Minimal Flavor Violation: from quarks to leptons

Vincenzo Cirigliano
Theoretical Division, Los Alamos National Laboratory
Outline

- The “Flavor Problem” and MFV in the quark sector
- MFV in the lepton sector?
  - Identify two ‘minimal’ scenarios
  - Signatures: the role of $\mu \rightarrow e$ and $\tau \rightarrow \mu, e$ processes
  - Role of mu-to-e conversion

The “Flavor Problem”: 

- Clash between *theoretical expectation* of “new physics” at the ~TeV scale and *experimental observations* in rare FCNC processes (K, B, μ, τ)

- Quark Sector: the unreasonable success of the CKM paradigm!

\[ \Lambda_{NP} > 10^{3-4} \text{ TeV} \]
The “Flavor Problem”:

- Clash between *theoretical expectation* of “new physics” at the \( \sim \)TeV scale and experimental observations in rare FCNC processes (K, B, \( \mu \), \( \tau \))

- Lepton sector: severe constraints from FCNC of charged leptons

\[ \mu \rightarrow e \gamma \text{ in SUSY} \]

\[ \frac{C_{\mu e}}{\Lambda^2} H^\dagger \bar{\mu} R \sigma^{\mu\nu} L_L F_{\mu\nu} \]

\[ \text{BR}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11} \]

\[ \Lambda / \sqrt{C_{\mu e}} > 2 \times 10^4 \text{ TeV} \]

(95% C.L.)
Evading the “Flavor Problem”

- $\Lambda \sim \text{TeV}$ + Symmetry Principle protecting FCNC $\rightarrow$ MFV hypothesis:

  \[
  \text{The irreducible sources of flavor-symmetry breaking are aligned to fermion mass matrices (Yukawa couplings + …)}
  \]


Most conservative of the “symmetry principles”: no additional source of flavor breaking beyond what is needed to generate observed fermion masses and mixings
Evading the “Flavor Problem”

- $\Lambda \sim \text{TeV} + \text{Symmetry Principle protecting FCNC} \rightarrow \text{MFV hypothesis:}$

- Flavor symmetry of $\mathcal{L}_{\text{Gauge}} \ [G_F = \text{SU}(3)^5]$ broken only by $\lambda_U$ and $\lambda_D$

$$\mathcal{L}_{\text{Gauge}} = \sum_i \bar{\psi}_i i \gamma^\mu \partial_\mu (A) \psi_i$$

$$\psi^i = \left( Q_L^i, u_R^i, d_R^i, L_L^i, e_R^i \right)$$

$$Q_L = \left( \begin{array}{c} u_L \\ d_L \end{array} \right) \quad L_L = \left( \begin{array}{c} v_L \\ e_L \end{array} \right)$$

$$\bar{Q}_L \lambda_D^i d_R^i H$$

$$\bar{Q}_L \lambda_U^i u_R^i H_c$$

$$\frac{m_D}{\nu} \quad V_C K M \frac{m_U}{\nu}$$
Evading the “Flavor Problem”

- $\Lambda \sim \text{TeV}$ + Symmetry Principle protecting FCNC $\rightarrow$ MFV hypothesis:

  - Flavor symmetry of $L_{\text{Gauge}}$ [$G_F = \text{SU}(3)^5$] broken only by $\lambda_U$ and $\lambda_D$

$$L_{\text{Gauge}} = \sum_i \bar{\psi}_i i \overleftrightarrow{D}(A) \psi_i$$

$$\psi^i = (Q^i_L, u^i_R, d^i_R, L^i_L, e^i_R)$$

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad L_L = \begin{pmatrix} v_L \\ e_L \end{pmatrix}$$

- Explore consequences of MFV in model-independent way:

