Harold G. Kirk
Brookhaven National Laboratory

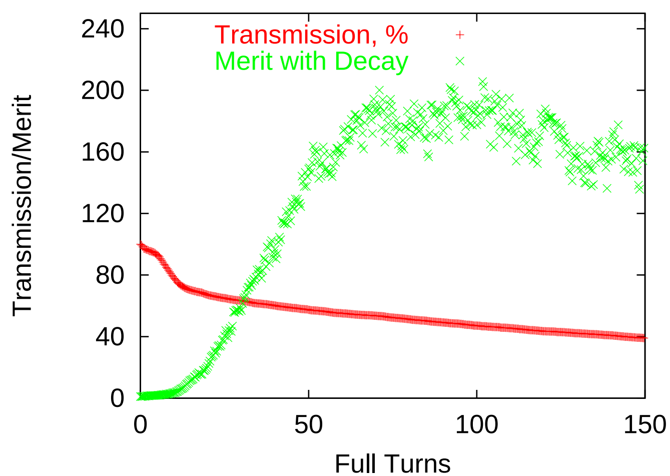
6 DIPOLE RING



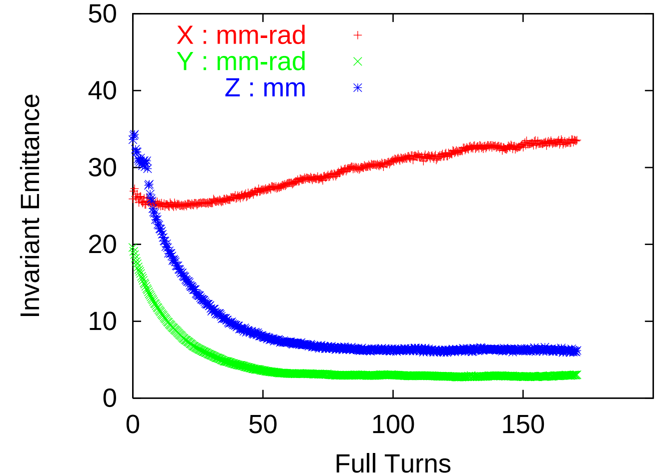
Recalculation with ICOOL V2.66

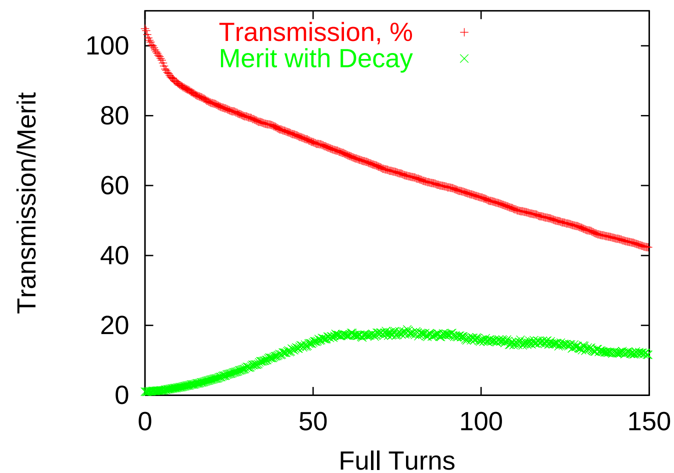
ICOOL V2.59





ICOOL V.266





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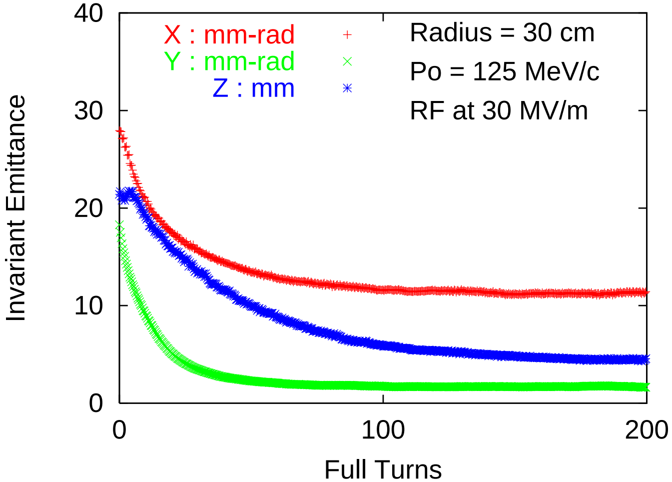
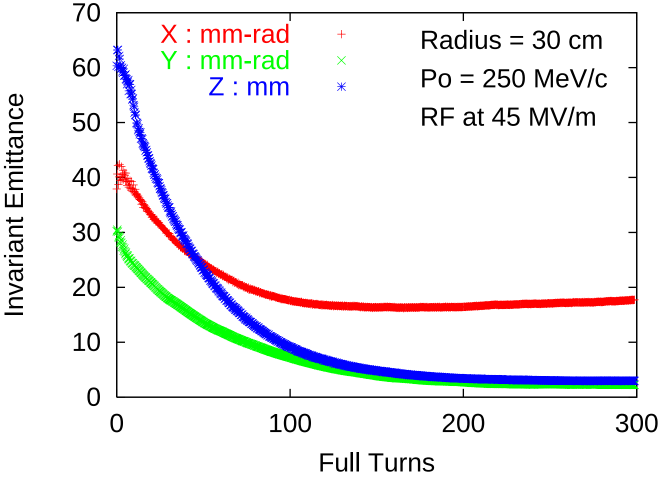
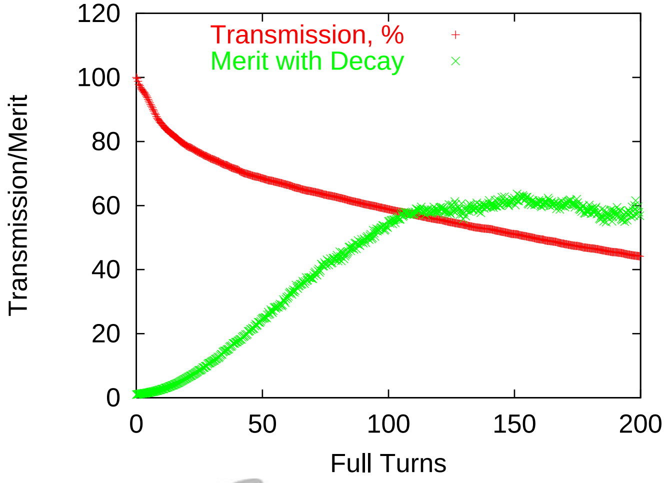
Reduced Radius Performance

