Space Charge Tune Shift in PIC

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- Introduction
- Space Charge Force
- Simulation of "Ideal" PIC
- Add space charge defocus
- Conclusion

1) Introduction

Parameters from the PIC/REMEX revised paper

		Pic 1	PIC 2
Cell lengths	cm	19	19
Momentum	MeV/c	100	100
Muons/bunch		10^{11}	10^{11}
Absorber thick	mm	6.4	1.6
Absorber Mat		Be	Be
Trans RMS emit	mm mrad	600	30
Sigma(theta)	Mrad	200	200
Sigma(r)	mm	3	0.15
β_{Beam}	mm	15	0.75
ϵ_o	mm mrad	118	6.0
RMS dp/p	%	3	3
Sigma(z)	cm	0.5	0.5
Long RMS emittance	cm	0.015	0.015

The blue numbers differ from the original paper

The red numbers are calculated on right

$$\beta_\perp = \frac{\sigma_{x,y}}{\sigma_{\theta x,\theta y}}$$
$$= 15 \rightarrow 0.75 \quad (\text{mm})$$

material	T ^{o}K	density kg/m^3	dE/dx MeV/m	L_R m	C_o 10^{-4}
Liquid H_2	20	71	28.7	8.65	38
Liquid He	4	125	24.2	7.55	51
LiH	300	820	159	0.971	61
Li	300	530	87.5	1.55	69
Be	300	1850	295	0.353	89
Al	300	2700	436	0.089	248

$$\epsilon_o = \frac{\beta_\perp}{\beta_v} C_{\text{Be}} \frac{dE/dx(min)}{dE/dx(p)}$$
$$\approx 118 \rightarrow 6.0 \quad (10^{-6}\text{m})$$

Introduction Continued

- There has been considerable confusion about the space charge tune shift in the proposed PIC lattices
- A PIC lattice operates on an unstable half integer resonance
- There appears to be no self-consistent definition of tune shift for a lattice operating in an unstable stop band
- I will thus just compute the spot size at the absorber centers, with and without space charge
- For this purpose I will simulate an "ideal" PIC with:
  - A continuous solenoid (B=10.4 T)
  - A focusing thin lens 3/4 along each cell
  - With no spherical aberrations

$$\Delta(\tan\theta) \propto \; -K(s)\, r\, \Delta s$$

  - Fixed momentum and thus no Chromatic aberrations
  - An infinitely thin absorber with incorporated re-acceleration
  - Cooling decrements set to those found in reported ICOOL simulation
    That agrees with or is better than the PIC paper's performance

2) Space Charge Force

The defocus radial force $$F(r) = \left(\frac{2mc^2\, r_\mu}{\gamma^2}\right) \frac{(dN(r)/ds)}{r}$$

where $dN(r)/ds$ is the total line charge density inside a radius r

$$(dN(r)/ds) = \int_o^r 2\pi r\ (dn(r)/ds)\ dr$$

for a flat distribution up to a radius a

$$(dN(r)/ds) = (dn(o)/ds)\ \pi\ r^2 = (dN(\infty)/ds)\ \frac{r^2}{a^2}$$

for a Gaussian distribution in r

$$(dN(r)/ds) = (dN(\infty)/ds) \left(e^{\frac{r^2}{2\sigma_{x,y}^2}}\right) \frac{r^2}{2\sigma_{x,y}^2}$$

For small r and flat (as given in SY Lee p109)

$$F(r) = \left(\frac{2mc^2\, r_\mu}{\gamma^2}\right) \left(\frac{(dN(\infty)/ds)}{a^2}\right) r$$

For small r and gaussian

$$F(r) = \left(\frac{2mc^2\, r_\mu}{\gamma^2}\right) \left(\frac{(dN(\infty)/ds)}{2\sigma_{x,y}^2}\right) r$$