  1) Observe that mass terms are formally invariant if

  $$\lambda_D \rightarrow V_L \lambda_D V_D^+$$

  $$\lambda_U \rightarrow V_L \lambda_U V_U^+$$

  2) Construct local operators (BSM physics) that are formally invariant under $G_F$

$$\bar{Q}^i_L \lambda_D^i d^i_R H$$

$$\bar{Q}^i_L \lambda_U^i u^i_R H_c$$

$$m_D^{\text{diag}}$$

$$V_{\text{CKM}}$$
Evading the “Flavor Problem”

- $\Lambda \sim \text{TeV} + \text{Symmetry Principle protecting FCNC} \rightarrow \text{MFV hypothesis}$

- Flavor symmetry of $\mathcal{L}_{\text{Gauge}} \ [G_F = \text{SU}(3)^5]$ broken only by $\lambda_U$ and $\lambda_D$

\[ \Lambda_{FB} \gg \Lambda \]

\[ \Lambda \ (\sim \text{TeV}) \]

Breaking of $G_F$ occurs ONLY via $\lambda$ insertions

Flavor-blind interactions of particles with $m > \Lambda$

Local operator* involving SM fields and $\lambda$

Group Theory + Effective Field Theory $\Rightarrow$ investigate consequences of MFV hypothesis in great generality
How does it work for quarks?

- Typical MFV operator mediating FCNC

\[ O_{F1} = H^\dagger \bar{D}_R \sigma^{\mu\nu} \left( \lambda_D \lambda_U \lambda_U^\dagger \right) Q_L F_{\mu\nu} \rightarrow \bar{d}_R \sigma^{\mu\nu} m_D^{ij} \Delta_{FC}^{ij} d_L F_{\mu\nu} \]

\[(\Delta_{FC})_{ij} = (\lambda_U \lambda_U^\dagger)_{ij} \simeq \left( \frac{m_t}{v} \right)^2 V_{3i}^* \, V_{3j} \]

\[ \text{Normalization} \quad \text{Mixing pattern} \]

1. FCNC suppression follows from Cabibbo hierarchy. Flavor problem essentially “solved”: \( \Lambda \sim \text{TeV} \) is now allowed

2. Highly predictive (=testable) framework, relates various \( d_i \rightarrow d_j \) transitions. Tool to investigate structure of flavor-breaking. Far from being verified.
A “geometric” point of view

- Mass matrices (Yukawas) select two distinct eigen-bases in $Q_L$ flavor space (related by $V_{CKM}$)

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

- MFV(q): new physics flavor structures do not select new “eigen-bases” in $Q_L$ flavor space $\rightarrow$ FCNC controlled by masses and $V_{CKM}$ (GIM + predictive)

Not diagonalized simultaneously
MFV in the lepton sector?

- If MFV reflects a deep principle, it is worth exploring extensions to leptons

  - Does MFV(ℓ) alleviate the leptonic FCNC problem?

  - What pattern of FCNC is predicted? Can we test it?

In some sense, we want to use MFV(ℓ) as a tool to learn about the flavor-breaking structures in the lepton sector.
MFV in the lepton sector?

- Our definition of MFV(ℓ) [based on mass matrices]:
  - $m_\nu$ and $m_\ell$ select two distinct eigen-bases in $L_L$ space (related by $U_{PMNS}$)
  
  $$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

- New physics flavor structures do not select new “eigen-bases” in $L_L$ flavor space [$\rightarrow$ FCNC controlled by lepton mass eigenvalues and $U_{PMNS}$]
Discussion

- This scheme is predictive but quite restrictive \([\supseteq \text{few explicit models}].\) Other definitions are possible [Davidson-Palarini 2006]

- Even with our restrictive definition, several options are available:

  - Replica of quark MFV
    \[\lambda_D \rightarrow \lambda_e\]
    \[\lambda_U \rightarrow \lambda_\nu \approx m_\nu / v < 10^{-11}\]

  - SM field content \((L_L, e_R)\)

  - Extended field content \((L_L, e_R, \nu_R)\)

