Space Charge Tune Shift in PIC

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- Introduction
- Space Charge Force
- Simulation of "Ideal" PIC
- Add space charge defocus
- Conclusion
1) Introduction
Parameters from the PIC/REMEX revised paper

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pic 1</th>
<th>PIC 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell lengths</td>
<td>cm</td>
<td>19</td>
</tr>
<tr>
<td>Momentum</td>
<td>MeV/c</td>
<td>100</td>
</tr>
<tr>
<td>Muons/bunch</td>
<td></td>
<td>$10^{11}$</td>
</tr>
<tr>
<td>Absorber thick</td>
<td>mm</td>
<td>6.4</td>
</tr>
<tr>
<td>Absorber Mat</td>
<td></td>
<td>Be</td>
</tr>
<tr>
<td>Trans RMS emit</td>
<td>mm mrad</td>
<td>600</td>
</tr>
<tr>
<td>Sigma(theta)</td>
<td>Mrad</td>
<td>200</td>
</tr>
<tr>
<td>Sigma(r)</td>
<td>mm</td>
<td>3</td>
</tr>
<tr>
<td>$\beta_{\text{Beam}}$</td>
<td>mm</td>
<td>15</td>
</tr>
<tr>
<td>$\epsilon_o$</td>
<td>mm mrad</td>
<td>118</td>
</tr>
<tr>
<td>RMS dp/p</td>
<td>%</td>
<td>3</td>
</tr>
<tr>
<td>Sigma(z)</td>
<td>cm</td>
<td>0.5</td>
</tr>
<tr>
<td>Long RMS emittance</td>
<td>cm</td>
<td>0.015</td>
</tr>
</tbody>
</table>

The blue numbers differ from the original paper
The red numbers are calculated on right

$$\beta_\perp = \frac{\sigma_{x,y}}{\sigma_{\theta x,\theta y}}$$
$$= \frac{15}{0.75} (\text{mm})$$

<table>
<thead>
<tr>
<th>material</th>
<th>$T_{^0K}$</th>
<th>density $kg/m^3$</th>
<th>$dE/dx (\text{MeV/m})$</th>
<th>$L_R$</th>
<th>$C_o$ $10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid H$_2$</td>
<td>20</td>
<td>71</td>
<td>28.7</td>
<td>8.65</td>
<td>38</td>
</tr>
<tr>
<td>Liquid He</td>
<td>4</td>
<td>125</td>
<td>24.2</td>
<td>7.55</td>
<td>51</td>
</tr>
<tr>
<td>LiH</td>
<td>300</td>
<td>820</td>
<td>159</td>
<td>0.971</td>
<td>61</td>
</tr>
<tr>
<td>Li</td>
<td>300</td>
<td>530</td>
<td>87.5</td>
<td>1.55</td>
<td>69</td>
</tr>
<tr>
<td>Be</td>
<td>300</td>
<td>1850</td>
<td>295</td>
<td>0.353</td>
<td>89</td>
</tr>
<tr>
<td>Al</td>
<td>300</td>
<td>2700</td>
<td>436</td>
<td>0.089</td>
<td>248</td>
</tr>
</tbody>
</table>

$$\epsilon_o = \frac{\beta_\perp}{\beta_v} C_{\text{Be}} \frac{dE/dx(\text{min})}{dE/dx(p)}$$
$$\approx \frac{118}{6.0} (10^{-6} \text{m})$$
Introduction Continued

• There has been considerable confusion about the space charge tune shift in the proposed PIC lattices
• A PIC lattice operates on an unstable half integer resonance
• There appears to be no self-consistent definition of tune shift for a lattice operating in an unstable stop band
• I will thus just compute the spot size at the absorber centers, with and without space charge
• For this purpose I will simulate an "ideal" PIC with:
  – A continuous solenoid (B=10.4 T)
  – A focusing thin lens 3/4 along each cell
  – With no spherical aberrations

\[ \Delta (\tan \theta) \propto - K(s) \cdot r \cdot \Delta s \]

  – Fixed momentum and thus no Chromatic aberrations
  – An infinitely thin absorber with incorporated re-acceleration
  – Cooling decrements set to those found in reported ICOOL simulation
    That agrees with or is better than the PIC paper’s performance
2) Space Charge Force

