Physics at e^+e^- Linear Colliders

2. W^+W^- and $t\,\bar{t}$

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In addition to e^+e^- annihilation to fermion pairs, the SM contains important processes of e^+e^- annihilation to vector bosons.

These include the relatively simple process of e^+e^- annihilation to photons, but also the process of W^+W^- production, which will turn out to have unexpected subtlety.

In learning how to model this process, we will learn important details that are more generally relevant to massive particle production at the LC.

For massive particles, we should be more careful about our description of the polarization state. I will label these states by helicity:

$$h = \hat{p} \cdot \vec{S}$$

Helicity is invariant with respect to rotations and with respect to boosts along the direction of motion.

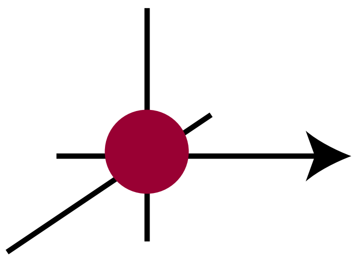
For massless particles, the polarization states with the simplest description are states of definite helicity:

Massless fermions are best described as 2-component (chiral) fermions; these have definite helicity $h = \pm\frac{1}{2}$

Massless vector bosons such as photons have transverse polarization; this is definite helicty $h = \pm 1$

For massive vector particles, the story is more complicated.

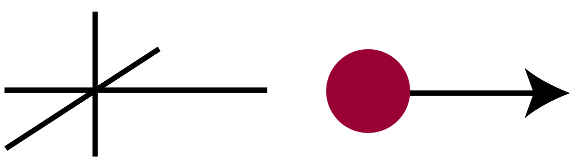
Begin in the rest frame. A spin-1 particle has 3 polarization states:



$$\epsilon_+ = \frac{1}{\sqrt{2}}(0, 1, i, 0) \qquad \epsilon_- = \frac{1}{\sqrt{2}}(0, 1, -i, 0)$$

$$\epsilon_0 = (0, 0, 0, 1)$$

Now boost along the 3 axis. The transverse polarization vectors are unchanged, but the longitudinal (h=0) polarization vector now takes the form:



$$\epsilon_0 \to (\frac{p}{m}, 0, 0, \frac{E}{m})$$

Notice that, still, the ϵ_i are the three 4-vectors that satisfy

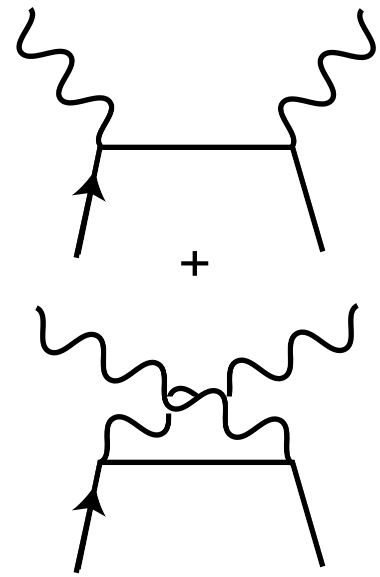
$$p^\mu \epsilon_\mu = 0$$

amplitudes for $e^+e^- \to$ 2 bosons are simplest for states of definite helicity:

$$\mathcal{M}(e^-_L e^+_R \to \gamma_L \gamma_R) = 2e^2 \left(\frac{1+\cos\theta}{1-\cos\theta}\right)^{1/2}$$

$$\mathcal{M}(e^-_L e^+_R \to \gamma_R \gamma_L) = 2e^2 \left(\frac{1-\cos\theta}{1+\cos\theta}\right)^{1/2}$$

(otherwise = 0)

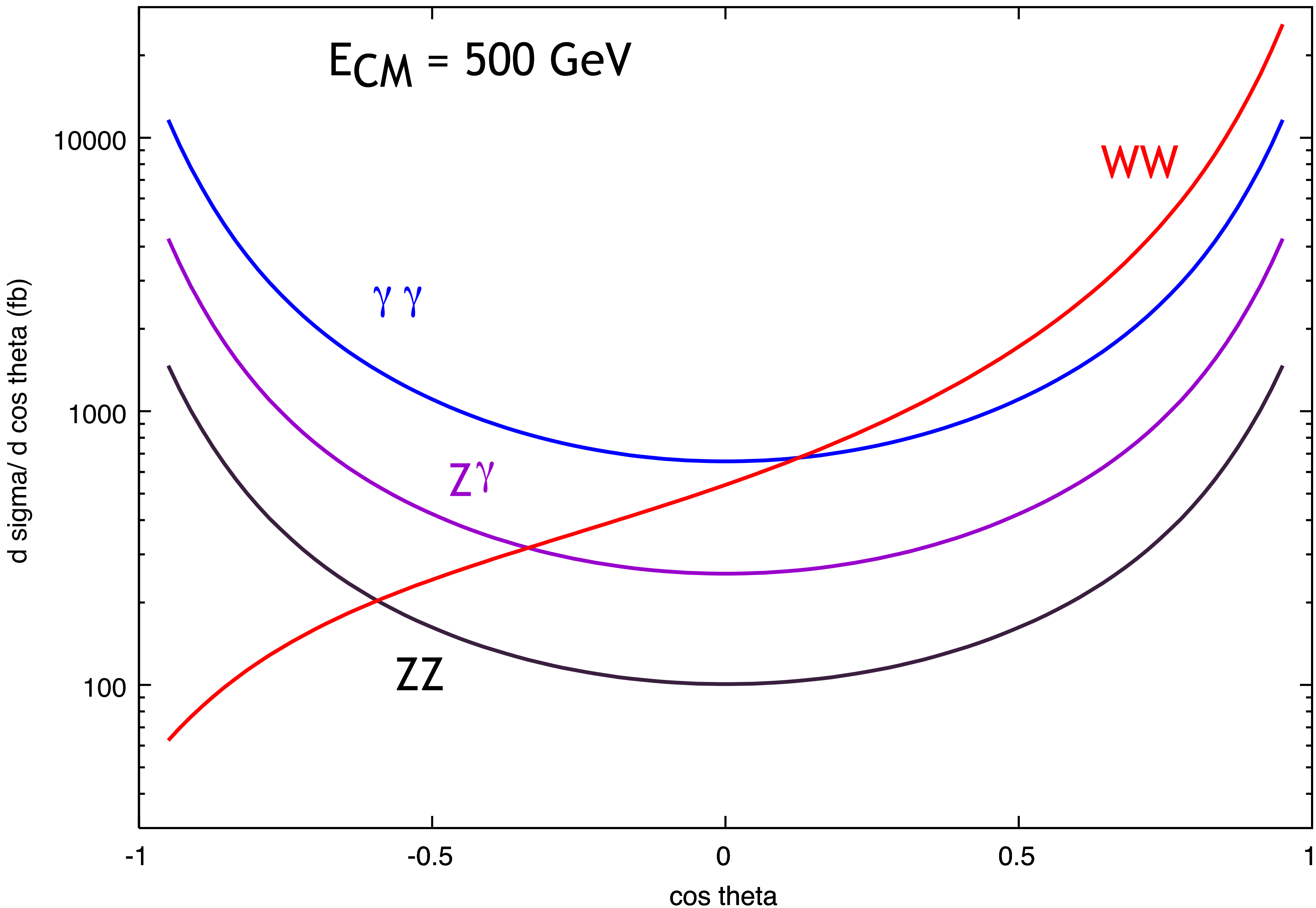


for $e^+e^- \to Z\gamma$, ZZ, find similar expressions, with

$$\mathcal{M}(e^-e^+ \to Z^0_0 \gamma) \sim \frac{m_Z}{E} \qquad \mathcal{M}(e^-e^+ \to Z^0_L \gamma_L) \sim \frac{m_Z^2}{E^2}$$

