Design Low-kappa HCC

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NFMCC Friday meeting
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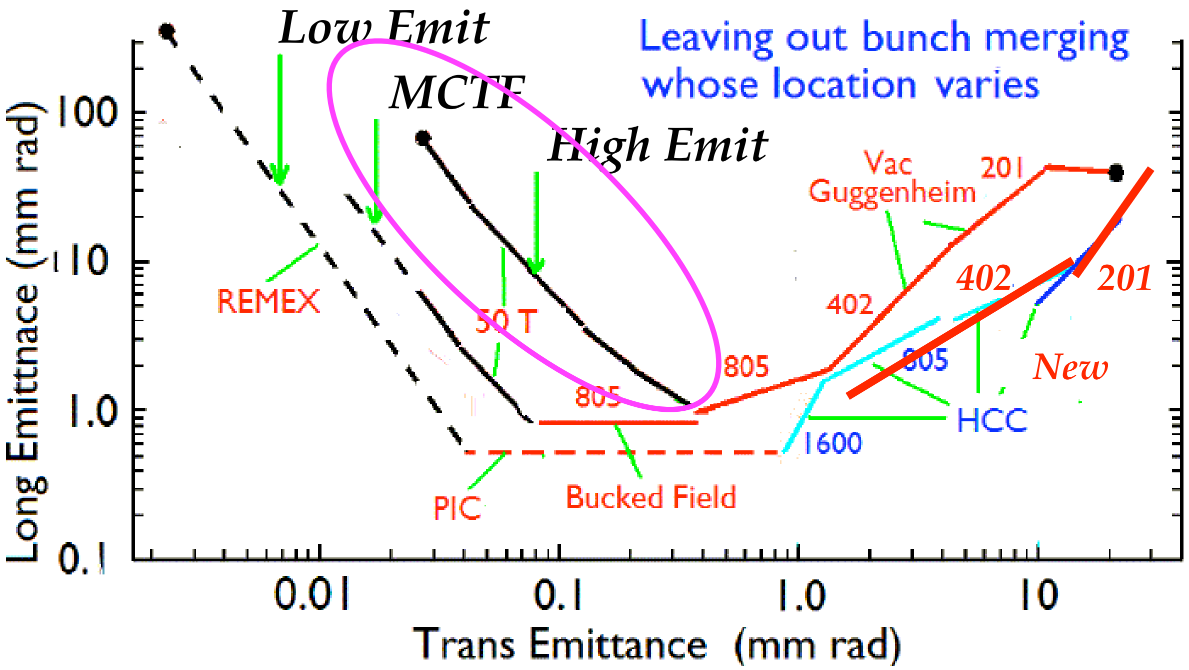
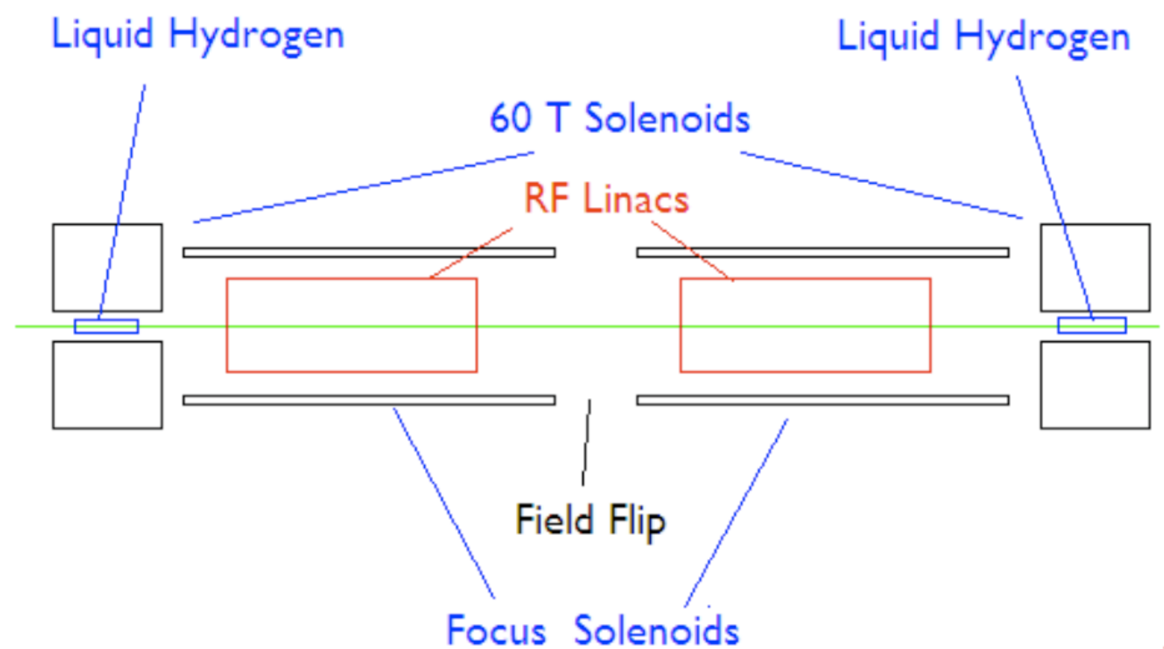
Final cooling channel