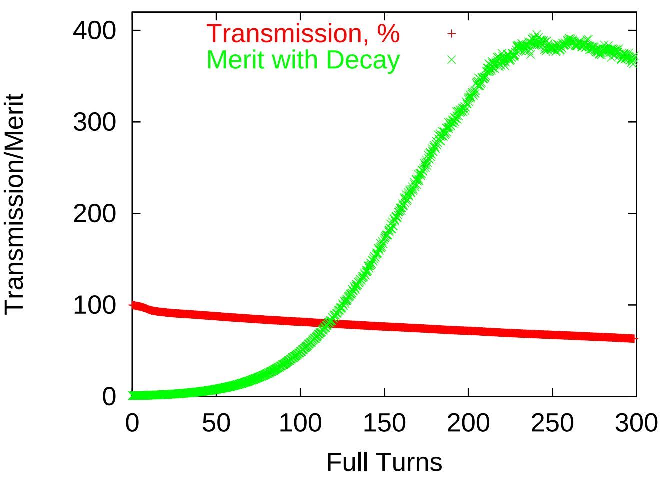
B = 2.6T Po = 125 MeV/c

B = 5.2T Po = 250 MeV/c









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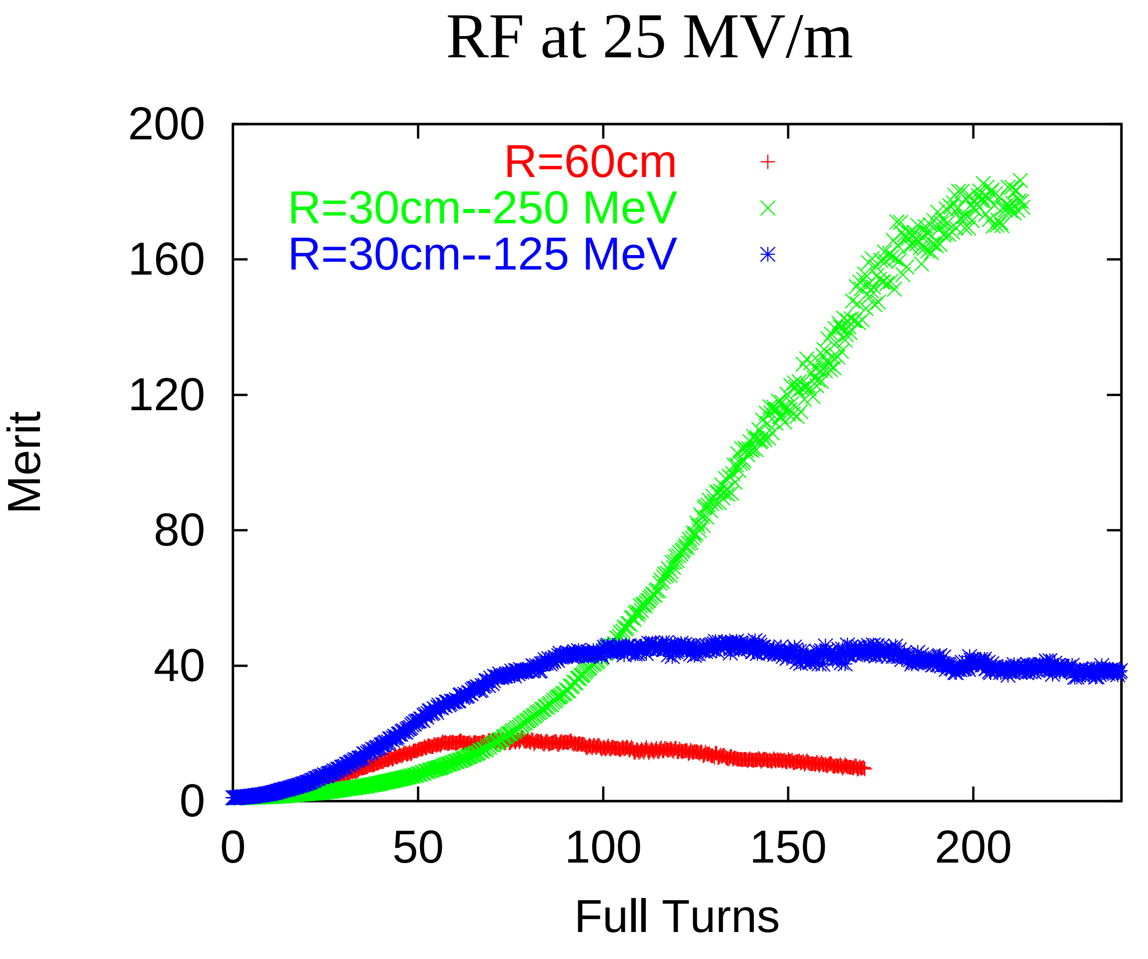
Merit Factor Comparison