Focus on Majorana case(s)
Majorana mass: 

\[ m^{ij}_\nu = g^{ij}_\nu \frac{v^2}{\Lambda_{LN}} \]

Throughout, I assume that \( U(1)_{LN} \) is broken at scale \( \Lambda_{LN} > v_{ew} \) (so that EFT description in terms of dim5 operator is appropriate)
- Majorana mass: \[ m_{ij}^\nu = g_{ij}^\nu \frac{v^2}{\Lambda_{LN}} \quad \text{+} \quad \frac{g_{ij}^\nu}{\Lambda_{LN}} (L_{iL}^{T}C H_{c}^{*})(H_{c}^{+}L_{iL}^{j}) \]

- Identify irr. sources of \( G_{LF} = SU(3)_L \times SU(3)_E \) breaking satisfying MFV

Dim 4 Yukawa

\[ \bar{L}^{i}_L \chi^{ij}_e e^{j}_R H \]

Dim 5 \(|\Delta L|=2\)

\[ \frac{g_{ij}^\nu}{\Lambda_{LN}} (L_{iL}^{T}C H_{c}^{*})(H_{c}^{+}L_{iL}^{j}) \]
- **Majorana mass:** 
  \[ m_{\nu}^{ij} = g_{\nu}^{ij} \frac{v^2}{\Lambda_{LN}} \]

  \[ \frac{g_{\nu}^{ij}}{\Lambda_{LN}} (L_{L}^{T} C H_{c}^{*})(H_{c}^{*} L_{L}^{j}) \]

- Identify irr. sources of \( G_{LF} = SU(3)_{L} \times SU(3)_{E} \) breaking satisfying MFV

  Dim 4 Yukawa

  \[ \bar{L}^{i}_{L} \lambda_{e}^{ij} e_{R}^{j} H \]

  Dim 5 \(|\Delta L|=2\)

  \[ \frac{g_{\nu}^{ij}}{\Lambda_{LN}} (L_{L}^{T} C H_{c}^{*})(H_{c}^{*} L_{L}^{j}) \]

  Treat \( g_{\nu} \) as irreducible structure. Most natural if underlying theory has SM lepton field content.
- **Majorana mass:**
  \[ m_{\nu}^{ij} = g_{\nu}^{ij} \frac{v^2}{\Lambda_{LN}} \]
  \[ \frac{g_{\nu}^{ij}}{\Lambda_{LN}} (L^T_L C H_c^*)(H_c^\dagger L_L^j) \]

- Identify irr. sources of $G_{LF} = SU(3)_L \times SU(3)_E$ breaking satisfying MFV
  
  **Dim 4 Yukawa**
  
  \[ \bar{L}_L^i \lambda_e^{ij} e_R^j H \]

  **Dim 5** $|\Delta L|=2$
  
  \[ \frac{g_{\nu}^{ij}}{\Lambda_{LN}} (L^T_L C H_c^*)(H_c^\dagger L_L^j) \]

  Treat $g_{\nu}$ as irreducible structure.
  Most natural if underlying theory has SM lepton field content

\[ g_{\nu} = \frac{\Lambda_{LN}}{v^2} U_{PMNS}^* \hat{m}_\nu U_{PMNS}^\dagger \]
- **Majorana mass:** \[ m^{ij}_\nu = g^{ij}_\nu \frac{v^2}{\Lambda_{LN}} \]

- Identify irr. sources of \( G_{\text{LF}} = \text{SU}(3)_L \times \text{SU}(3)_E \) breaking satisfying MFV

Dim 4 Yukawa

\[
\bar{L}^i_L \lambda^{ij}_e e^j_R H
\]

Dim 5 \(|\Delta L|=2\)

\[
\frac{g^{ij}_\nu}{\Lambda_{LN}} (L^T_L \bar{C} H_c^*) (H_c^\dagger L^j_L)
\]

Treat \( g_\nu \) as irreducible structure.
Most natural if underlying theory has
SM lepton field content