Final cooling section

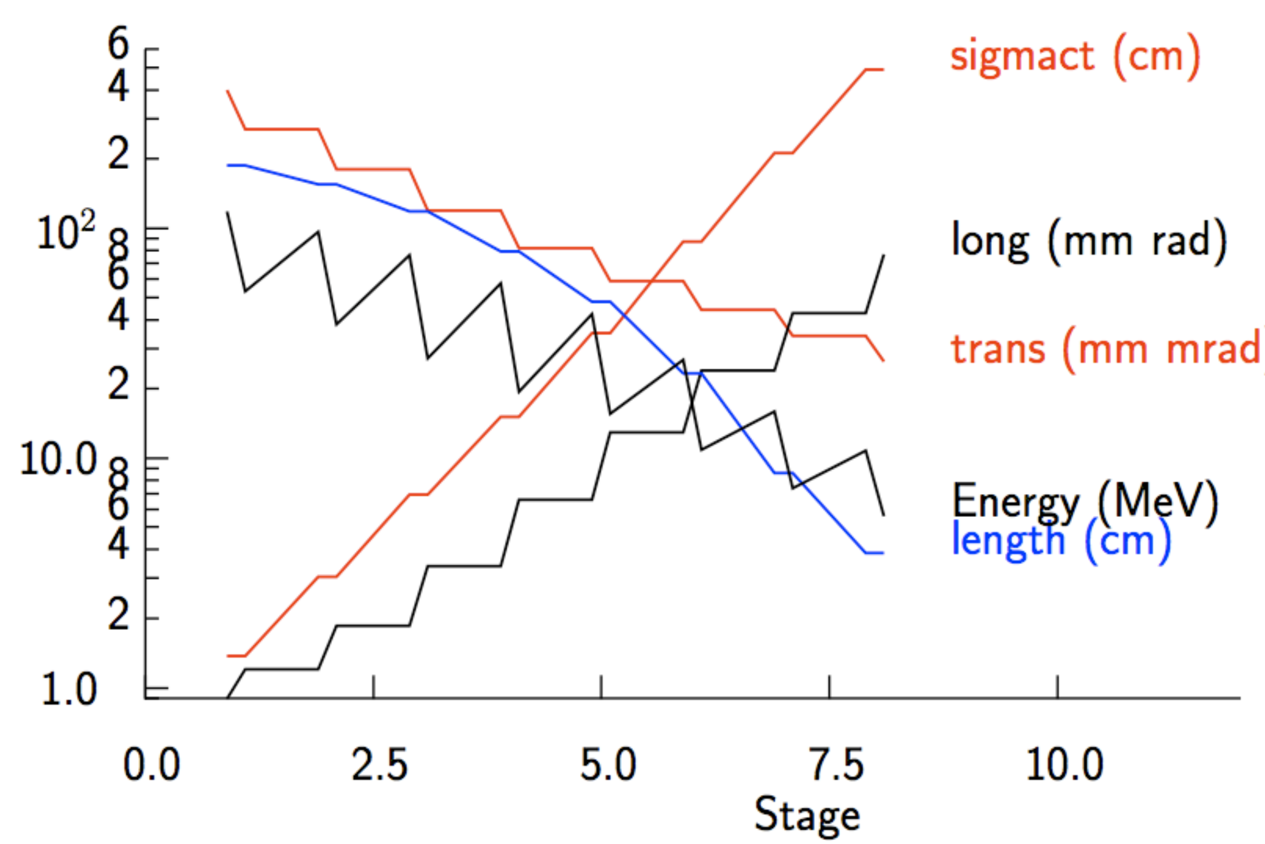


Phase advance in various scenarios



High field cooling channel

Final longitudinal beam size will be 6 m!



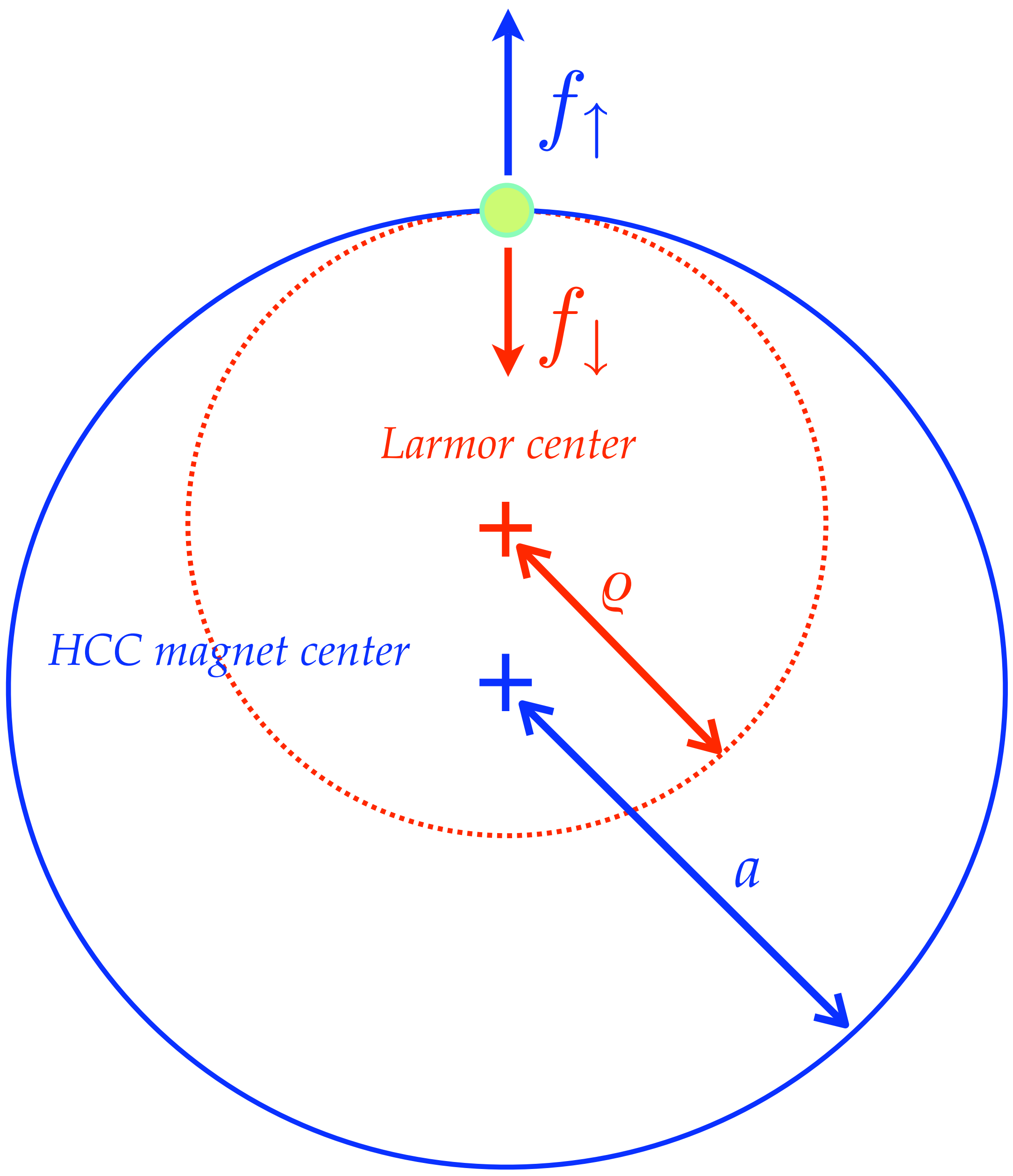
Emittance exchange in high field solenoid

Possibility of using Low kappa HCC for final cooling

Positive aspects in this channel:

- Reduce longitudinal beam size growth by emittance exchange in HCC
- It will be easy to build a matching section for low kappa HCC
- Hence, it will be easy to include RF structure
  - Inside? or Outside?
  - This study I assume the acceleration structure will be outside from HCC (MANX type channel)

Transverse stability in low kappa HCC



Larmor motion in pure solenoid

$$f_\downarrow = -\frac{e}{m_\mu} p_\varphi \cdot b_z$$

$$\rho = \frac{p_\varphi}{b_z}$$

Radial equation of motion with helical dipole

$$f = f_\uparrow + f_\downarrow$$
$$= \frac{e}{m_\mu}\left(p_z b_\varphi - p_\varphi b_z\right)$$

$$f = \gamma m r'' - \gamma m r \omega^2$$

$$p(a) = \frac{\sqrt{1+\kappa^2}}{k}\left(b_z - \frac{1+\kappa^2}{\kappa} b_\varphi\right)$$

Particle motion in stable orbit

Correct field gradient makes stabilize the transverse particle motion

Beam parameters

Dispersion factor

$$\widehat{D}^{-1} = \left(\frac{a}{p}\frac{dp}{da}\right) = \frac{\kappa^2 + (1-\kappa^2)q}{1+\kappa^2} + g$$

$$g = -\frac{(1+\kappa^2)^{3/2}}{pk^2}\frac{\delta b_\varphi}{\delta a}$$

$$q = \frac{k_c}{k} - 1 = \frac{(1+\kappa^2)^{3/2}}{kp\kappa} b_\varphi = \frac{a}{\rho} - 1$$

Important parameter to design HCC: $\widehat{D}, q$

Stability condition is

$$0 < \left(\frac{2q+\kappa^2}{1+\kappa^2} - \widehat{D}^{-1}\right)\widehat{D}^{-1} < \frac{1}{4}\left(1 + \frac{q^2}{1+\kappa^2}\right)^2$$

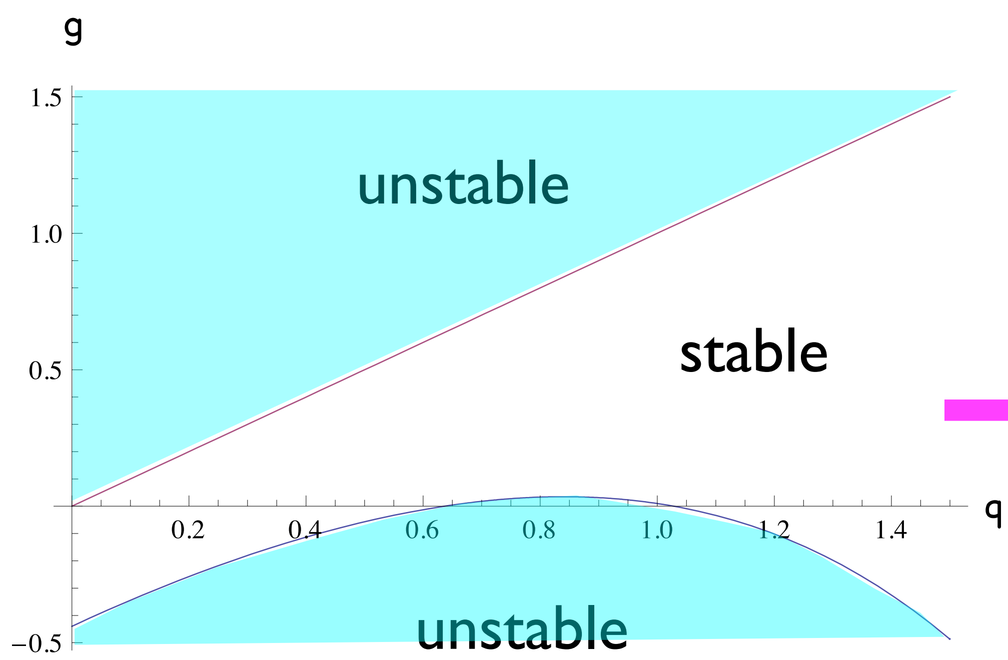
Above inequalities come from betatron motion in HCC

See eqs. (3.18) to (3.24) in Slava & Rol's paper (PRSTAB 8, 041002 (2005))