Defocus Strength K

$F(r)$ introduces a 'quadrupole' like defocus, in both x and y, of strength $K_{sc}(r)$:

$$K_{sc}(r) \quad = \quad \frac{(dN(\infty)/ds)\ r_\mu}{\sigma_{x,y}^2\ \beta_v^2\ \gamma^3}$$

For a bunch with Gaussian longtudinal shape

$$K_{sc}(r) \quad = \quad \left(\frac{N_\mu}{\sqrt{2\pi}\ \sigma_z}\right)\ \left(\frac{r_\mu}{\beta_v^2\ \gamma^3}\right)\ \left(\frac{e^{-\left(\frac{r^2}{2\sigma_{x,y}^2}\right)}}{\sigma_{x,y}^2}\right)$$

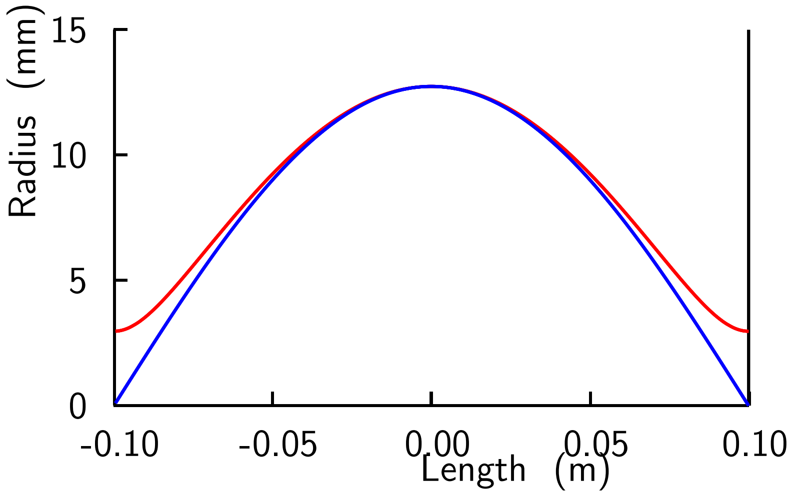
$r_\mu = 1.35\ 10^{-17}$ (m)

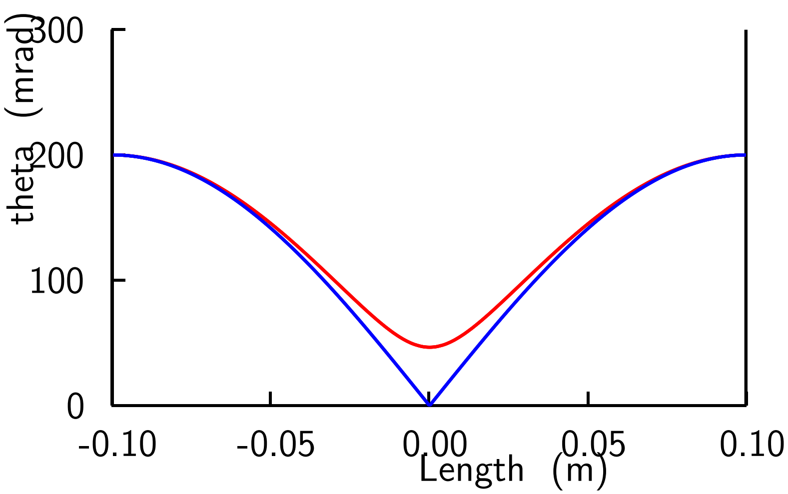
- This is the maximum tune shift along an assumed Gaussian distribution in s, including tune spread from radial non-linearity
- There will be additional tune spread from this distribution in s that is not included in the following simulations