Treat \( g_\nu \) as reducible.
Consider class of models with heavy \( \nu_R \):
\[ g_\nu \sim \lambda^T \nu \text{ } M_R^{-1} \lambda \nu \rightarrow \]
treat \( \lambda \nu \) and \( M_R \) as irreducible

\[ g_\nu = \frac{\Lambda_{LN}}{v^2} U_{\text{PMNS}}^* \hat{m}_\nu U_{\text{PMNS}}^\dagger \]
- **Majorana mass:**

\[ m_{ij}^{\nu} = g_{ij}^{\nu} \frac{v^2}{\Lambda_{LN}} \]

- **Identify irr. sources of** \( G_{LF} = SU(3)_L \times SU(3)_E \) **breaking satisfying MFV**

- **Dim 4 Yukawa**

\[ \bar{L}^i_L \lambda_e^{ij} e_R^j H \]

- **Dim 5** \(|\Delta L|=2\)

\[ \frac{g_{ij}^{\nu}}{\Lambda_{LN}} (L^T L C H^*_c) (H^*_c L^L_L) \]

- **Treat** \( g^{\nu} \) **as irreducible structure.**

Most natural if underlying theory has SM lepton field content

- **Treat** \( g^{\nu} \) **as reducible.**

Consider class of models with heavy \( \nu_R \):

\[ g^{\nu} \sim \lambda^{\nu T} M^{-1}_R \lambda^{\nu} \rightarrow \]

\[ \text{treat} \ \lambda^{\nu} \ \text{and} \ M_R \ \text{as irreducible} \]

- **Satisfies MFV “alignment” only if**

\[ M_R = M^{\nu} \times I \ \text{and} \ \lambda^{\nu} = \lambda^{\nu*} \]

\[ \lambda^{\nu} = \frac{M^{\nu 1/2}}{v} \hat{m}^{\nu 1/2} U^\dagger_{PMNS} \]
Effective operator analysis

Typical dim=6 operators:

\[ O_{LL}^{(3)} = \bar{L}_L \gamma^\mu \Delta L_L \bar{Q}_L \gamma_\mu Q_L \rightarrow \bar{\ell}_L^i \gamma^\mu \Delta_{FC}^{ij} \ell_L^j \bar{Q}_L \gamma_\mu Q_L \]
\[ O_{RL}^{(F)} = H^\dagger \bar{e}_R \sigma^{\mu\nu} (\lambda_e \Delta) L_L F_{\mu\nu} \rightarrow \bar{\ell}_R^i \sigma^{\mu\nu} (m_\ell^i \Delta_{FC}^{ij}) \ell_L^j F_{\mu\nu} \]

\[ \Delta \rightarrow V_L \Delta V_L^\dagger \quad [\Delta = \lambda_e^\dagger \lambda_e, \ g_\nu^\dagger g_\nu, \ \lambda_\nu^\dagger \lambda_\nu, \ldots] \]

Effective coupling governing \( \ell_i \rightarrow \ell_j \) transitions:

\[
\Delta_{FC} = \begin{cases} \frac{\Lambda_{LN}^2}{v^4} \hat{U} m_\nu^2 \hat{U}^\dagger \equiv \frac{\Lambda_{LN}^2}{v^2} \hat{a} \quad & \leftarrow \quad g_\nu^\dagger g_\nu \quad [\text{minimal}] \\ \frac{M_\nu}{v^2} \hat{U}^* m_\nu \hat{U}^\dagger \equiv \frac{M_\nu}{v} \hat{b} \quad & \leftarrow \quad \lambda_\nu^\dagger \lambda_\nu \quad [\text{extended}] \end{cases}
\]