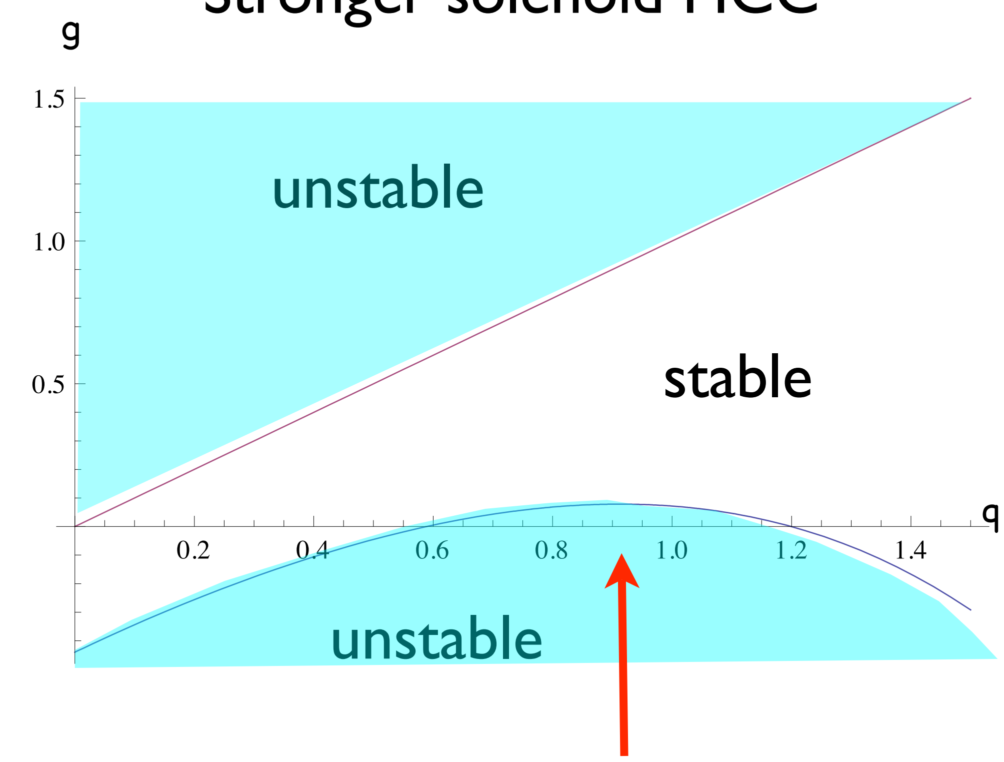
Example

Higher bz → kappa > 1

Optimum HCC



Stronger solenoid HCC



Stability area gets narrower in stronger solenoid condition

Stability condition with various kappa

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Table[qgplot[κ, Dhat = 1.8], {κ, 0.0, 1.1, 0.1}]
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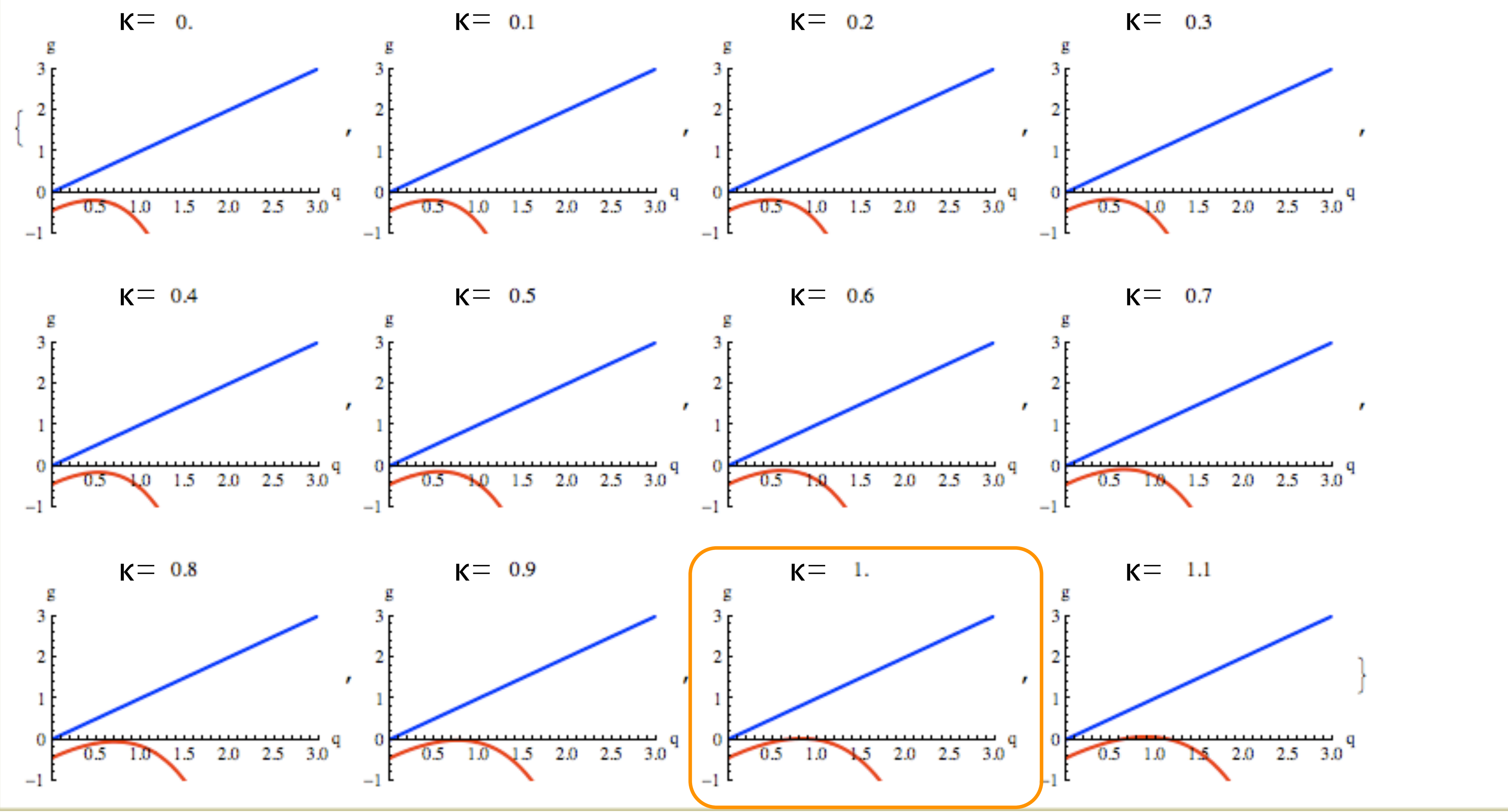
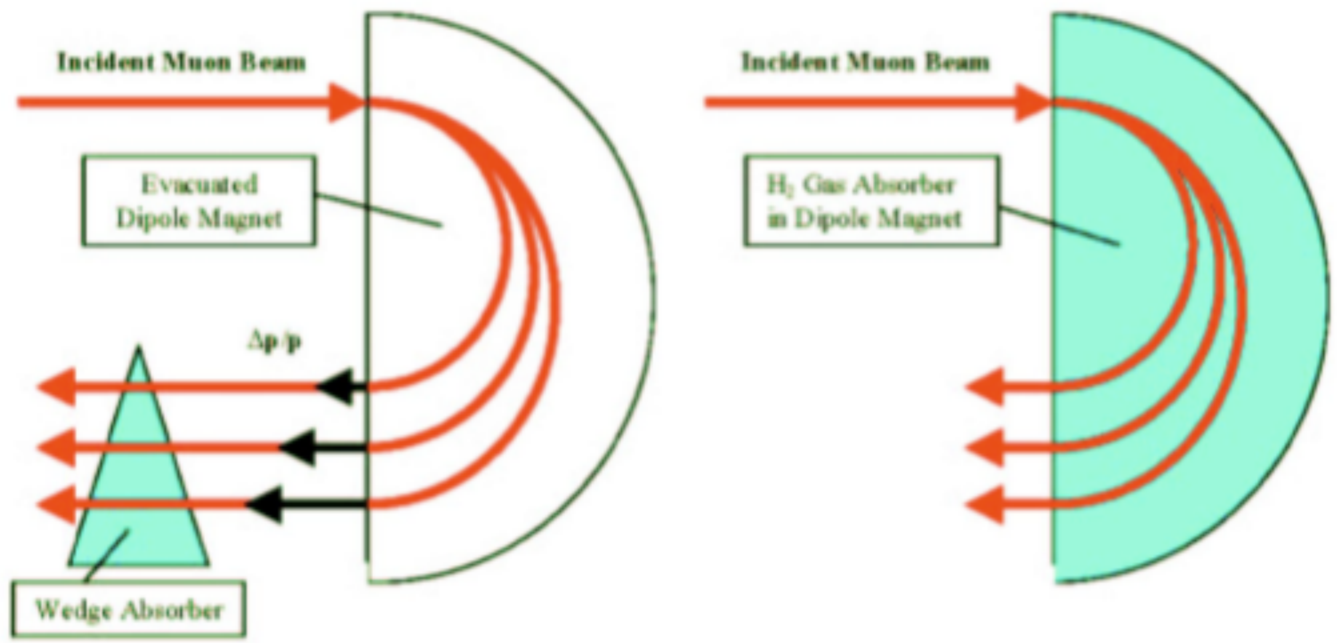