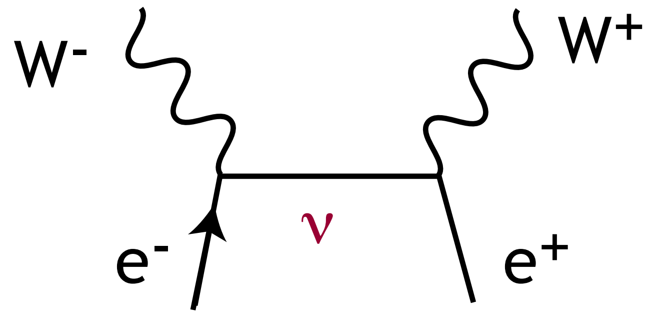
Plot the cross sections at 500 GeV for:

$$e^+e^- \to \gamma\gamma\ ,\ Z^0\gamma\ ,\ Z^0Z^0\ ,\ W^-W^+$$



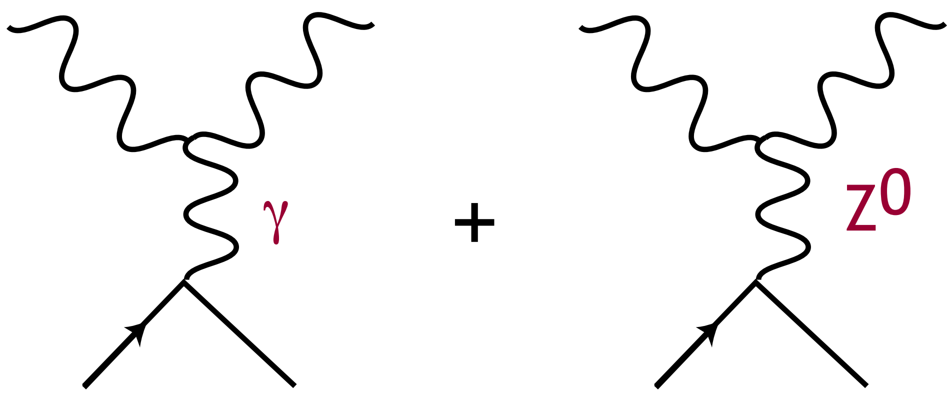
E_CM = 500 GeV
WW
γ γ
Zγ
ZZ
10000
1000
100
d sigma/ d cos theta (fb)
-1
-0.5
0
0.5
1
cos theta

$e^+e^- \rightarrow W^+W^-$ is strongly forward-peaked because of the ν -exchange diagram



The forward peak is dominantly $e^-_L e^+_R \rightarrow W^-_L W^+_R$

There is no corresponding u-channel diagram. Instead, there are diagrams with the Yang-Mills vertex:



The Yang-Mills diagrams give problems when evaluated with longitudinal polarization. We would expect that

$$\mathcal{M} \sim e^2 \sin\theta \cdot \epsilon^* \cdot \epsilon'^*$$

but, inserting $\epsilon = \frac{1}{m}(p, 0, 0, E) \qquad \epsilon' = \frac{1}{m}(p, 0, 0, -E)$

we find: $|\mathcal{M}|^2 \sim e^4 \cdot \frac{2E^2}{m^2}$

This gives $\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s} \cdot \left(\frac{s}{2m^2}\right)^2$

However, unitarity requires that, in each partial wave:

$$\frac{d\sigma}{d\cos\theta} \sim \frac{1}{s}$$

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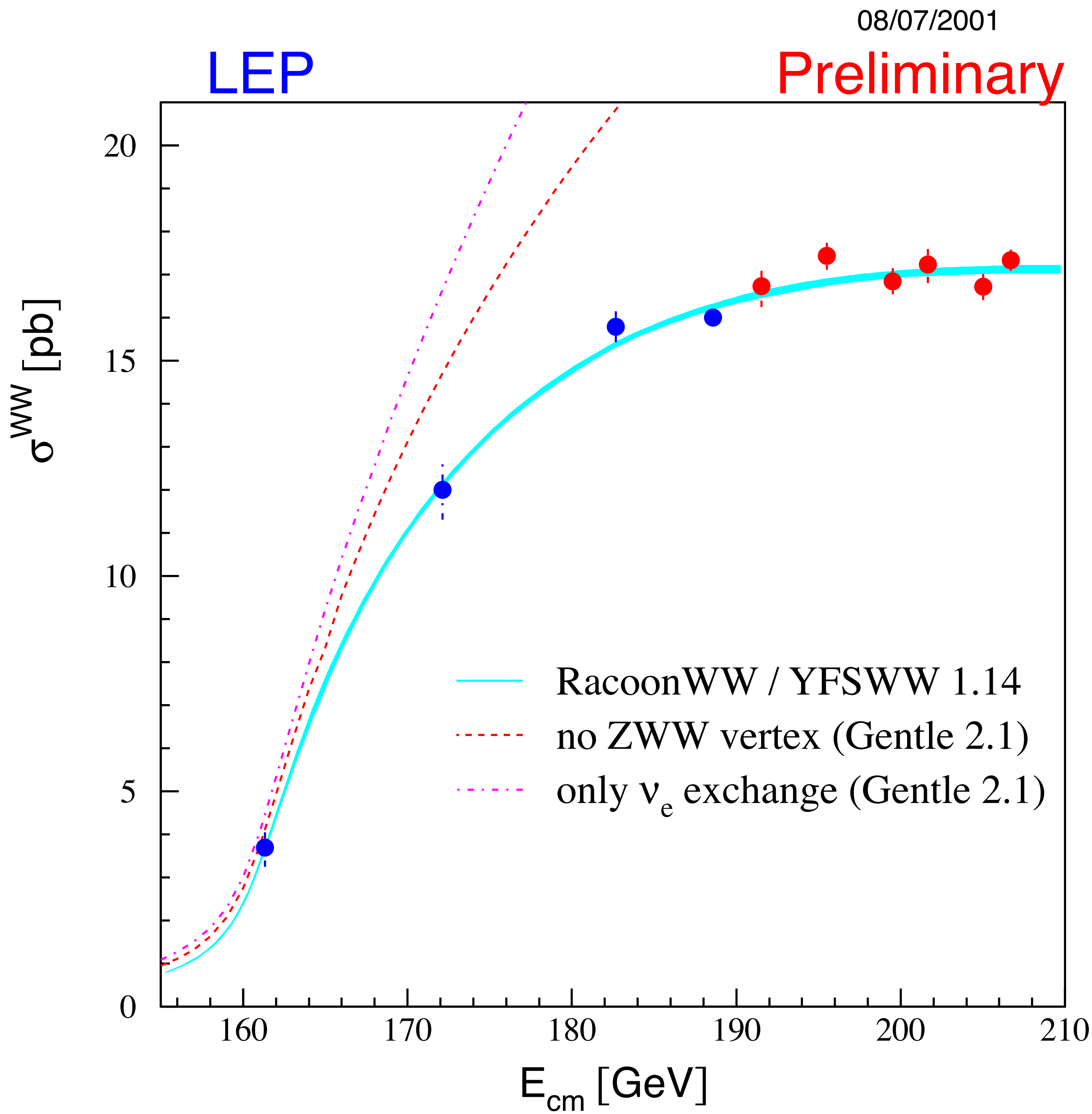
For $e^-_R\ e^+_L \rightarrow W^-W^+$, this behavior is removed by a cancellation between the γ and Z diagrams.

For $e^-_L\ e^+_R \rightarrow W^-W^+$, the bad behavior is cancelled against similar bad behavior of the ν diagram.

This cancellation has been observed experimentally at LEP 2.



08/07/2001
LEP
Preliminary
σ^WW [pb]
E_cm [GeV]
RacoonWW / YFSWW 1.14
no ZWW vertex (Gentle 2.1)
only ν_e exchange (Gentle 2.1)
0
5
10
15
20
160
170
180
190
200
210

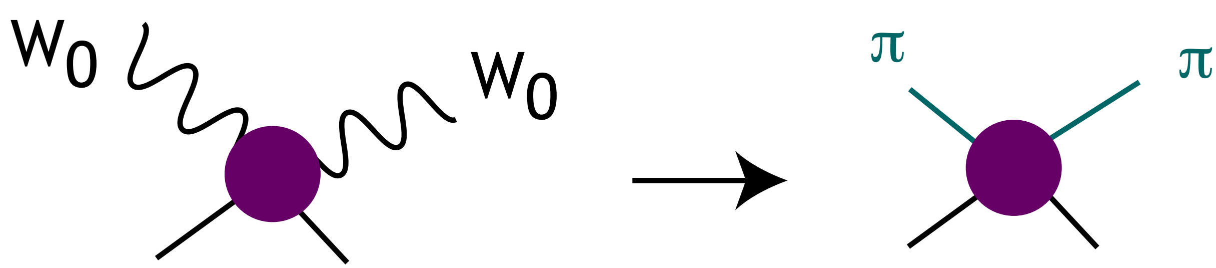
What nonzero terms survive after the cancellation?

Recall that W obtains mass only through electroweak symmetry breaking. The original massless gauge field had no longitudinal polarization state. To obtain this state, the W must steal the Goldstone boson degree of freedom from the Higgs field.

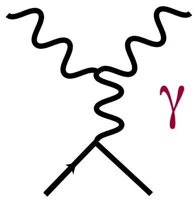
So, we might guess that, in the high energy limit

$$\mathcal{M}(e^+e^- \to W_0^+W_0^-) \to \mathcal{M}(e^+e^- \to \pi^+\pi^-)$$

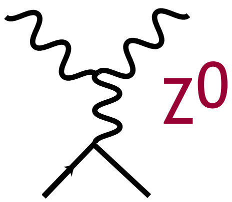
where π is the charged Goldstone boson from the Higgs sector.