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LEAR ANTI-CYCLOTRON

Don Summers and Romulus Godang

(University of Mississippi-Oxford)

Muon Ring Cooler Mini-Workshop

Tucson, Arizona

15-16 December 2003

B-Field

- **Magnet 1.6 Tesla**
  4 concentric coils
  Weak focusing
  Azimuthally symmetric field

- **Injection radius = 110 mm,**
  p = 202 MeV/c, 0.3 mbar hydrogen

- **Anti-protons adiabatically spiral to the center**

- **dE/dx cannot be too high**

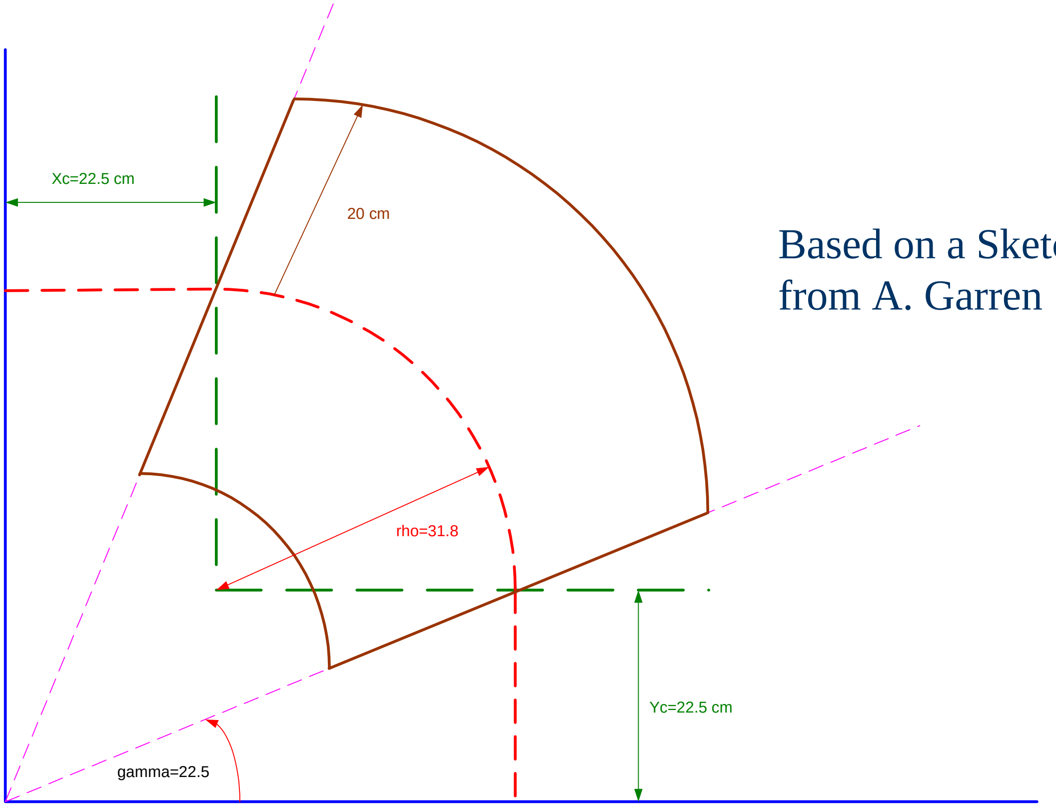
Examining the Garren-Kirk Dipole Cooling Ring with Realistic Fields