Use this condition to obtain the best cooling performance

Those plots indicate that a large stable phase space exists in transverse plane

Dynamics in longitudinal direction

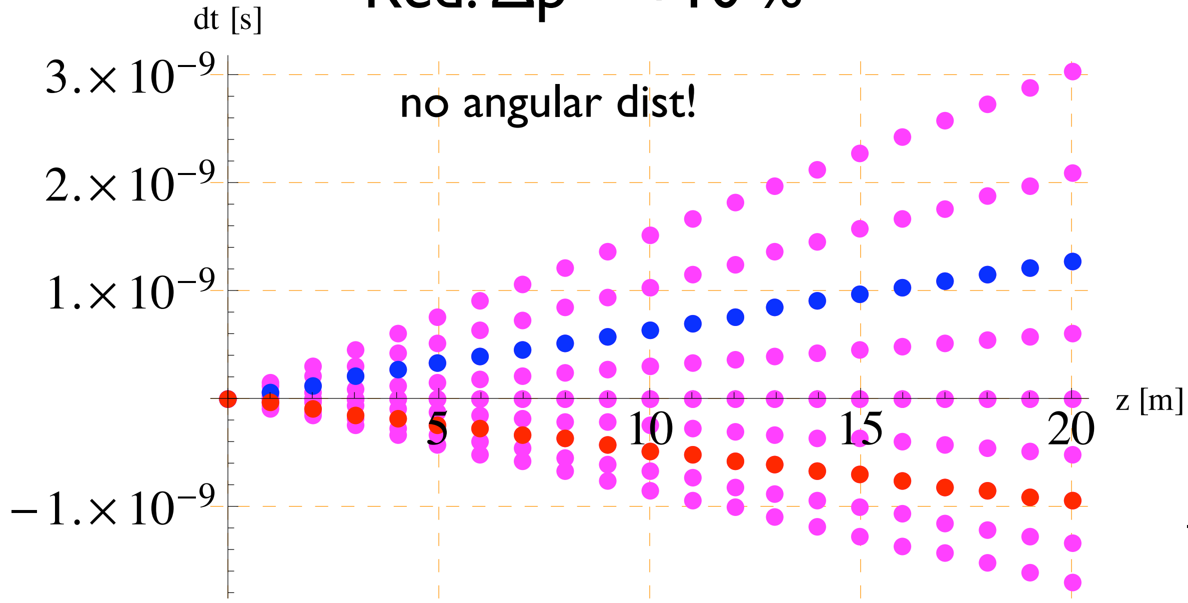
Emittance exchange is achieved by manipulating the path length as a function of momentum



Time of flight in optimum HCC and pure solenoid

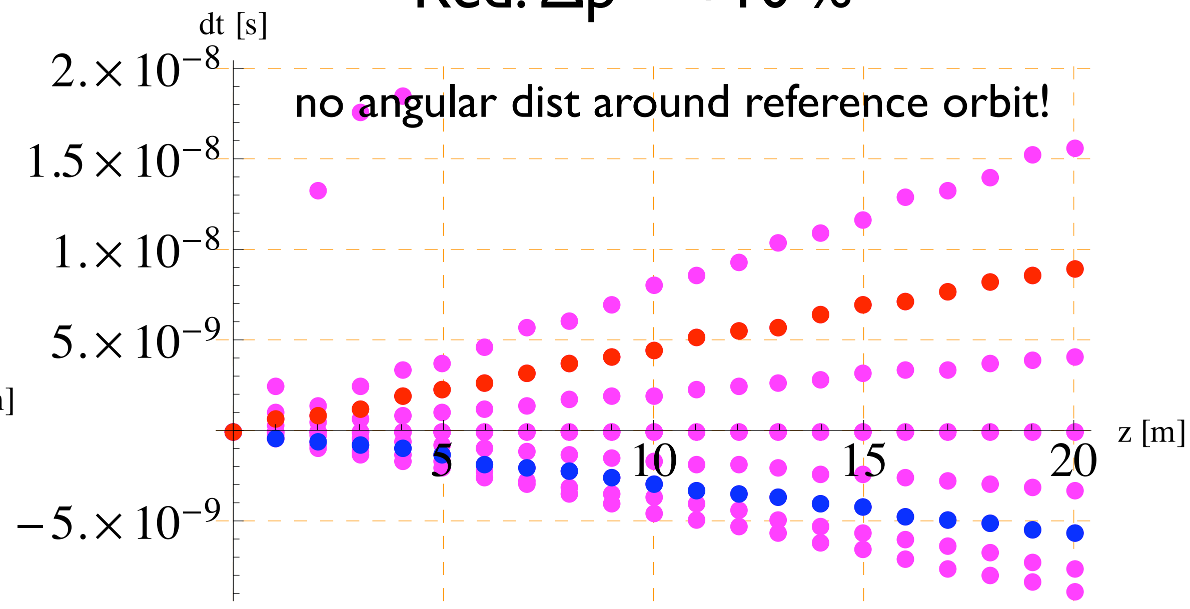
In a pure solenoid channel, a tof of slow particle takes more time to reach the other end of channel and vise versa for fast particle
This picture is completely opposite in HCC

Blue: Δp = -10 %
Red: Δp = +10 %



Pure solenoid

Blue: Δp = -10 %
Red: Δp = +10 %



HCC (kappa =1)