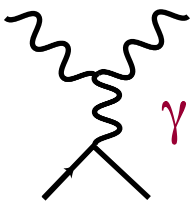
Explicitly, for $e^-_R\, e^+_L$:



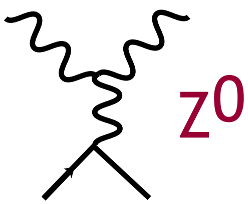
$$= -ie\sqrt{s}\;(0,1,i,0)^\mu \left(\frac{-i}{s}\right)\cdot ie[\epsilon_0^* \cdot \epsilon_0'^*(p-p')_\mu]$$



$$= (-1)\frac{s}{s-m_Z^2}\cdot(\text{same})$$

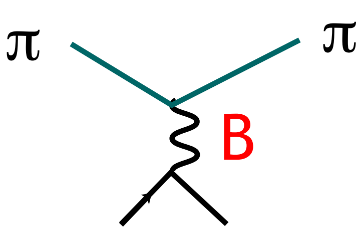


+



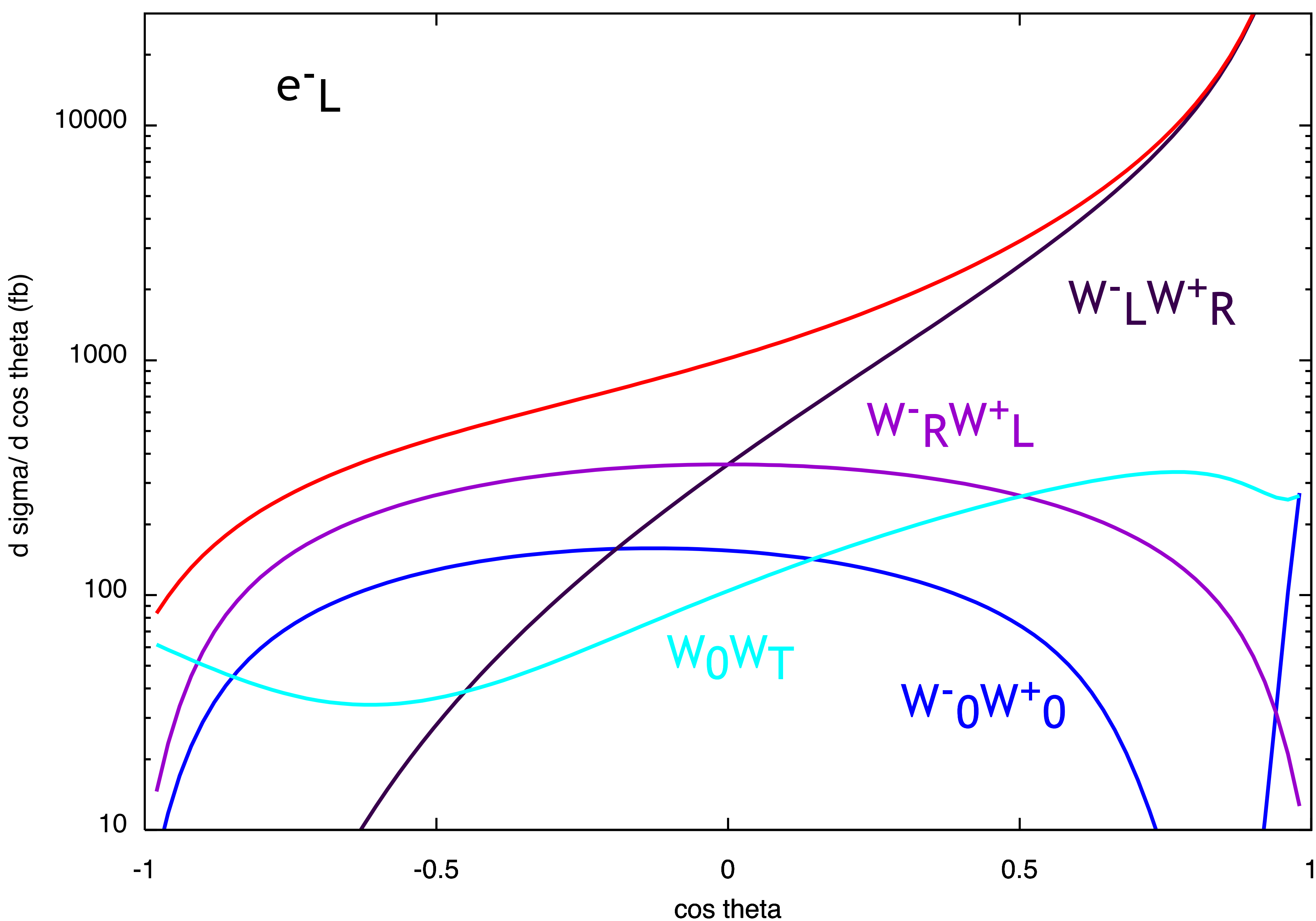
$$= e^2\sin\theta\,\frac{p}{E}\cdot\frac{s}{2m_W^2}\cdot\frac{m_Z^2}{s-m_Z^2}\quad\longrightarrow\quad\frac{e^2}{2c_w^2}\sin\theta$$

which agrees nicely with

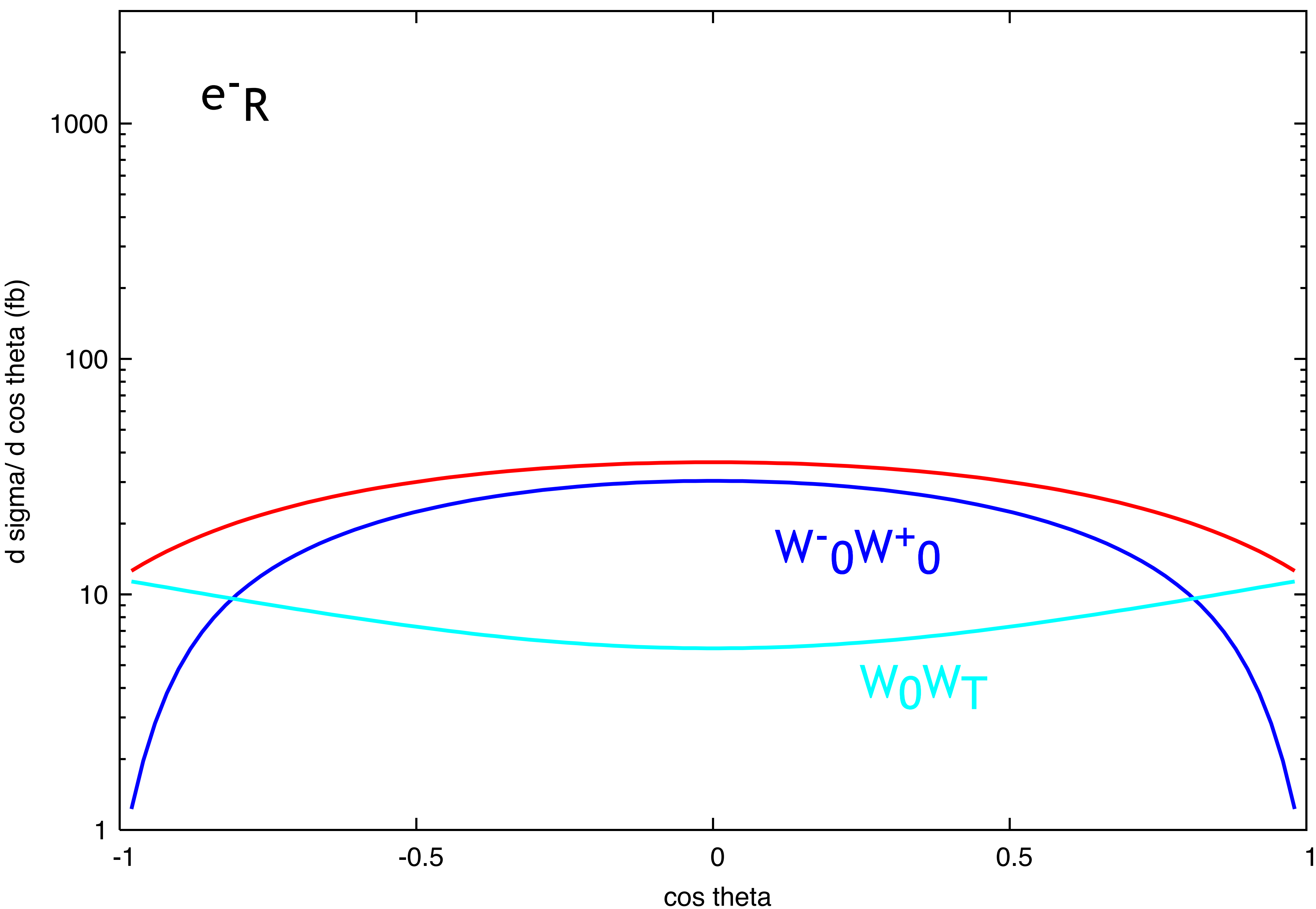


$$= \frac{e^2}{c_w^2}\cdot 1\cdot\frac{1}{2}\cdot\sin\theta$$

A more intricate cancellation gives the corresponding result for $e^-_L\, e^+_R$.