3) Simulation of Ideal PIC without space charge

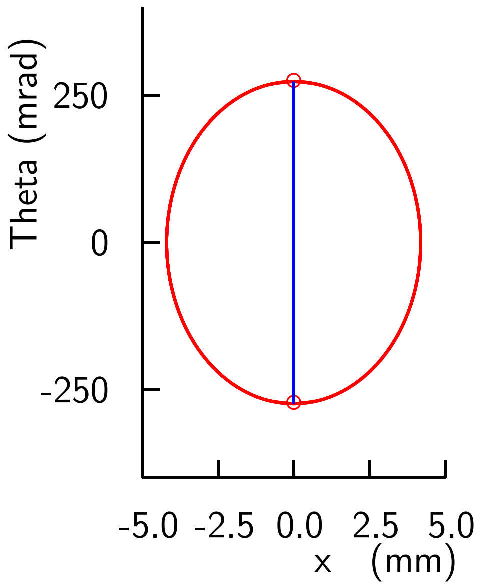
Unperturbed 1/2 integer focus in ideal solenoid

spherical aberation magically removed



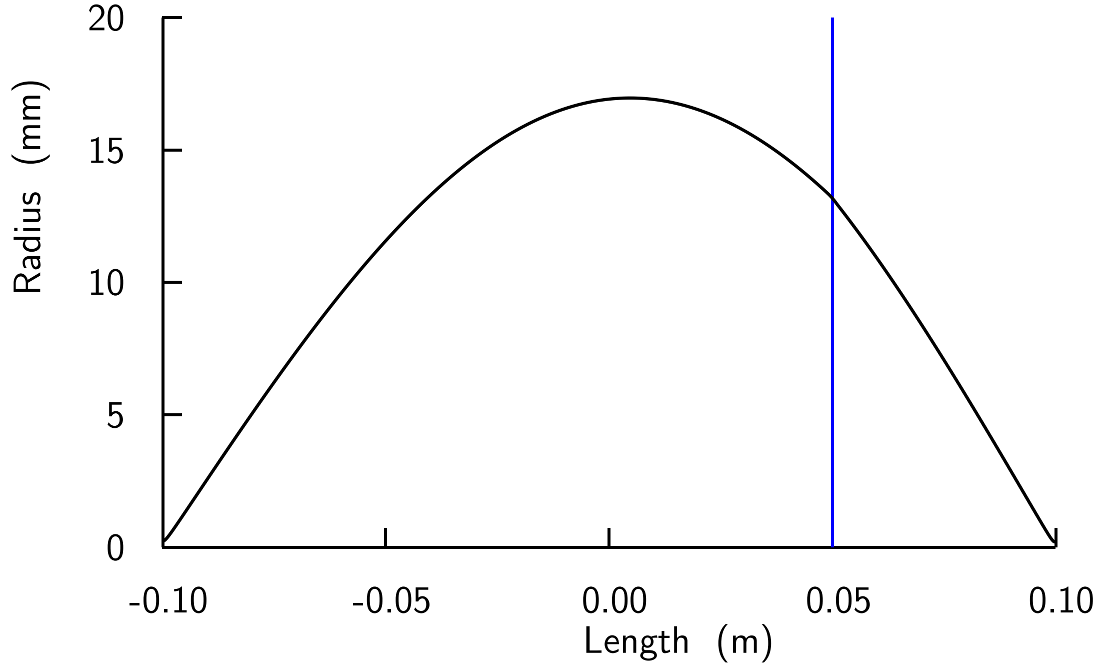


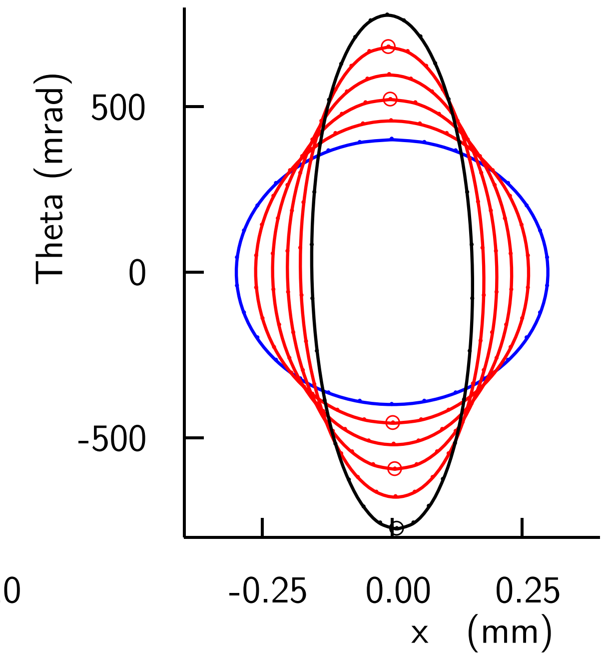
- Blue: $\sigma_\theta = 200$ (mrad)
  $\sigma_x = 0$