Momentum slip factor in HCC

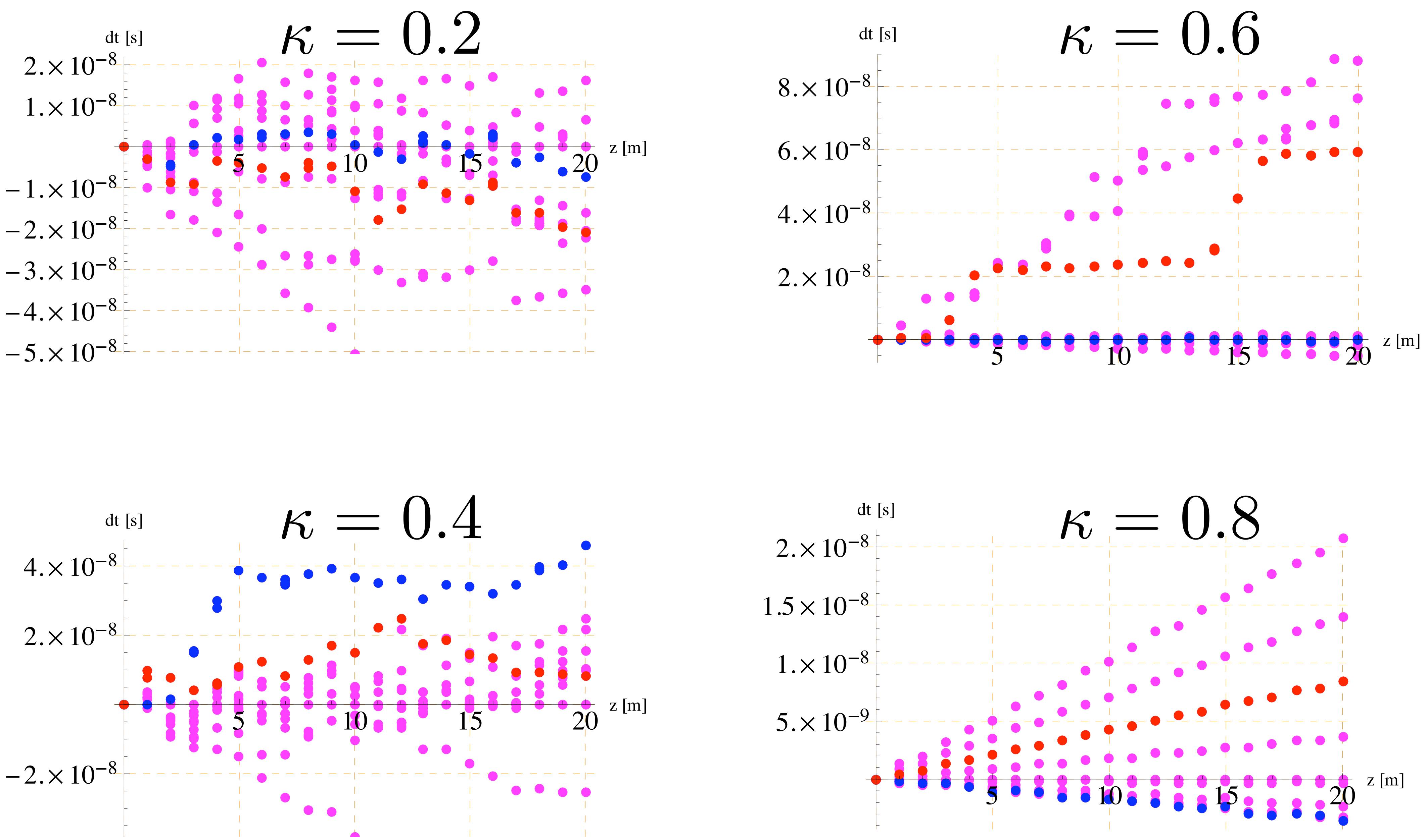
$$\eta = \frac{\sqrt{1+\kappa^2}}{\gamma\beta^3}\left(\frac{\kappa^2}{1+\kappa^2}\hat{D} - \frac{1}{\gamma^2}\right)$$

Y. Derbenev & R. Johnson, PRSTAB 8, 0410020 (2005), Eq. (3.52)

For optimum cooling condition, momentum slip factor is always positive
This condition can be realized even in low kappa HCC

ToF in various kappa

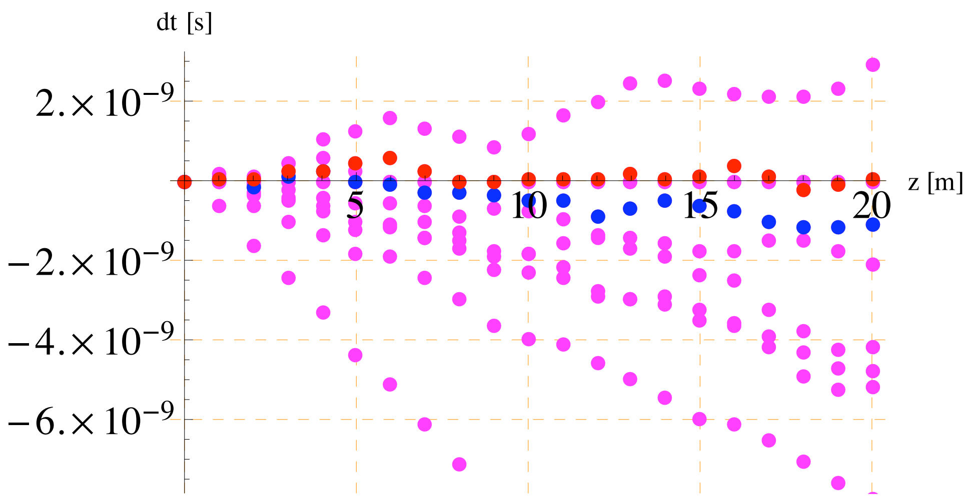
Blue: Δp = -10 %
Red: Δp = +10 %



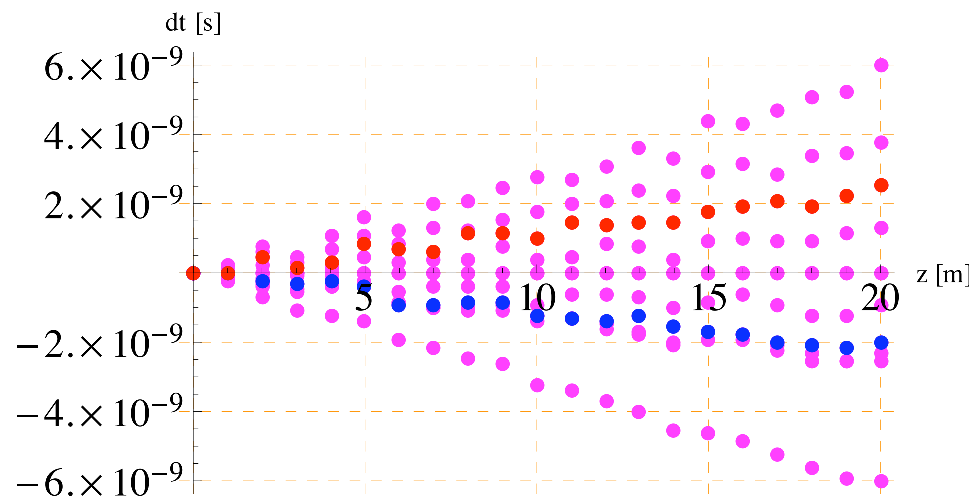
Proper condition can be seen from kappa = 0.6