e^-_L
10000
1000
100
10
d sigma/ d cos theta (fb)
-1
-0.5
0
0.5
1
cos theta
$W^-_L W^+_R$
$W^-_R W^+_L$
$W_0 W_T$
$W^-_0 W^+_0$



e⁻R
W⁻0W⁺0
W0WT
1000
100
10
1
-1
-0.5
0
0.5
1
d sigma/ d cos theta (fb)
cos theta

This analysis has a number of important consequences:

- The forward peak in $e^+e^- \rightarrow W^+W^-$ is dominantly $e^-_L e^+_R \rightarrow W^-_L W^+_R$. It is insensitive to new physics in the Higgs sector.

- The process $e^-e^+ \rightarrow W^-_0 W^+_0$ is central:

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow W_0^- W_0^+) \sim \sin^2\theta$$

This process is directly influenced by new physics in the Higgs sector.

- Because of the cancellation between diagrams in $e^-_L e^+_R \to W^- W^+$, the cross section is very sensitive to modifications of the Yang-Mills vertex. LC measurements are expected to reach a sensitivity to the $WW\gamma$, WWZ couplngs of order

$$10^{-3} e \sim e \cdot \left(\frac{m_W}{3\ \mathrm{TeV}}\right)^2$$

- The cancellation in $e^-_R e^+_L \to W^-_0 W^+_0$ suppresses the whole cross section. So, if WW production is a background, we can control it by using an e^-_R beam.

Now I would like to discuss the modelling of a reaction such as $e+e- \rightarrow W+W-$. In an event such as

$$e^-e^+ \longrightarrow W^- \quad W^+ \longrightarrow q\,\bar{q}$$

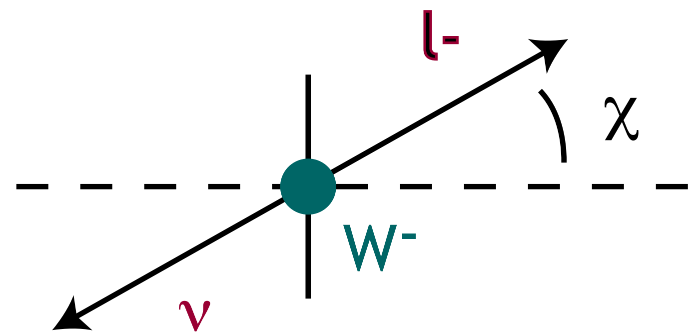
$$W^- \longrightarrow l^-\nu$$

the entire event can be reconstructed and all angles characterizing the final state can be measured. (With ISR, there is a two-fold ambiguity.)

All of these angles contain information about the W polarizations. How do we get it out ?

for a W- at rest, there are characteristic decay distributions

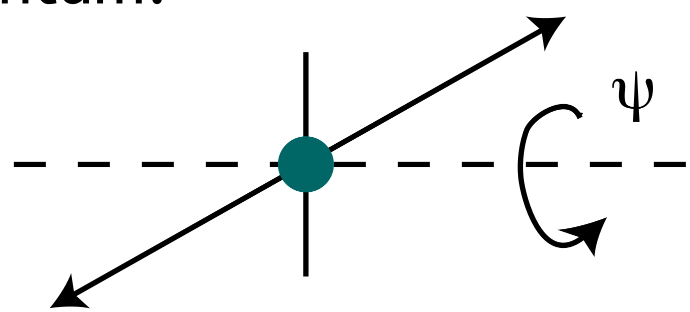
$$\mathcal{M}(W_L^- \to \ell^- \overline{\nu}) \sim (1 + \cos\chi)$$
$$\mathcal{M}(W_0^- \to \ell^- \overline{\nu}) \sim \sqrt{2}\sin\chi$$
$$\mathcal{M}(W_R^- \to \ell^- \overline{\nu}) \sim (1 - \cos\chi)$$



reflecting conservation of angular momentum.

A rotation by ψ adds a factor

$$\exp(ih_W\psi)$$



To put the W$^-$ in motion, boost along the axis, then rotate it into the correct direction. An initial W$^-$ of definite angular momentum goes into a moving W$^-$ of definite helicity.

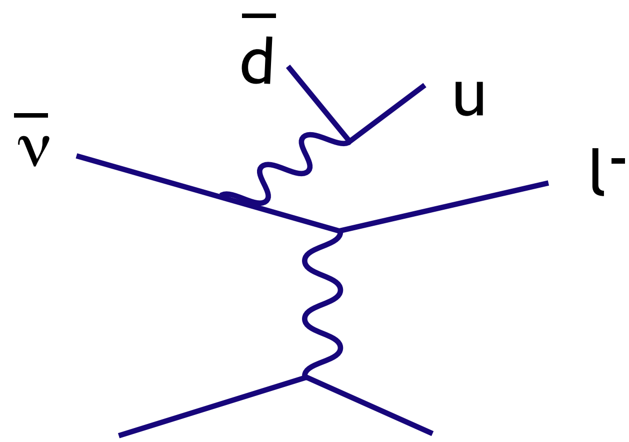
We can now build up the amplitude for a complete process by summing coherently over the W$^-$ and W$^+$ helicity states:

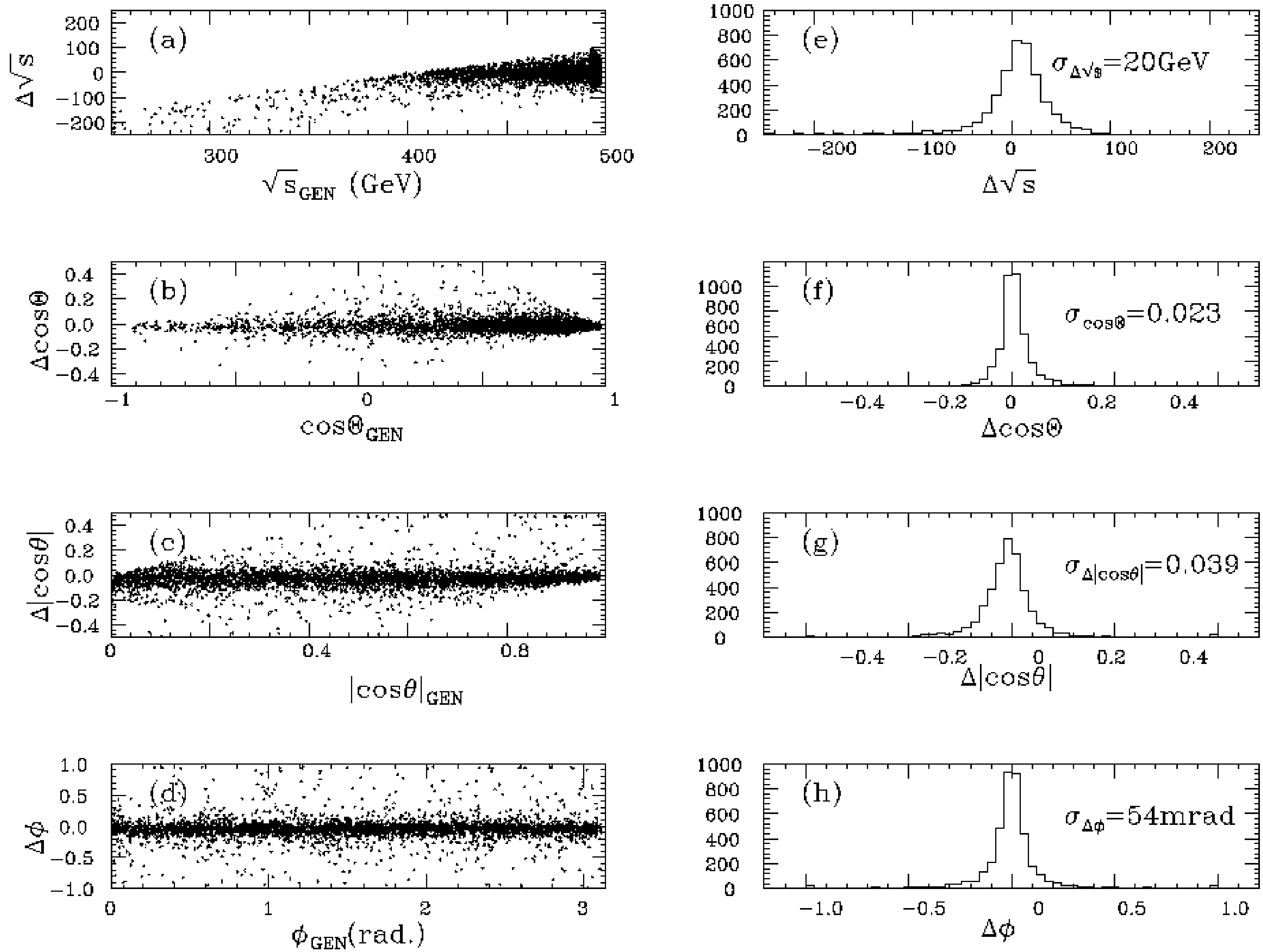
$$\mathcal{M}(e^-e^+ \to \ell^-\overline{\nu}u\overline{d}) = \sum_{h_+,h_-} \mathcal{M}(e^+e^- \to W^-(h_-)W^+(h_+)) \\ \cdot e^{ih_-\psi_-}\mathcal{M}(W^-(h_-) \to \ell^-\overline{\nu}) \\ \cdot e^{ih_+\psi_+}\mathcal{M}(W^+(h_+) \to u\overline{d})$$