Steve Kahn

Alper Garren

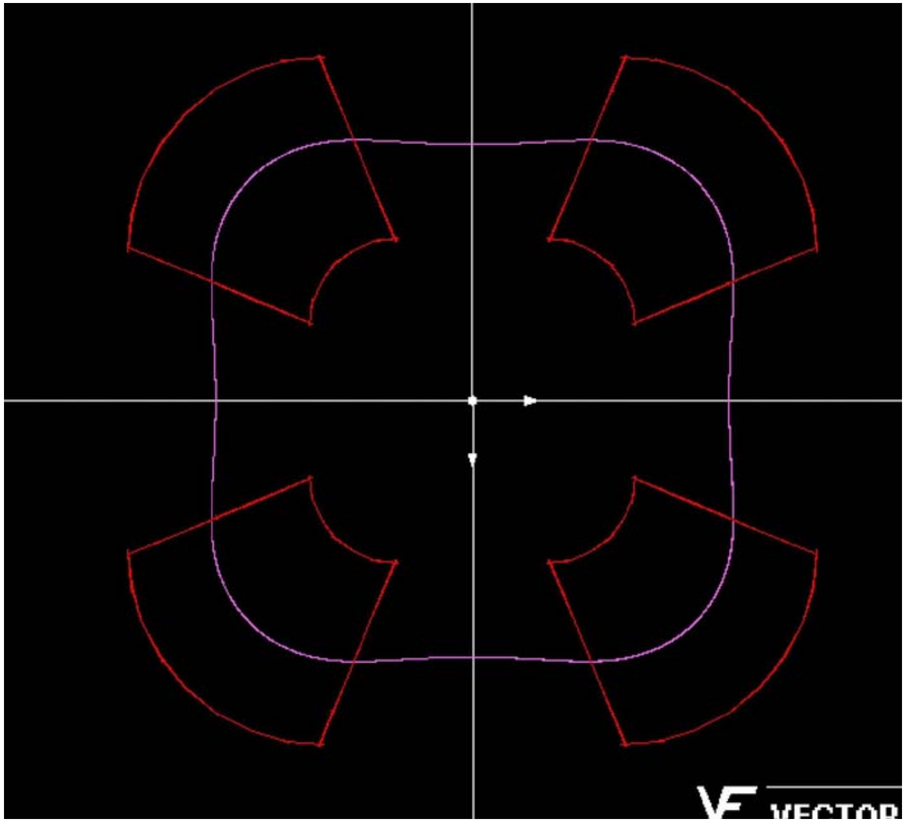
Harold Kirk

Half Cell Geometry Description



Based on a Sketch from A. Garren

Closed Orbit

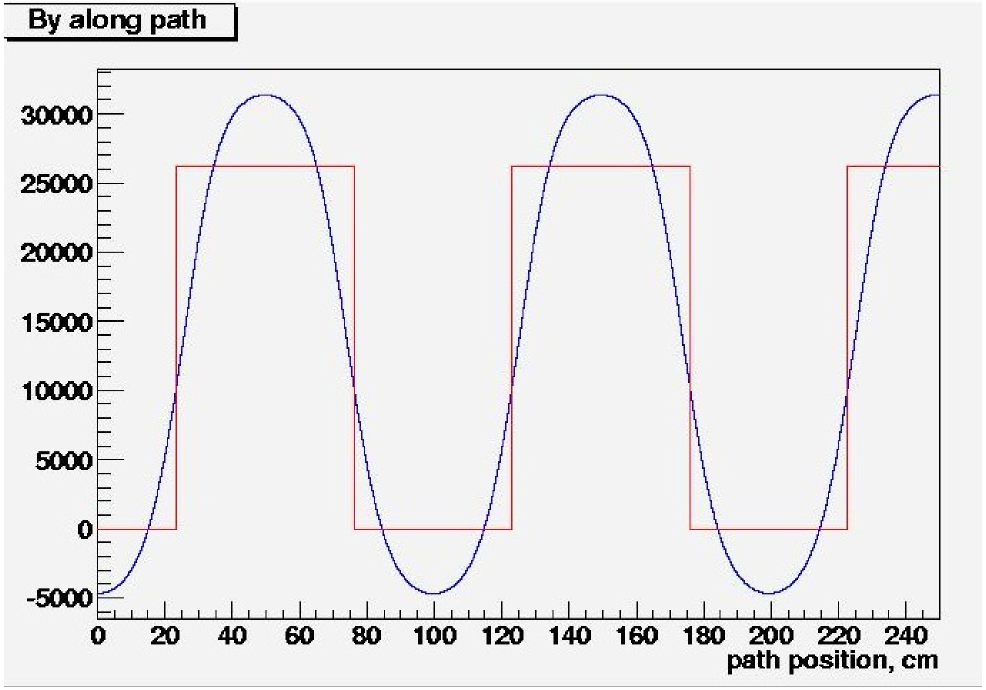


Closed orbit trajectory for 250 MeV/c μ started at *x*=55.02994 cm.

Note that there is curvature in region between magnets since there is still a significant field.

Field Along the Reference Path

- Figure shows B_y along the 250 MeV/c reference path.
  - The blue curve indicates the field from the Tosca field map.
  - The red curve is the hard edge field.
- Note the –0.5 T field in the gap mid-way between the magnets.