Adjusting field to optimize cooling performance

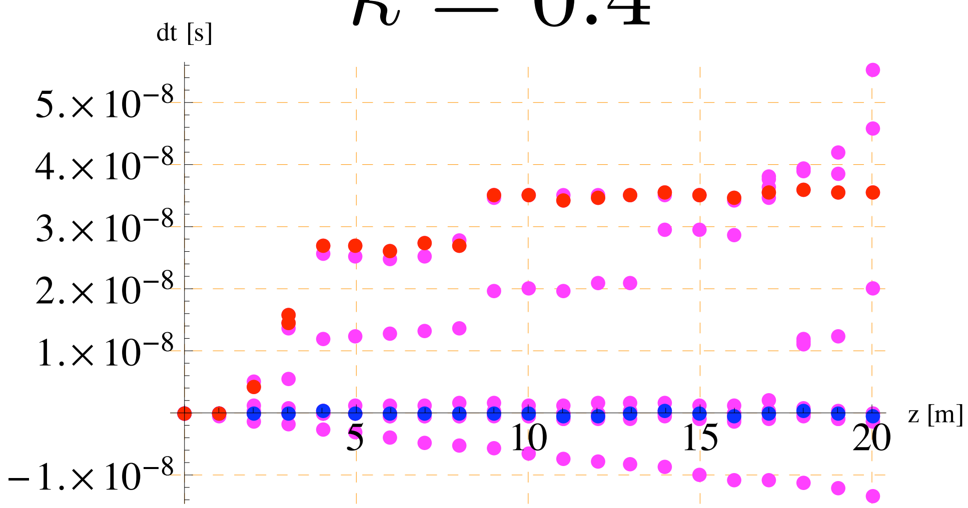
$\kappa = 0.2$



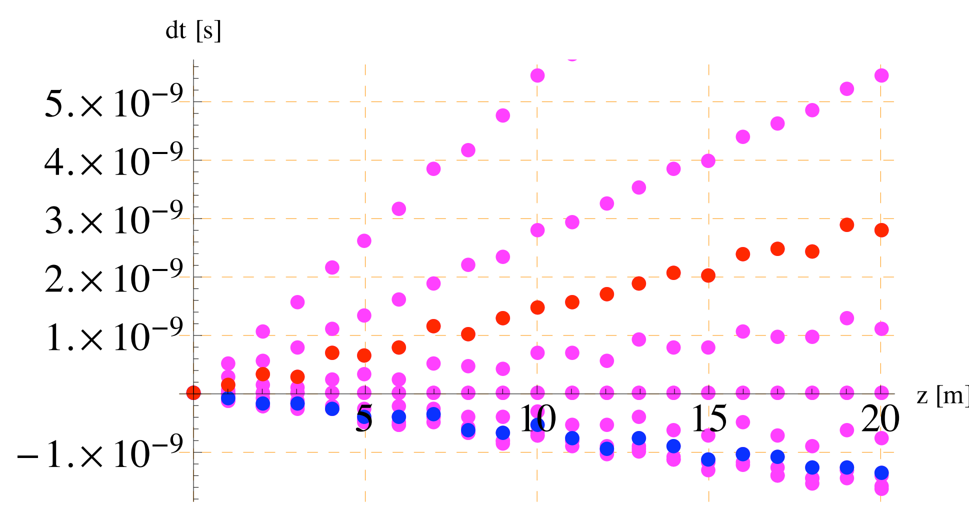
$\kappa = 0.6$



$\kappa = 0.4$



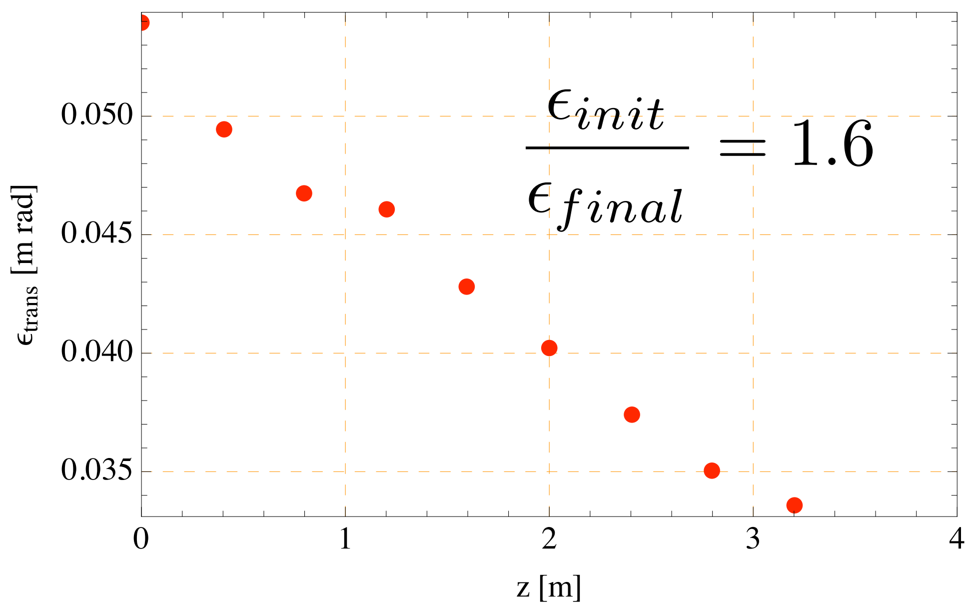
$\kappa = 0.8$

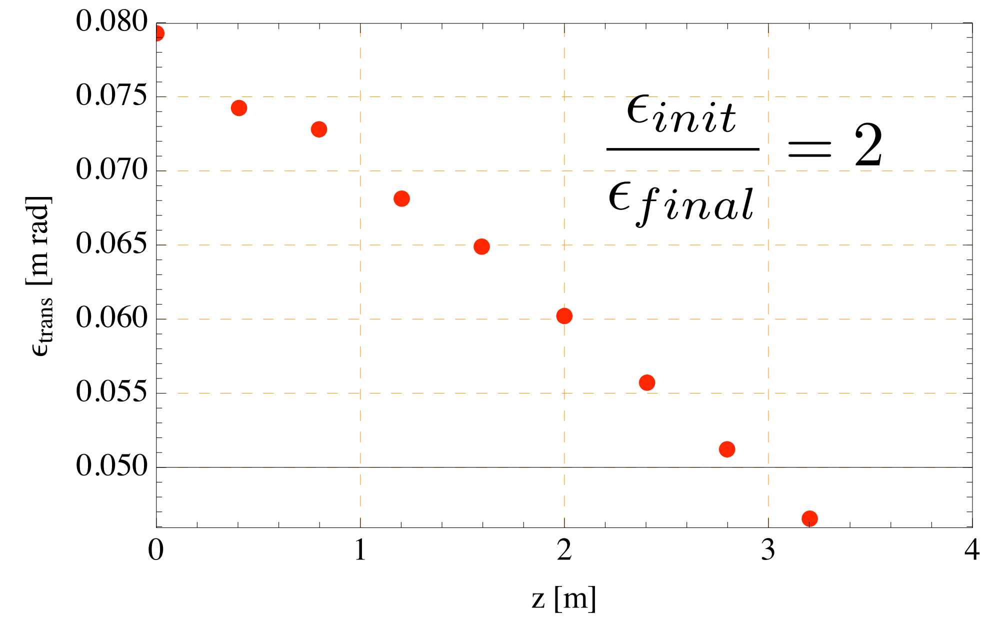


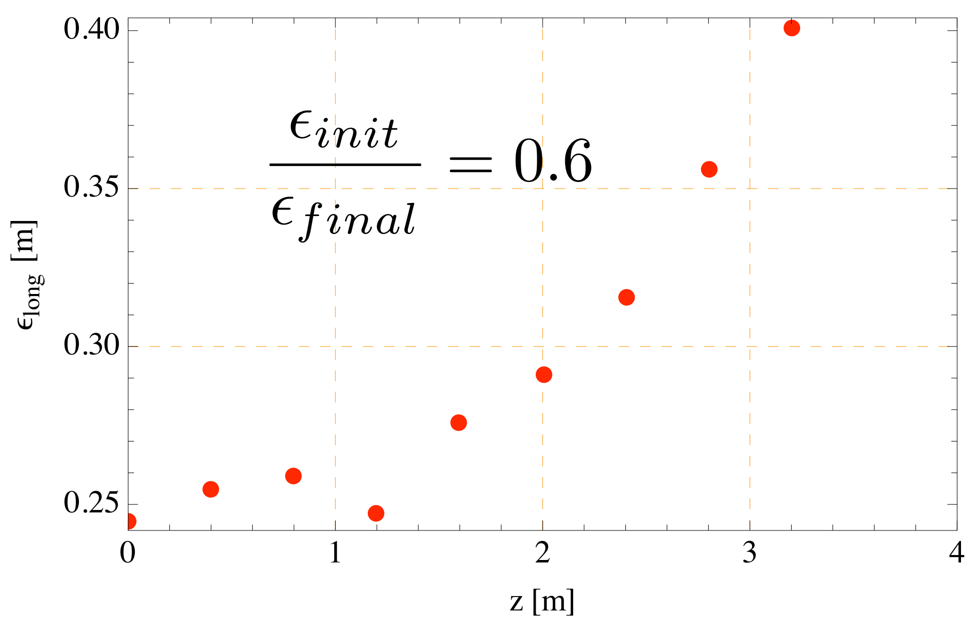
Proper condition can be seen from kappa = 0.4

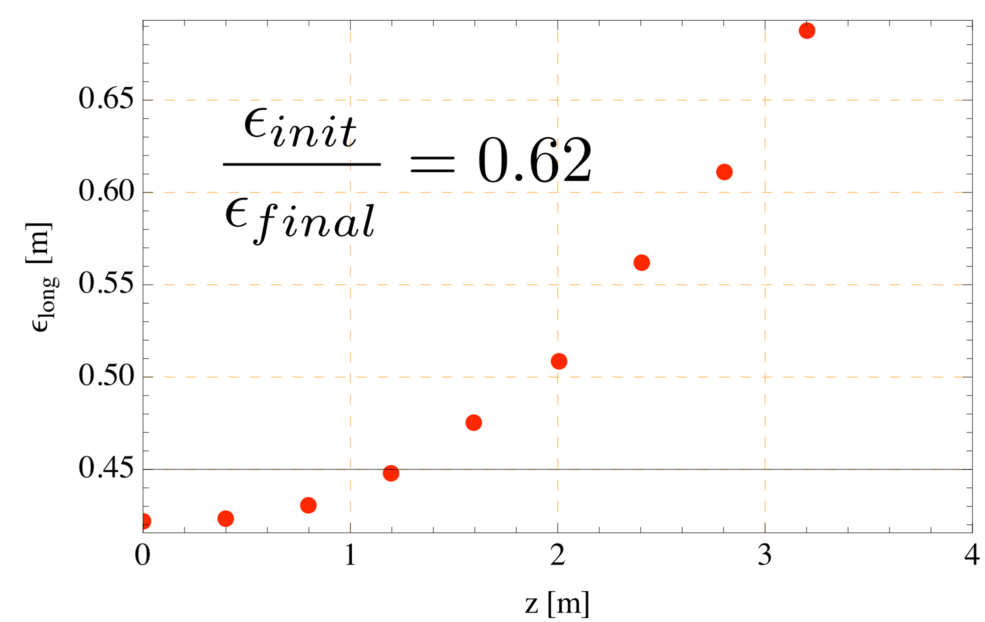
Example result in kappa=0.2 MANX

No correction | w correction









Phase advance is improved in both planes

Conclusion

- Investigate stable condition in HCC as a function of kappa
- HCC theory works well for transverse phase space stability (checked not only me but also other people)
- But, HCC theory did not well represent in longitudinal phase space, especially for low kappa case
- Nevertheless, we found the tuning knob to optimize the HCC field even for low kappa case
- Now fine tuning is on going