(averaging over ψ gives the incoherent sum over helicity states)

For precision work, and for computation of backgrounds with off-shell W's, it is necessary to take into account also diagrams that do not have W$^+$W$^-$ intermediate states:

e.g.,

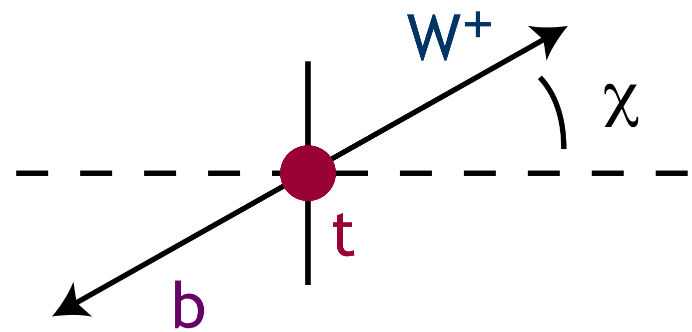



(a)
Δ√s
√s_GEN (GeV)
(e)
σ_Δ√s=20GeV
Δ√s
(b)
Δcosθ
cosθ_GEN
(f)
σ_cosθ=0.023
Δcosθ
(c)
Δ|cosθ|
|cosθ|_GEN
(g)
σ_Δ|cosθ|=0.039
Δ|cosθ|
(d)
Δφ
φ_GEN(rad.)
(h)
σ_Δφ=54mrad
Δφ

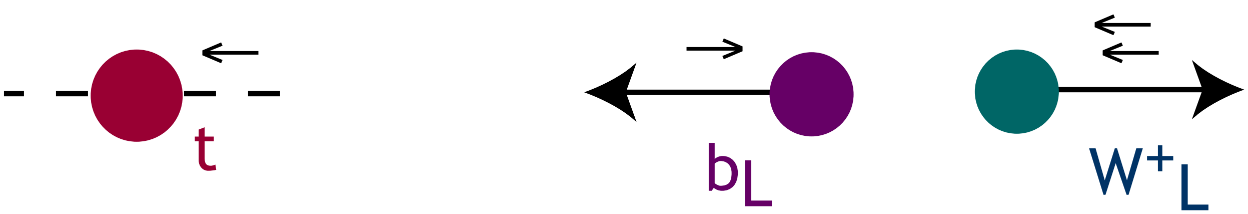
Miyamoto

The heaviest particle of the SM is the top quark. The top quark is also influenced in a not-so-intuitive way by its coupling to the symmetry-breaking sector.

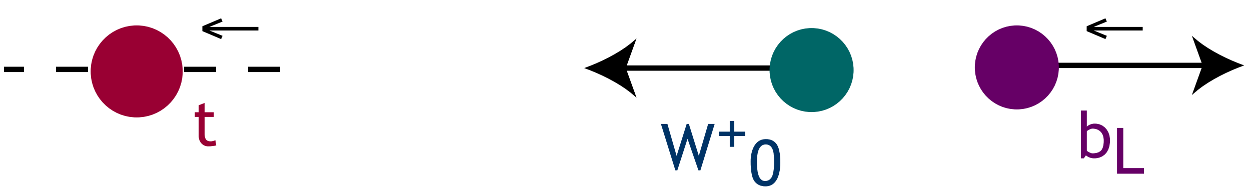
Consider the process of t decay: $t \to bW^+$.
The final quark is a b_L (if we ignore m_b).
There are two possible W^+ helicity states.
Define the decay angle χ as for W:



a t_L can decay to



$$\mathcal{M} \sim \frac{g}{\sqrt{2}} \cos\frac{\chi}{2}$$



$$\mathcal{M} \sim \sin\frac{\chi}{2}$$

The matrix element for the second decay mode is enhanced.

Taking $\epsilon_0^\mu \approx k^\mu / m_W$,

$$\epsilon_0^* \overline{u}_{bL} \gamma_\mu u_t \approx \overline{u}_{bL} \frac{\not{k}}{m_W} u_t = \frac{m_t}{m_W} \overline{u}_{bL} u_t$$

and so

$$\mathcal{M}(t \to bW_0^+) \sim \frac{m_t}{2m_W} \sin\frac{\chi}{2}$$

Assembling all of the pieces, we find the top quark width:

$$\Gamma_t = \frac{\alpha m_t}{8 s_w^2}\left(1 - \frac{m_W^2}{m_t^2}\right)^2 \left(1 + \frac{m_t^2}{2m_W^2}\right) \approx \frac{\alpha}{16 s_w^2} \frac{m_t^3}{m_W^2}$$

Γ_t is large and is dominated by $t \to b\, W^+_0$

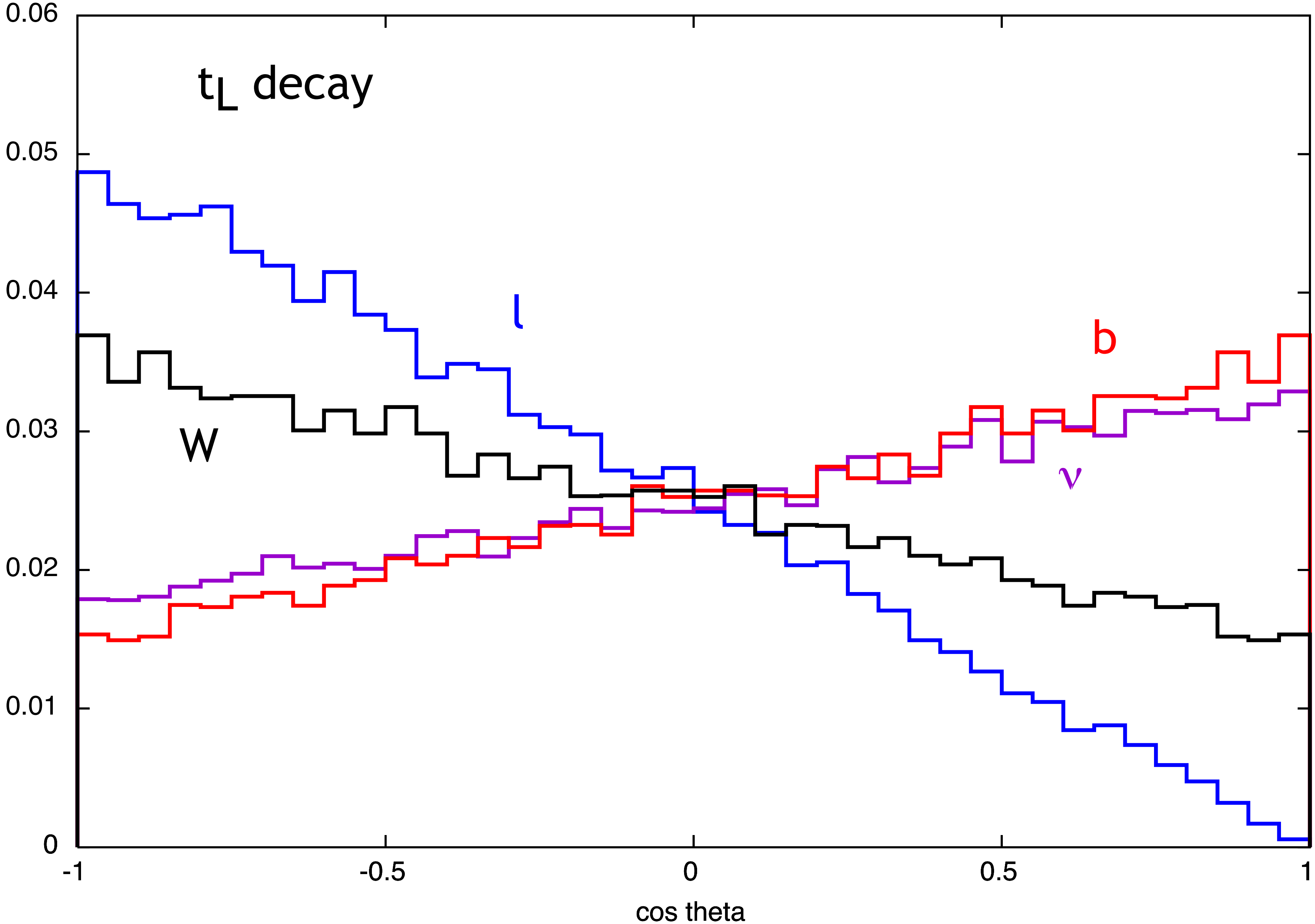
Γ_t = 1.5 GeV and $$P_0 = \frac{\Gamma(\to W_0)}{\Gamma(\to W_0) + \Gamma(\to W_T)} = 0.7$$