- Red: $\sigma_\theta = 200$ (mrad)
  $\sigma_x = 3$ (mm)
  as at start of PIC



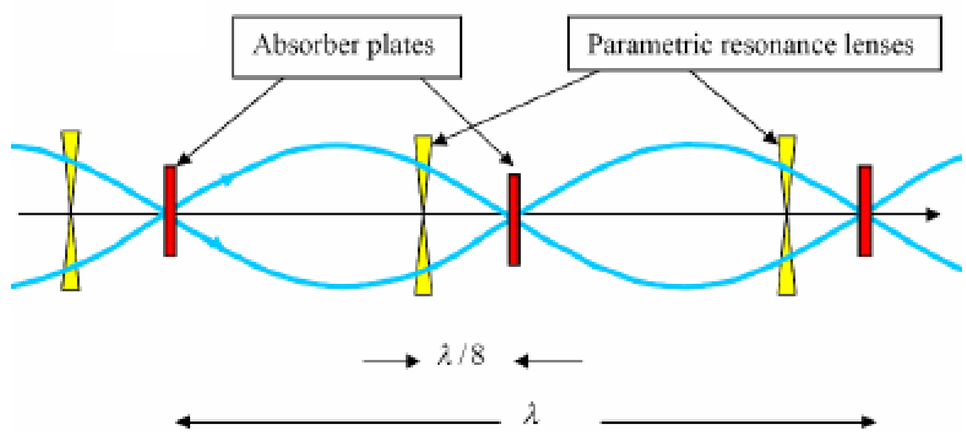
With perturbation

Added positive thin lens makes lattice unstable





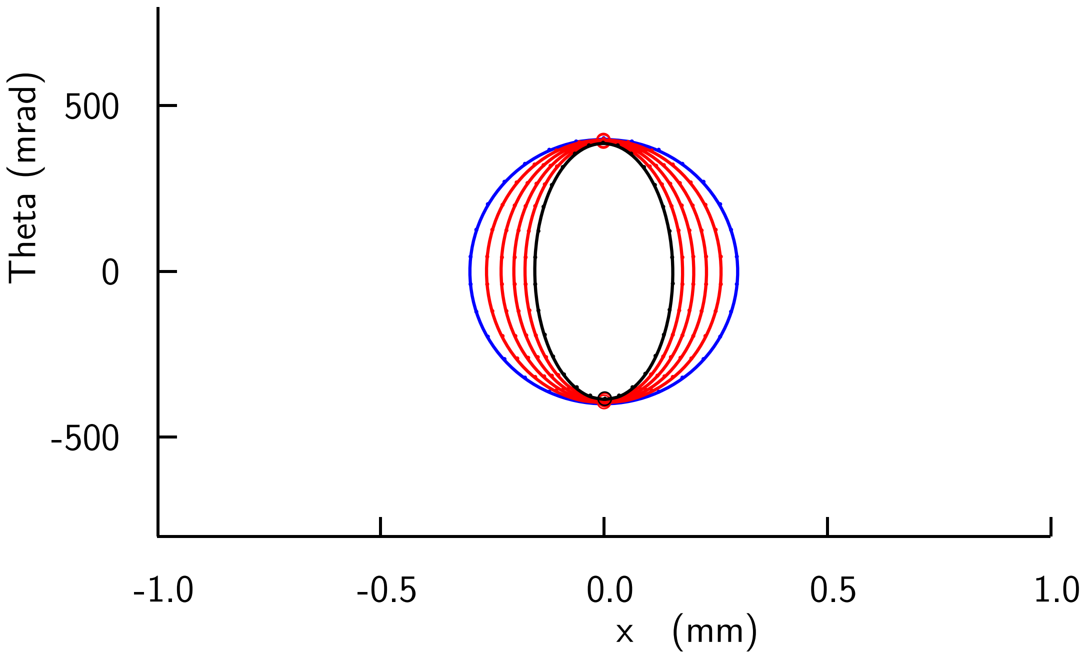
- σ_θ rises without limit
- σ_x falls without limit
  (As in proposed PIC)