90° Transfer Matrix

- This is the transfer matrix for transversing a quarter turn:

$$\begin{bmatrix} \delta x \\ \delta x' \\ \delta y \\ \delta y' \end{bmatrix} = \begin{bmatrix} -0.29145 & 31.965 & 0 & 0 \\ -0.0287 & -0.289 & 0 & 0 \\ 0 & 0 & -0.18336 & 52.9949 \\ 0 & 0 & -0.01823 & -0.1853 \end{bmatrix} \begin{bmatrix} \delta x_0 \\ \delta x'_0 \\ \delta y_0 \\ \delta y'_0 \end{bmatrix}$$

- This should be compared to the 2×2 matrix to obtain the twiss variables:

$$\begin{bmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ \gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{bmatrix}$$

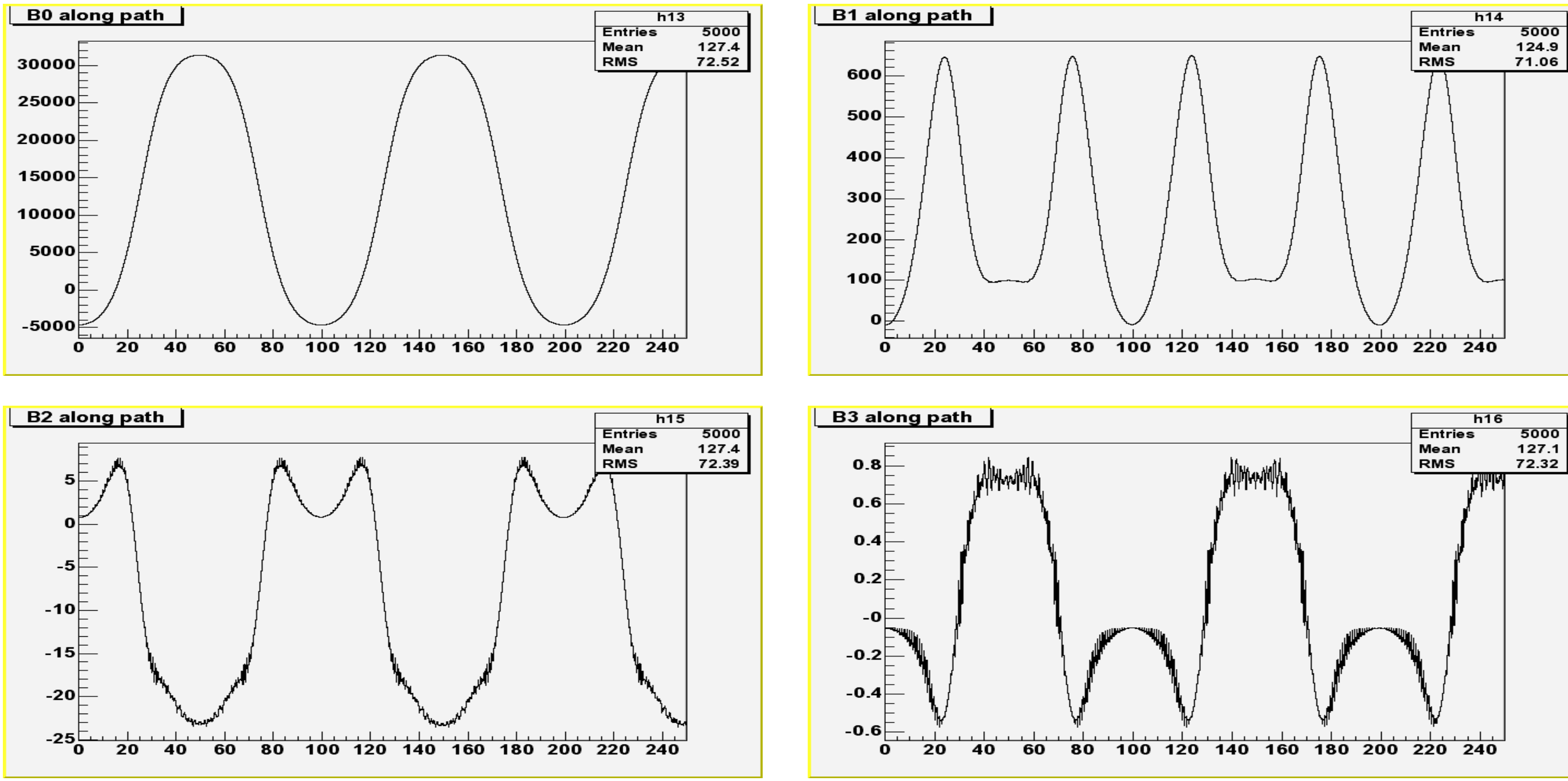
Twiss Variables Half Way Between Magnets

Parameter	Tosca	A. Garren Synch
μ_x	98.38°	99.8784°
β_x	32.3099 cm	37.854 cm
α_x	-0.00124	0
μ_y	100.62°	92.628°
β_y	53.9188 cm	56.891 cm
α_y	0.0009894	0