The b and W^+ directions in the top rest frame are indicators of the polarization.

However, the best individual polarimeter is the l^+ or $\bar{d}$ quark from the W+ decay, which has the distribution

$$\frac{d\Gamma}{d\cos\theta_\ell}(t_L) \sim (1 - \cos\theta_\ell)$$

For all-hadronic t decays, c tagging will be useful to increase the statistics for t polarization measurements.



0.06
0.05
0.04
0.03
0.02
0.01
0
-1
-0.5
0
0.5
1
cos theta
t_L decay
l
b
W
ν

There are two different sets of issues to study in top quark pair production, associated conceptually with production energies just at the threshold and somewhat above.

The large value of Γ_t has important effects in both regimes.

Above threshold, we can study fermion pair production just as for lighter quarks.

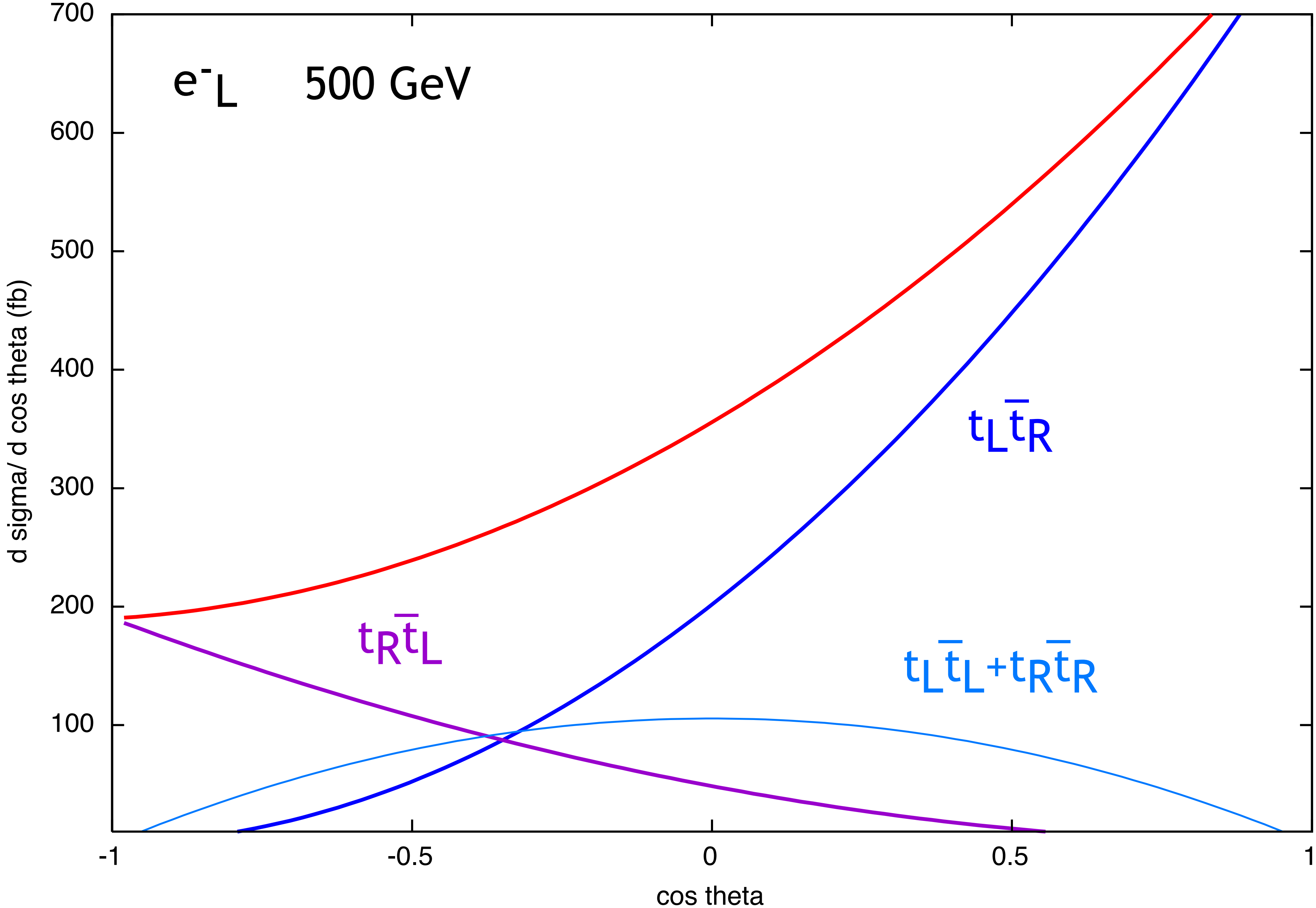
Recall that, for a u quark,

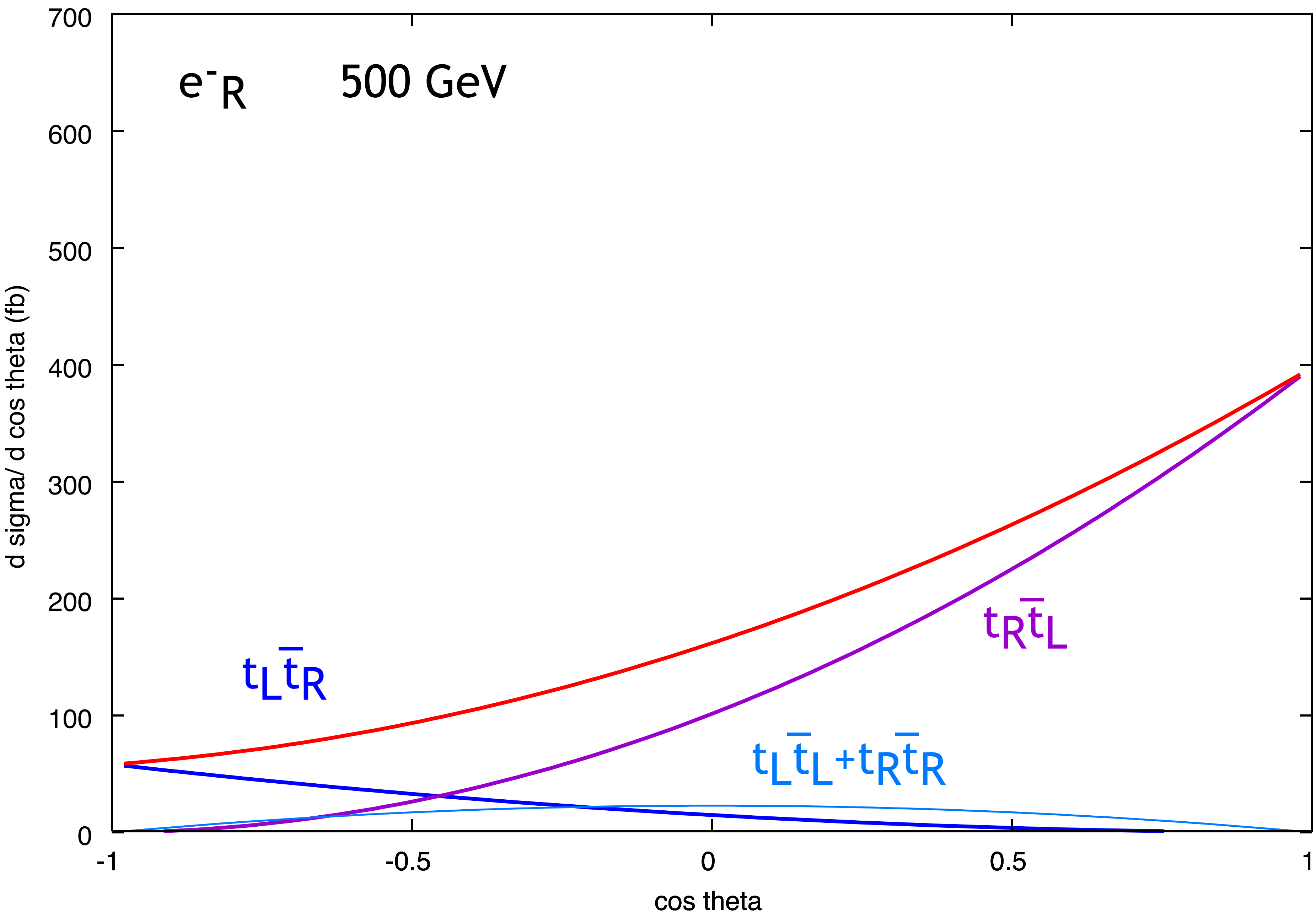
$$|f_{LL}|^2 = 1.42, \qquad |f_{RR}|^2 = 0.75, \quad \text{other } |f|^2\text{'s are small}$$