Added combination material and acceleration

Effect reduces angular spread without changing size

Effect falls as cooling continues because absorber thickness must be reduced



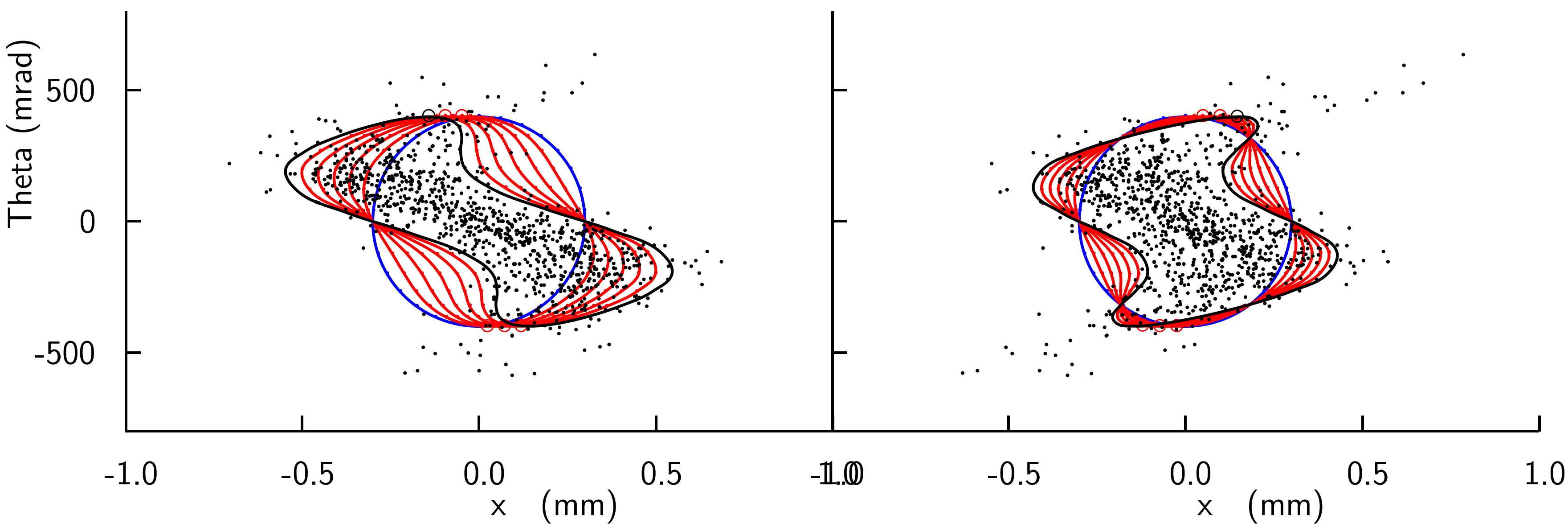
- If solenoid and thin lens strength adjusted, cell by cell, so that $-\Delta_\theta$ from cooling, cancelling $+\Delta_\theta$ from instability
  Phase advance = pi

- Then: $\sigma_\theta \; = \;$ constant
  and: $\sigma_x \; \rightarrow \;$ zero