Controlled by \( U_{PMNS} \) and \( m_\nu^{\text{diag}} \) (up to overall normalization)
Phenomenology of $\ell_i \rightarrow \ell_j \gamma$

$$B_{\ell_i \rightarrow \ell_j \gamma} = |C_{RL}^{(F)}|^2 I_{PS} \times \left\{ \left( \frac{\Lambda_{LN}}{\Lambda} \right)^4 \cdot |a_{ij}(U_{PMNS}, \Delta m^2_\nu)|^2 \right\}$$

$$\frac{v^2 M^2_\nu}{\Lambda^4} \cdot |b_{ij}(U_{PMNS}; m_{\text{min}}; \Delta m^2_\nu)|^2$$

Investigate: (i) overall normalization, size of CLFV rates

(ii) MFV signatures, falsifiable predictions
Phenomenology of $\ell_i \rightarrow \ell_j \gamma$

\[
B_{\ell_i \rightarrow \ell_j \gamma} = |C_{RL}^{(F)}|^2 I_{PS} \times \left\{ \left( \frac{\Lambda_{LN}}{\Lambda} \right)^4 \cdot |a_{ij}(U_{PMNS}, \Delta m^2_\nu)|^2 \right. \\
\left. \frac{v^2 M^2_\nu}{\Lambda^4} \cdot |b_{ij}(U_{PMNS}; m_{\text{min}}; \Delta m^2_\nu)|^2 \right\}
\]

ia) Flavor problem “solved” for $\Lambda_{LN} < 10^{12-13}$ GeV (normalization of $g_\nu$ and $\lambda_\nu$)

ib) Signals within reach of future facilities are expected only for large hierarchy between scale of $U(1)_{\text{LN}}$ breaking and $\Lambda$

\[
B_{\mu \rightarrow e(\gamma)} \sim 10^{-13} \quad \Leftrightarrow \quad \Lambda_{LN} \sim 10^{12}$ GeV \times $\Lambda/1$ TeV
\]

$M_\nu \sim 10^{12}$ GeV \times $(\Lambda/10$ TeV$)^2$

$c_i \sim O(1)$
Phenomenology of $\ell_i \rightarrow \ell_j \gamma$

$B_{\ell_i \rightarrow \ell_j \gamma} = |C_{RL}^{(F)}|^{2} I_{PS} \times \begin{cases} \left( \frac{\Lambda_{LN}}{\Lambda} \right)^{4} \cdot |a_{ij}(U_{PMNS}, \Delta m_{\nu}^{2})|^{2} \\ \frac{v^{2}M_{\nu}^{2}}{\Lambda^{4}} \cdot |b_{ij}(U_{PMNS}, m_{\text{min}}, \Delta m_{\nu}^{2})|^{2} \end{cases}$

ia) Flavor problem “solved” for $\Lambda_{LN} < 10^{12-13}$ GeV (normalization of $g_{\nu}$ and $\lambda_{\nu}$)

ib) Signals within reach of future facilities are expected only for large hierarchy between scale of $U(1)_{LN}$ breaking and $\Lambda$

$B_{\mu \rightarrow e(\gamma)} \sim 10^{-13} \Leftrightarrow c_{i} \sim O(1)$

$\Lambda_{LN} \sim 10^{12}$ GeV $\times \Lambda/1$ TeV

$M_{\nu} \sim 10^{12}$ GeV $\times (\Lambda/10$ TeV$)^{2}$

ii) MLFV predicts ratios of $B(\ell_{a} \rightarrow \ell_{b} \gamma)$ (c_{RL} and $\Lambda$ cancel out, PS is known)

$B(\tau \rightarrow \mu \gamma) >> B(\tau \rightarrow e \gamma) \sim B(\mu \rightarrow e \gamma)$

(with $\mu \rightarrow e/\tau \rightarrow \mu$ suppression increasing as $s_{13} \rightarrow 0$)
Illustration: minimal field content

\[ \frac{B_{\mu \rightarrow e\gamma}}{B_{\tau \rightarrow \mu\gamma}} \]

\[ \frac{B_{\mu \rightarrow e\gamma}}{B_{\tau \rightarrow e\gamma}} \]