For a massive quark, we have, in addition,

$$e^+e^- \to t_L\,\bar{t}_L\,, \quad t_R\,\bar{t}_R\,, \quad \text{with} \qquad \mathcal{M} \sim \frac{m_t}{E}$$

The cross sections for these modes are related by CP.


e-L   500 GeV
tLt̄R
tRt̄L
tLt̄L+tRt̄R
d sigma/ d cos theta (fb)
cos theta
700
600
500
400
300
200
100
-1
-0.5
0
0.5
1



e^-_R
500 GeV
$t_L\bar{t}_R$
$t_R\bar{t}_L$
$t_L\bar{t}_L+t_R\bar{t}_R$
d sigma/ d cos theta (fb)
cos theta
0
100
200
300
400
500
600
700
-1
-0.5
0
0.5
1

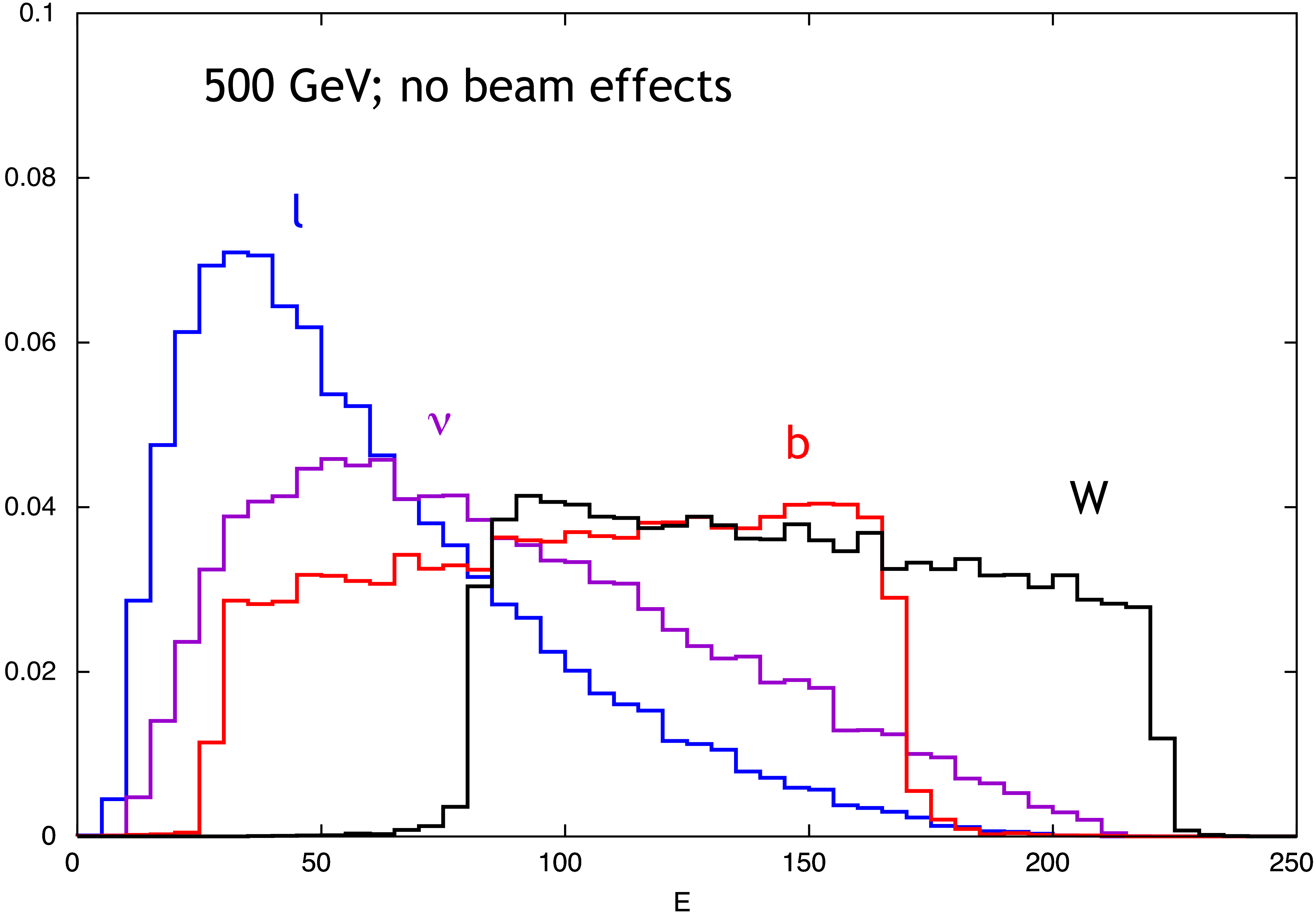
The polarization of the t quark at its production is reflected in its decay.

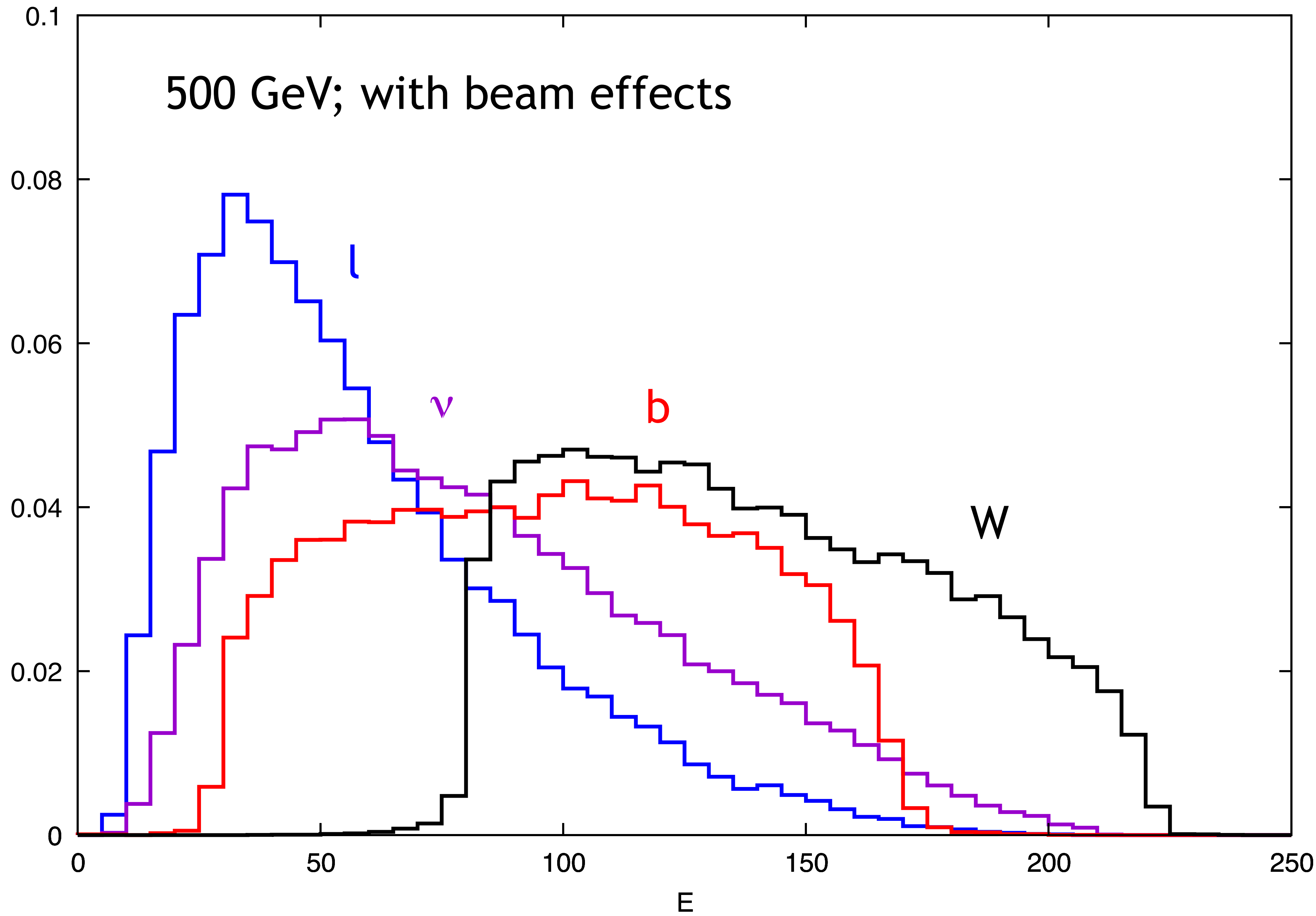
This is not true of any other quark. Lighter quarks form hadrons which then decay to state of lower spin (e.g. D, B mesons). However, top decays before hadronization and long before its spin can flip.