Using the Field Map

- We can produce a 3D field map from TOSCA.
  - We could build a GEANT model around this field map however this has not yet been done.
  - We have decided that we can provide a field to be used by ICOOL.
    - ICOOL works in a beam coordinate system.
      - We know the trajectory of the reference path in the global coordinate system.
        - We can calculate the field and its derivatives along this path.

b_n along the path



Fourier Expansion of $b_n(s)$

- The $b_n(s)$ can be expanded with a Fourier series:

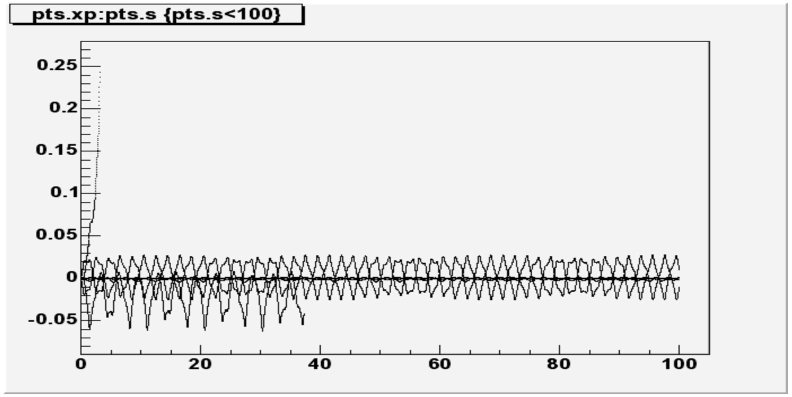
$$b_n = \Re \sum_{k=0}^{N-1} c_{k,n} e^{-ik\frac{s}{T}} \quad \text{where} \quad c_{k,n} = \frac{1}{T}\int_0^T b_n(s) e^{ik\frac{s}{T}}$$

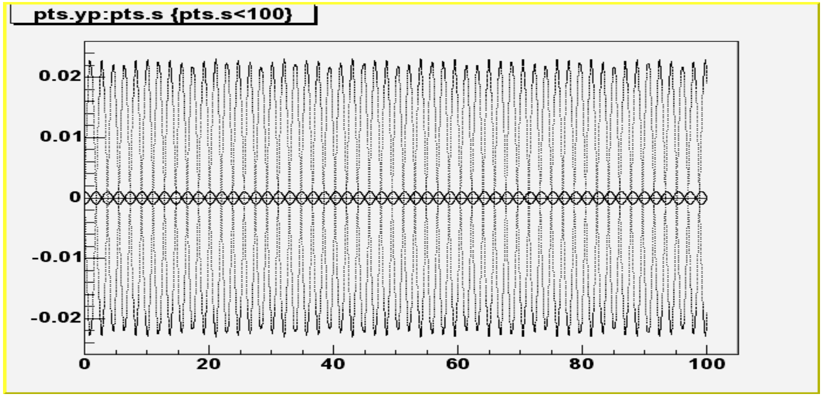
- These Fourier coefficients can be fed to ICOOL to describe the field with the *BSOL 4* option.
- We use the b_n for $n=0$ to 5.

Storage Ring Mode

- Modify Harold Kirk's ICOOL deck to accept the Fourier description of the field.
  - Scale the field to 250 MeV/c on the reference orbit.
    - This is a few percent correction.
- Verify the configuration in storage ring mode.
  - RF gradient set to zero.
  - Material density set to zero.
- Use a sample of tracks with:
  - $\delta x=\pm 1$ mm; $\delta y=\pm 1$ mm; $\delta z=\pm 1$ mm;
  - $\delta p_x=\pm 10$ MeV/c; $\delta p_y=\pm 10$ MeV/c; $\delta p_z=\pm 10$ MeV/c;
  - Also the reference track.