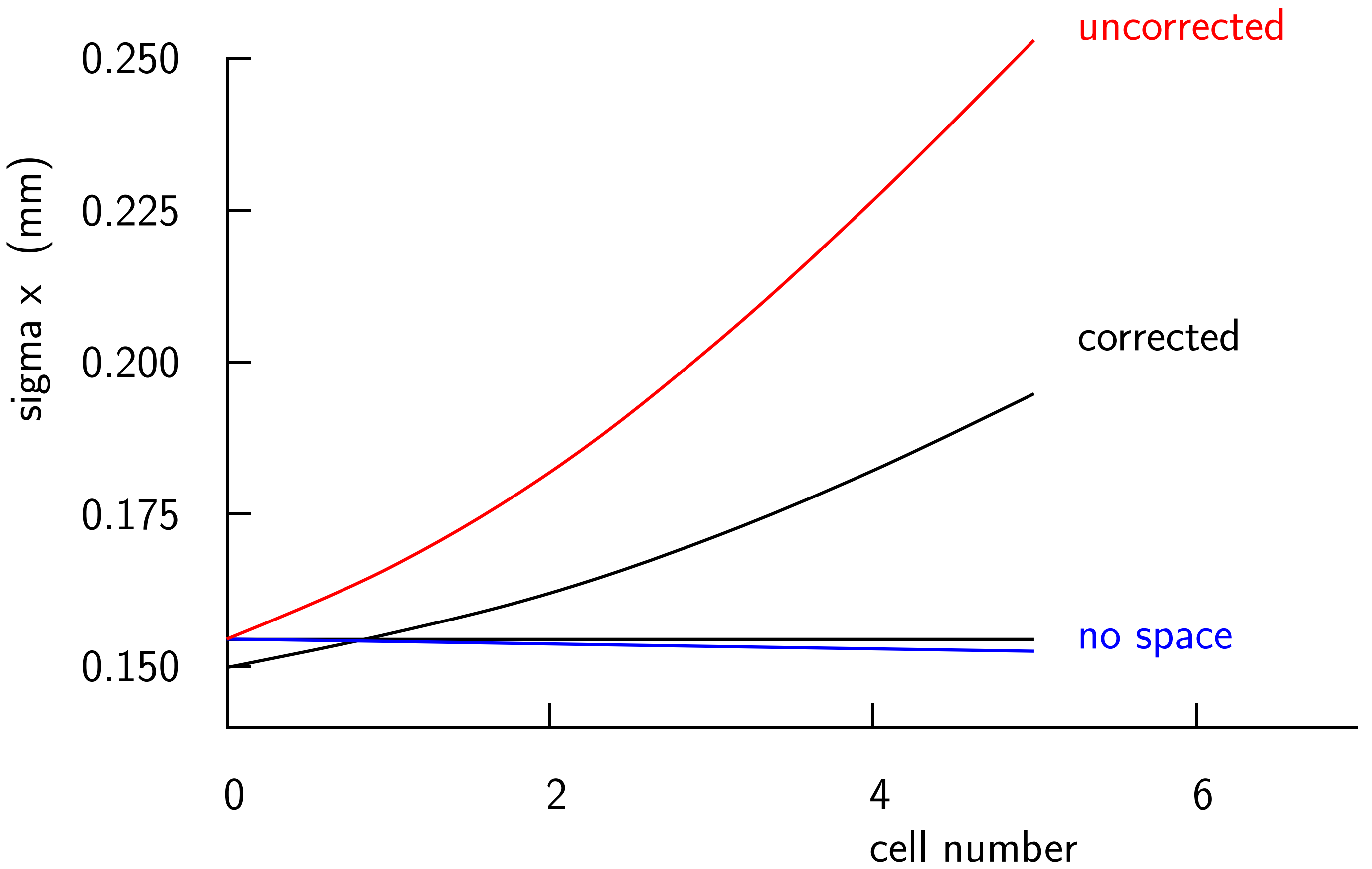
4) With Space Charge
4.1) at end of PIC

- Emittance = 30 pi mm
- Sigma x = 0.15 mm
- Cooling decrement is only 0.3 %
  (from MCTF May 3 talk, including exchange to keep dp/p=constant)
- "Correction" by adjusting solenoid strength to minimize growth in σ_x