So, we can assemble complete events such as

$$\begin{tikzcd} e^+e^- \arrow[r] & t\;\bar{t} \arrow[r] \arrow[dr] & \bar{b}\,W^- \arrow[r] & \bar{b}\;l^-\;\bar{\nu} \\ & & bW^+ \arrow[r] & b\;c\;\bar{s} \end{tikzcd}$$

using the helicity formalism just as for W^+W^- events.


500 GeV; no beam effects
l
ν
b
W
0.1
0.08
0.06
0.04
0.02
0
0
50
100
150
200
250
E



500 GeV; with beam effects
0.1
0.08
0.06
0.04
0.02
0
0
50
100
150
200
250
E
l
ν
b
W

Using all of the observables that the LC makes available - t and t angular distributions, decay distributions, and polarization asymmetries - we can search for anomalous t production and decay form factors. These include:

t anomalous magnetic moment

$t \rightarrow b_R \, W^+$ decay

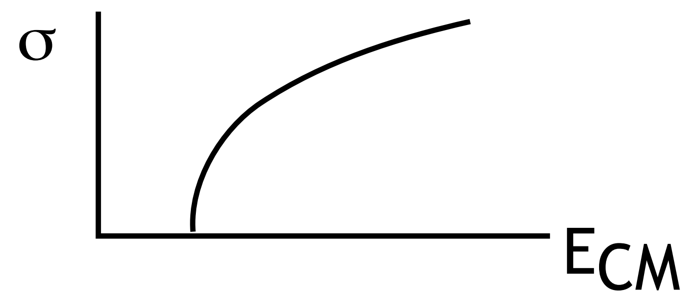
anomalous $t\bar{t}Z$ coupling

The last of these effects is generated in technicolor models and other models in which the t mass comes from strong Higgs sector interactions.

Now consider measurements very close to the threshold for $e^+e^- \to t\,\bar{t}$.

For a weakly interacting particle such as τ, the most accurate determination of the mass comes from the measurement of the cross section near threshold, since

$$\sigma = \frac{2\pi\alpha^2}{s}\left(1 - \frac{4m^2}{s}\right)^{1/2}$$



In light $q\,\bar{q}$ systems, the threshold behavior is hidden by nonperturbative quarkonium resonances.

But, for $t\,\bar{t}$, there is a different story.

Because the top quark is heavy, its lowest bound states fall into the perturbative part of the QCD potential. So we can estimate:

strong coupling: $\alpha_s(a_0) \sim 0.15$

Bohr radius: $a_0 = \frac{4}{3}\alpha_s m_t \sim 35 \text{ GeV}$

Rydberg energy: $Ry = \frac{1}{2}(\frac{4}{3}\alpha_s)^2 m_t \sim 3.5 \text{ GeV}$

In addition, the large t width acts as an infrared cutoff. The t and $\bar{t}$ decay before they reach the nonperturbative region of the potential.

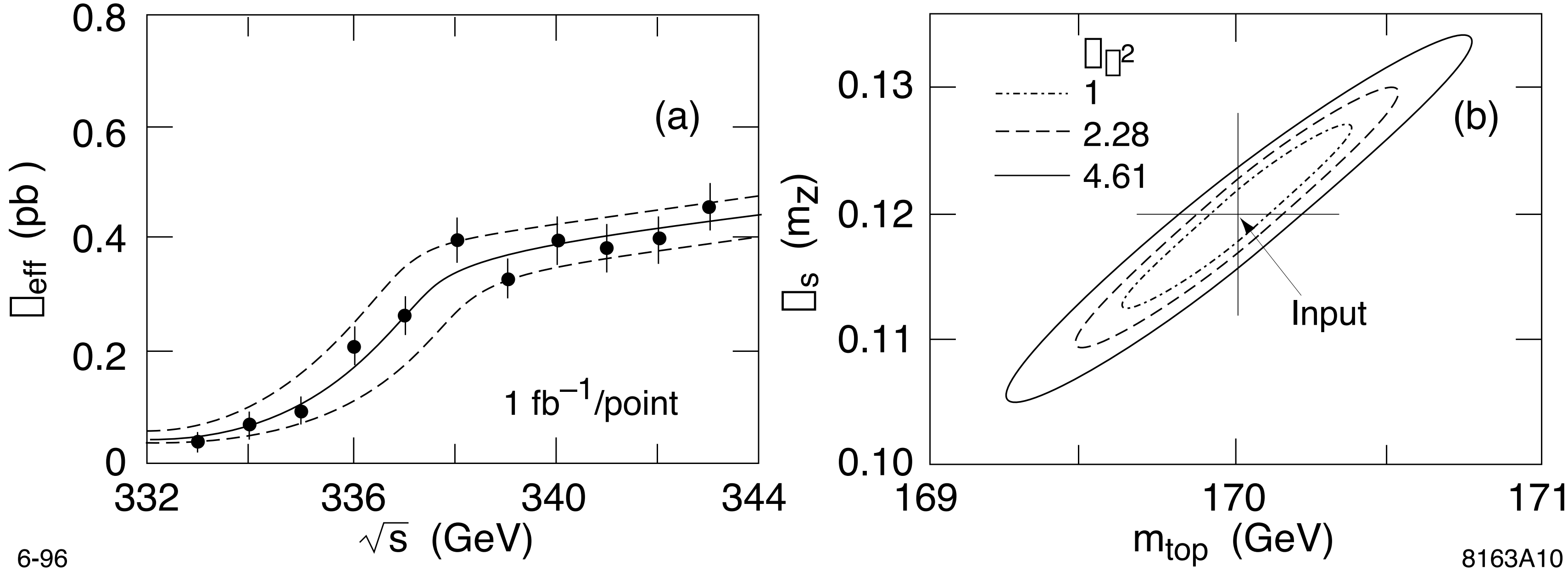
Thus, the t $\bar{t}$ threshold cross section can be predicted in QCD.

Because the $t\bar{t}$ threshold is relatively sharp and calculable, it can be the basis for a precision determination of the t quark mass. If α_s is known from other measurements, the error could be of order 100 MeV.

Further, the threshold position - unlike the kinematic t quark mass measured at the Tevatron - is a short-distance quantity with a simple relation to $m_{t\overline{MS}}$.

ISR and beamstrahlung reduce the size of the $t\bar{t}$ threshold cross section by a factor of 2. Measurement of the e^+e^- luminosity spectrum is an essential part of the threshold study.

Other observables of the $t\bar{t}$ threshold allow a few-percent determination of Γ_t. If the Higgs boson is light, the $t\bar{t}h$ Yukawa coupling affects the threshold cross section at the 10% level and should be measurable through this effect.



0.8
0.6
0.4
0.2
0
(a)
□eff (pb)
1 fb−1/point
332
336
340
344
√s (GeV)
6-96
□□2
1
2.28
4.61
(b)
0.13
0.12
0.11
0.10
□s (mZ)
Input
169
170
171
mtop (GeV)
8163A10

Sumino

One more heavy particle of the Standard Model remains to be discussed - the Higgs boson. We will consider its properties and its appearance in e^+e^- reactions tomorrow.