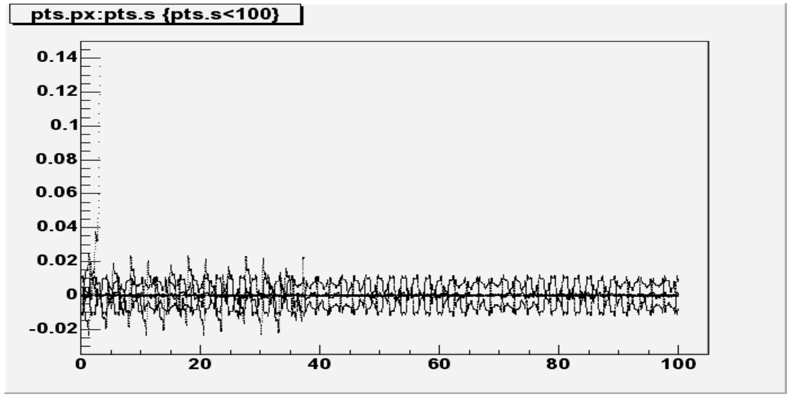
Spacial Deviations in Storage Mode

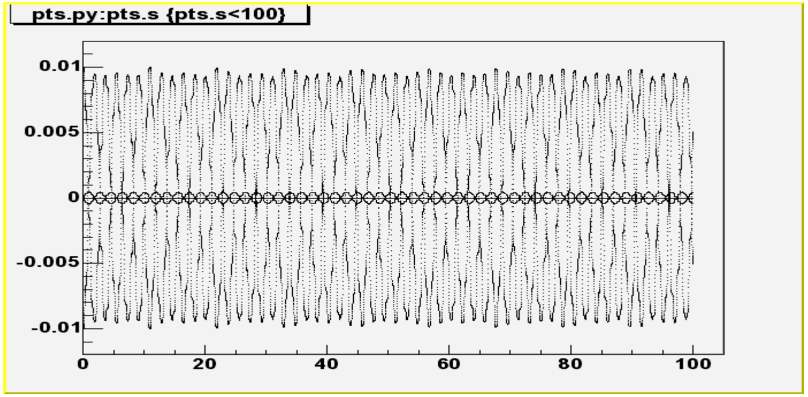




- The figures show δx and δy for the 13 sample tracks.
- Two tracks are lost. The others stay in a range of δx and δy = ±2 cm.
- Most of the track survive >100 m (25 turns).
  - The lost tracks are the two with $\delta p_z = \pm$ 10 MeV/c.

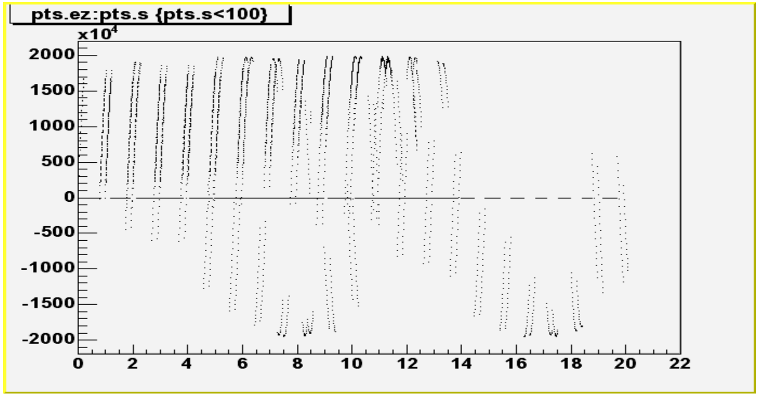
Transverse Momentum Deviation in Storage Ring Mode

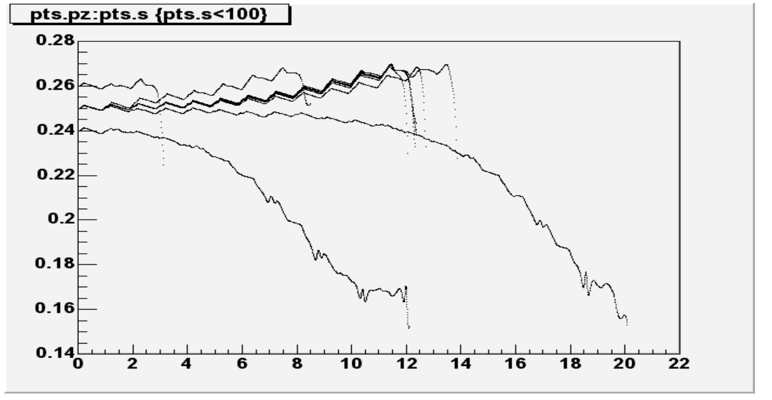




- The figures show δp_x and δp_y deviations.
- They stay in the range ± 10 MeV/c.

Now With RF

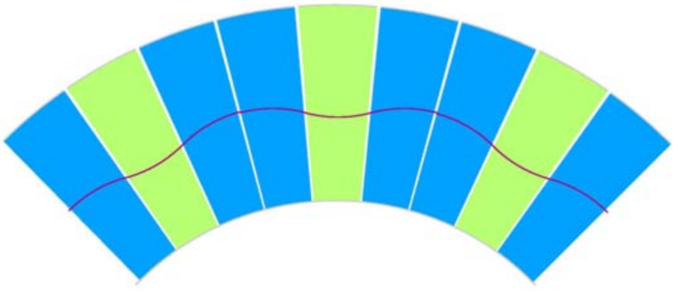




- We now set the material of the absorber to gaseous H_2 with 100 atm pressure.
- We need to optimize the RF phase relative to the reference particle.
- We must optimize the RF gradient such that the muon momentum is stable from period to period.
- Figures show plots of E_z and p_z vs. *s* for gradient of 20 mV/m and phase of 30°