Without correction With Correction



RMS x vs cell at end of PIC

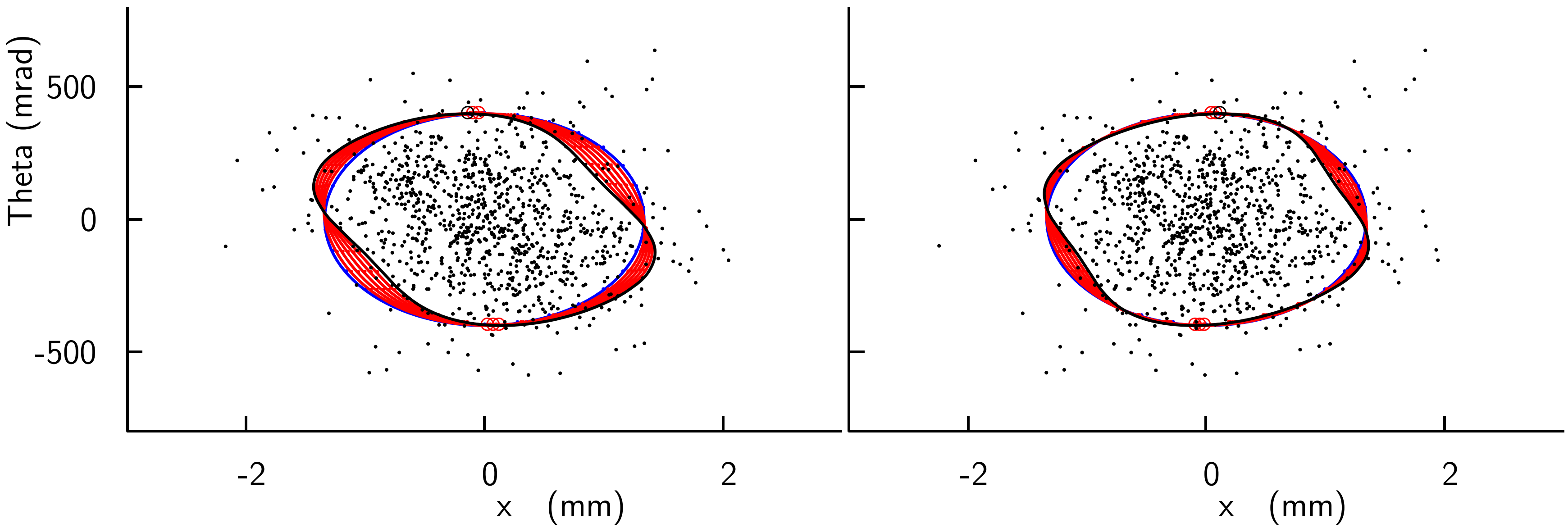


- Emittance rises with or without correction
- So PIC is clearly unable to cool to 30 pi mm