The defocus radial force

\[ F(r) = \left( \frac{2mc^2 r_\mu}{\gamma^2} \right) \left( \frac{dN(r)/ds}{r} \right) \]

where \( dN(r)/ds \) is the total line charge density inside a radius \( r \)

\[ (dN(r)/ds) = \int_0^r 2\pi r (dn(r)/ds) \, dr \]

for a flat distribution up to a radius \( a \)

\[ (dN(r)/ds) = (dn(o)/ds) \pi r^2 = (dN(\infty)/ds) \frac{r^2}{a^2} \]

for a Gaussian distribution in \( r \)

\[ (dN(r)/ds) = (dN(\infty)/ds) \left( \frac{r^2}{e^{2\sigma^2_{x,y}}} \right) \frac{r^2}{2\sigma^2_{x,y}} \]

For small \( r \) and flat (as given in SY Lee p109)

\[ F(r) = \left( \frac{2mc^2 r_\mu}{\gamma^2} \right) \left( \frac{(dN(\infty)/ds)}{a^2} \right) r \]

For small \( r \) and gaussian

\[ F(r) = \left( \frac{2mc^2 r_\mu}{\gamma^2} \right) \left( \frac{(dN(\infty)/ds)}{2\sigma^2_{x,y}} \right) r \]
Defocus Strength $K$

$F(r)$ introduces a 'quadrupole' like defocus, in both $x$ and $y$, of strength $K_{sc}(r)$:

$$K_{sc}(r) = \frac{(dN(\infty)/ds) \, r_{\mu}}{\sigma_{x,y}^2 \, \beta_v^2 \, \gamma^3}$$

For a bunch with Gaussian longitudinal shape

$$K_{sc}(r) = \left( \frac{N_{\mu}}{\sqrt{2\pi} \, \sigma_z} \right) \left( \frac{r_{\mu}}{\beta_v^2 \, \gamma^3} \right) \left( \frac{\left( \frac{r^2}{2\sigma_{x,y}^2} \right)}{\sigma_{x,y}^2} \right)$$

$r_{\mu} = 1.35 \times 10^{-17} \, (m)$

- This is the maximum tune shift along an assumed Gaussian distribution in $s$, including tune spread from radial non-linearity
- There will be additional tune spread from this distribution in $s$ that is not included in the following simulations
3) Simulation of Ideal PIC without space charge
Unperturbed 1/2 integer focus in ideal solenoid
spherical aberration magically removed

- Blue: $\sigma_\theta = 200 \text{ (mrad)}$
  $\sigma_x = 0$
- Red: $\sigma_\theta = 200 \text{ (mrad)}$
  $\sigma_x = 3 \text{ (mm)}$
  as at start of PIC
With perturbation
Added positive thin lens makes lattice unstable

- $\sigma_\theta$ rises without limit
- $\sigma_x$ falls without limit
  (As in proposed PIC)
Added combination material and acceleration
Effect reduces angular spread without changing size
Effect falls as cooling continues because absorber thickness must be reduced

- If solenoid and thin lens strength adjusted, cell by cell, so that $-\Delta \theta$ from cooling, cancelling $+\Delta \theta$ from instability
  Phase advance = $\pi$

- Then: $\sigma_\theta = \text{constant}$
  and: $\sigma_x \rightarrow \text{zero}$
4) With Space Charge
4.1) at end of PIC

- Emittance = 30 pi mm
- Sigma x = 0.15 mm
- Cooling decrement is only 0.3 %
  (from MCTF May 3 talk, including exchange to keep \( \frac{dp}{p} = \)constant)
- "Correction" by adjusting solenoid strength to minimize growth in \( \sigma_x \)

Without correction  With Correction
RMS x vs cell at end of PIC

- Emittance rises with or without correction
- So PIC is clearly unable to cool to 30 pi mm
4.2) Half way along PIC

- emittance = 150 πi mm
- \( \sigma_x = 0.75 \) mm
- Cooling decrement is 0.7 %

Without correction  With Correction

Effect is much smaller, and cooling is stronger, but
- Emittance rises without correction
- Emittance initially falls with correction
- But rises again after 10 cells
  **A 3 sigma cut does not help**
- So PIC is unable to cool to 150 pi mm, but it is close to working

- At start of PIC, space charge effect is negligible
5) Conclusion

- Simulation of PIC without space charge or other aberrations works fine

- Using parameters from PIC paper ($\sigma_z=5 \text{ mm, } N=10^{11}$):
  - PIC works fine, with Space Charge, (without other abberations) with the initial emittance (600 pi mm)
  - At (130 pi mm), with Space Charge, there is initial cooling followed by heating
  - At (30 pi mm), with Space Charge, there is only heating

- It thus appears that space charge limits PIC cooling to an emittance of the order of 200 pi mm

- These results did not include the additional tune spread from the bunch distribution in $s$

To Do

- Run similar simulation with SFOFO lattice with the same beam parameters, but at a stable phase advance