A Strong Focusing AG Lattice

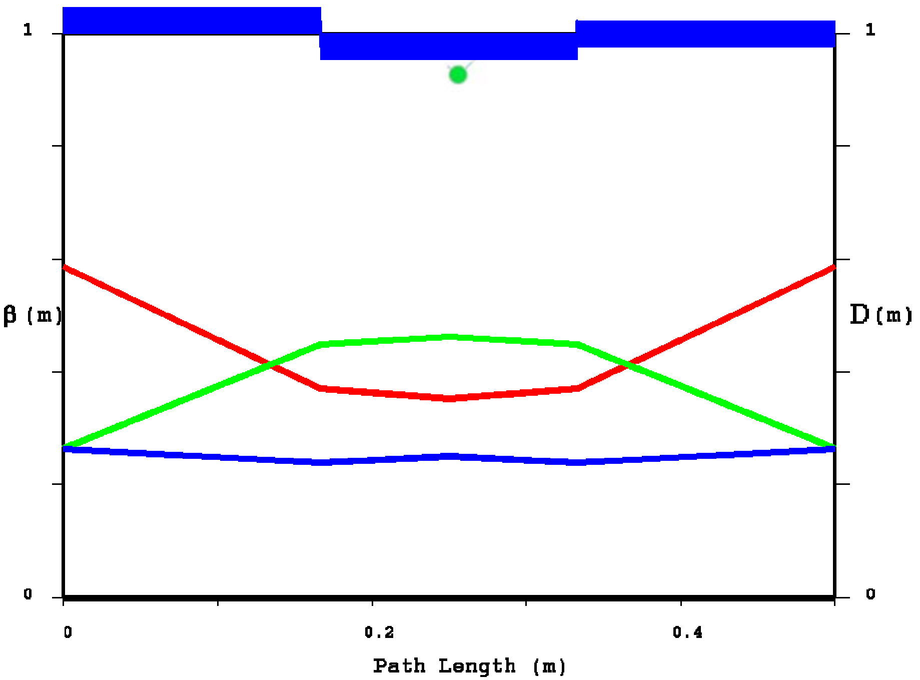
Lattice consists of alternating Horz. Defocusing and Horz. Focusing with $L_{HD} = ½ L_{HF}$. No drift cells between dipole elements.



3 Cells - 90°

Parameters

12 cells
Bend angles 30° and –15°
Circumference = 6m
B_o = 2.6T and P_o = 250 MeV/c
Dispersion = 25 cm



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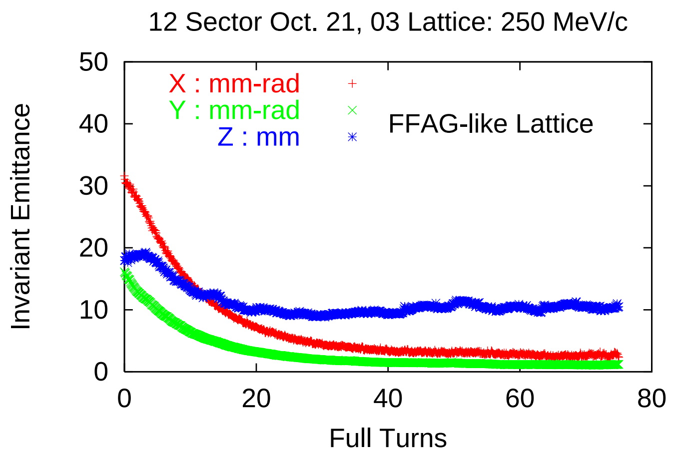
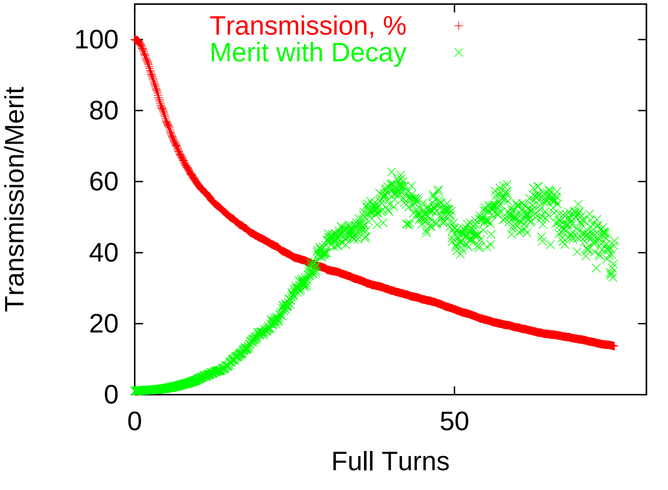
Strong Focusing Lattice Performance



RF at 25 MV/m over
60% of circumference

Horizontal Emittance Reduction Factor 10
Vertical Emittance Reduction Factor 11
Longitudinal Emittance Reduction Factor 2



BROOKHAVEN NATIONAL LABORATORY

Harold G. Kirk

Dec. 16, 2003

Preliminary Conclusions of the Tucson Workshop

1) Magnets are feasible for ring coolers at 2.6 T, 30 cm aperture, 60 cm radius. May be scaled to higher field magnets.

2)Weak Focusing /Strong Focusing Ring. Continue studying on both options.

3) Insertion of rf cells into dipole elements maybe feasible. Quarter wavelength coaxial rf cavities could do the job.

4)Helical Solenoid. Very promising method for 6D cooling.

5) We should consider designing and constructing a test muon ring cooler based on the gas filled model.

5) 6D cooling continues to look promising.