4.2) Half way along PIC

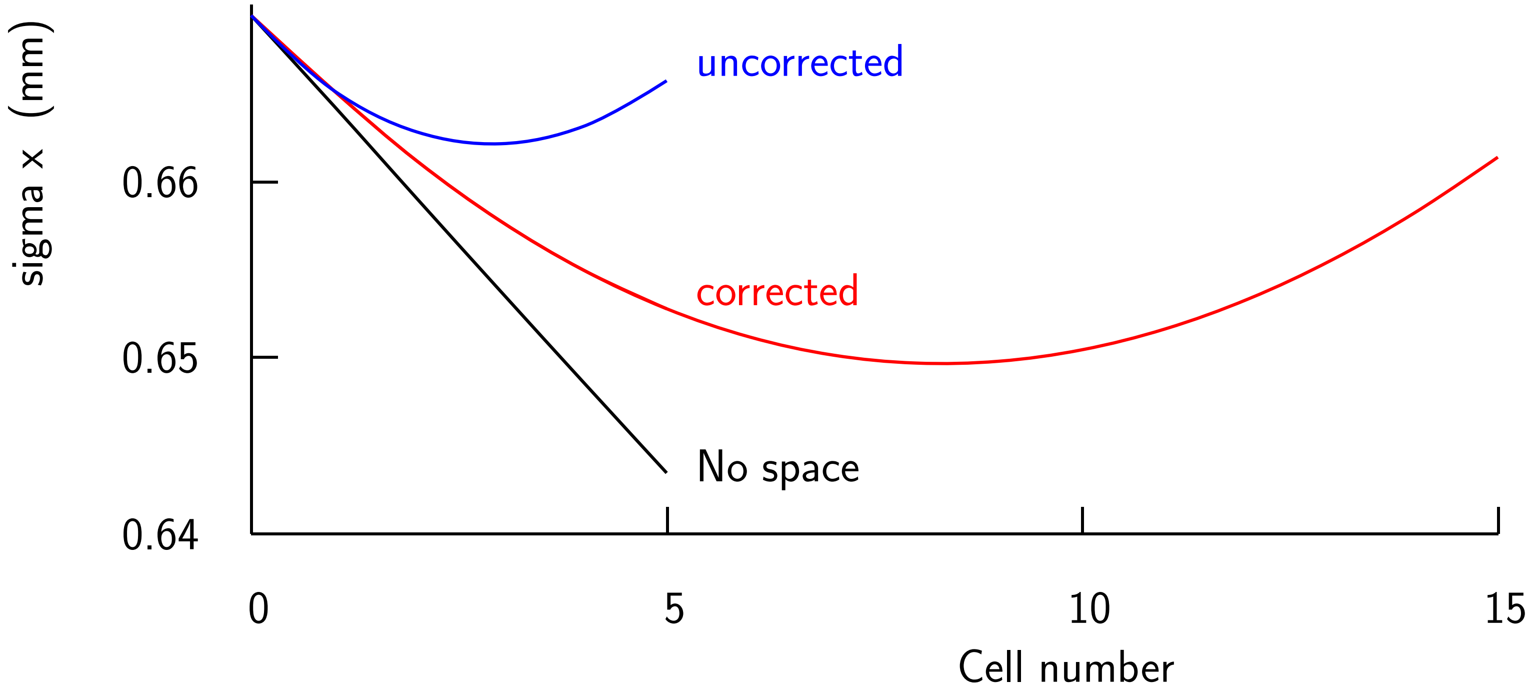
- emittance = 150 pi mm
- sigma x = 0.75 mm
- Cooling decrement is 0.7 %

Without correction　　　　With Correction



Effect is much smaller, and cooling is stronger, but

RMS x vs cell at middle of PIC



- Emittance rises without correction
- Emittance initially falls with correction
- But rises again after 10 cells
  A 3 sigma cut does not help
- So PIC is unable to cool to 150 pi mm, but it is close to working

- At start of PIC, space charge effect is negligible

5) Conclusion

- Simulation of PIC without space charge or other aberrations works fine
- Using parameters from PIC paper (σ_z=5 mm, N=10^{11}):
  - PIC works fine, with Space Charge, (without other abberations) with the initial emittance (600 pi mm)
  - At (130 pi mm), with Space Charge, there is initial cooling followed by heating
  - At (30 pi mm), with Space Charge, there is only heating
- It thus appears that space charge limits PIC cooling to an emittance of the order of 200 pi mm
- These results did not include the additional tune spread from the bunch distribution in s

To Do

- Run similar simulation with SFOFO lattice with the same beam parameters, but at a stable phase advance