1. Required magnetic fields for cooling in solenoid (or HCC)
2. Effects of single and double periodicity
3. RFOFO lattices for 201 & 402 MHz
4. Efficiency
5. Magnetic Insulation
6. 805 MHz lattice
7. Conclusion
Solenoid fields for Cooling

For cooling in hydrogen, without windows, at $\approx \gamma=2$ (chosen to avoid rapid increase in $dp/p$):

\[
\epsilon_\perp(\text{equilib}) = \frac{C\beta_\perp}{\beta_v} = \frac{2 \gamma C m_\mu}{c B}
\]

\[
B \approx \frac{10 \cdot 10^{-3}}{\epsilon_\perp(equ)}
\]

With emittance exchange giving $J_x = J_y = J_z$ then

\[
B \approx \left(\frac{3}{2}\right) \frac{10 \cdot 10^{-3}}{\epsilon_\perp(equ)}
\]

<table>
<thead>
<tr>
<th>Stage</th>
<th>rf freq</th>
<th>emit</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MHz</td>
<td>$\pi$ mm</td>
<td>T</td>
</tr>
<tr>
<td>1</td>
<td>201</td>
<td>2</td>
<td>3.75</td>
</tr>
<tr>
<td>2</td>
<td>401</td>
<td>1</td>
<td>7.5</td>
</tr>
<tr>
<td>3</td>
<td>805</td>
<td>0.32</td>
<td>23</td>
</tr>
</tbody>
</table>

- In HCC these are indeed the approximate axial fields needed
- But in a periodic lattice, the $\beta_\perp$ at the absorber can be less than the above
Decreasing beta in Solenoids by adding periodicity

In practice, the solenoid fields are usually altering to avoid a buildup of angular momentum - our homework will show how this occurs.
Super FOFO
Double periodicity

Radii (cm)

<p>| | | | |</p>
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<td>0.0</td>
<td>2.5</td>
<td>5.0</td>
<td>7.5</td>
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</table>

Length (m)

<p>| | | | |</p>
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<tbody>
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<td>0.0</td>
<td>2.5</td>
<td>5.0</td>
<td>7.5</td>
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</table>

Axial Field (T)

<p>| | | | |</p>
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<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
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</table>

Beta (m)

Polarities:
- shown:   | + + | + + |
- SFOFO:   | + + | − − |
- RFOFO:   | + − | + − |

- Beta lower over finite momentum range
- Beta lower by about 1/2 solenoid

RFOFO chosen for Ring/Guggenheim
because all cells identical
removes 1/2 the resonances
RFOFO Ring

Simulated with realistic Maxwellian Fields
But not fields from actual solenoids
Simulations with real fields give the same results

Lattices of this type ok at 201 and 402 MHz
ICOOL Simulated Performance

- Assume a Guggenheim will behave like the ring
- No Windows
An interesting detail

The emittance exchange is initially all in x.
But there is enough phase rotation in x,y to eventually give nearly symmetric cooling.
Efficiency vs. length for old RFOFO

Define: Efficiency \( Q = \frac{d\epsilon_3/\epsilon_3}{dn/n} \)

- Mismatch and Scraping losses at start
- Decay losses as emittances approach equilibrium at end
- Sweet region in between \((Q \approx 15)\)
- If tapered then the entire channel is operated in the sweet region

Required 6D cooling in RFOFO lattices from 280,000 to 2.1 \((\text{mm}^3)\) so expected transmission if tapered:

\[
\frac{n_{\text{final}}}{n_{\text{initial}}} = \left( \frac{2.1}{115,000} \right)^{1/15} = 0.48
\]

Good
**Mag Insulation of Guggenheim**

- Surface fields now $\approx 2$ times acceleration
- Shunt impedance worse
- Higher content of Fourier content in $B$ vs $z$
- Because used for so much cooling losses are unacceptable (3% vs 7% transmission)
805 MHz Guggenheim has to be different
Not practical to put 10-12 T solenoids outside rf

- Less efficient ($Q \approx 8$) but only needed for limited cooling so ok
Mag-Insulated version of 805 MHz lattice

Old simulated lattice

Mag Ins version Not yet simulated

Magnetically insulated version has similar fields to standard version and will probably show similar performance
Summary

<table>
<thead>
<tr>
<th>Stage</th>
<th>rf freq</th>
<th>emit(equ)</th>
<th>B(solenoid)</th>
<th>B(lattice)</th>
<th>ratio</th>
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<tbody>
<tr>
<td></td>
<td>MHz</td>
<td>π mm</td>
<td>T</td>
<td>T</td>
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<tr>
<td>3</td>
<td>805</td>
<td>0.32*</td>
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</tbody>
</table>

* Scaled from lattice shown

Conclusions

- Periodic lattices allow lower $\beta$s for same magnetic fields
- Double periodicity lattices give greater momentum acceptance
- Simulations of lattices with coils outside give good efficiency
- Magnetically insulated lattices have worse performance
- One hopes that Be Cavities will solve the problem at least for higher frequencies
- Alternatively: HCC or FOFO gas filled lattices are ok for 201 and 402 MHz stages, but these will not have factor 2 gain from focusing
- Magnetically insulated 805 HHz probably acceptable for cooling to lowest emittances