Sine and cosine of solar mixing angle

\[ a_{\mu e} = \frac{1}{\sqrt{2} v^2} \left( s_c \Delta m_{\text{sol}}^2 \pm s_{13} e^{i \delta} \Delta m_{\text{atm}}^2 \right) \]

\[ a_{\tau \mu} = \frac{1}{2 v^2} \left( -c^2 \Delta m_{\text{sol}}^2 \pm \Delta m_{\text{atm}}^2 \right) \]

Sine and cosine of solar mixing angle

Normal/inverted hierarchy

Pattern entirely determined by:

- \( \Delta m_{\text{atm}}^2 >> \Delta m_{\text{sol}}^2 \)
- \( \theta_{\text{atm}}, \theta_{\text{sol}} >> \theta_{13} \)
The framework can be tested:

If $s_{13} \geq 0.05$, limits on $B(\mu \rightarrow e\gamma)$ preclude observing $\tau \rightarrow \mu \gamma$ at B factories.

Shading corresponds to different values of the phase $\delta$ and normal/inverted spectrum.
Similar conclusion holds in the case of extended field content

Hierarchy is milder (lower power of $m_{\text{sol}}/m_{\text{atm}}$)

Shading corresponds to different values of the lightest neutrino mass
The role of $\mu \rightarrow e$ conversion

- $\mu \rightarrow e$ conversion rate depends on the UV details of the theory (relative strength of magnetic dipole vs 4-fermion operators)

- Z-dependence of conversion rates and/or comparison with $\mu \rightarrow e\gamma$ in principle allows one to disentangle relative size of operators: learn about underlying dynamics!

- In the best case scenario, the Z dependence alone would allow one to reconstruct size of Wilson coeff. $\Rightarrow$ test MFV by comparing $\mu \rightarrow e$ conversion with $\tau \rightarrow \mu/e \gamma$ transitions, without $\mu \rightarrow e\gamma$ !!!
Conclusions

- The notion of MFV can be extended to the lepton sector. Working hypothesis, to investigate the nature/structure of LFV sources:
  - are \( m_e \) and \( m_\nu \) the only irreducible sources of LFV?
  - if not, is \( M_R \) flavor blind?
  - …

- Two scenarios emerge (with/without \( \nu_R \)). Phenomenology highlights:
  - normalization of rates depends on \( \Lambda_{LN}/\Lambda \) (\( \mu \rightarrow e \) observable if \( \Lambda_{LN}/\Lambda > 10^{10} \))
  - pattern of predictions for ratios of LFV transitions \( \mu \rightarrow e/\tau \rightarrow \mu \), … is governed by measured leptonic mass matrices and mixing angles

- Role of mu-to-e conversion:
  - probe details of underlying UV dynamics (strength of different operators)
  - through \( Z \)-dependence of rates, extract \( \Delta_{\mu e} \) ⇒ test MFV with \( \tau \rightarrow \mu/e \gamma \)
Additional Material
MLFV: minimal field content

- $G_{LF} = SU(3)_{LL} \times SU(3)_{ER}$ broken only by $\lambda_e, g_\nu$

\[ \mathcal{L}_{\text{Sym.Br.}} = -\lambda_e^{ij} \bar{e}_R^i (H^\dagger L_L^j) - \frac{1}{2\Lambda_{LN}} g_\nu^{ij} (\bar{L}_L^c i \tau_2 H) (H^T \tau_2 L_L^j) \]

Formally invariant under

\[ \begin{align*}
L_L &\rightarrow V_L L_L \\
\nu_R &\rightarrow V_R \nu_R
\end{align*} \quad \text{if} \quad \begin{align*}
\lambda_e &\rightarrow V_R \lambda_e V_L^\dagger \\
g_\nu &\rightarrow V_L^* g_\nu V_L^\dagger
\end{align*} \]

- Effective coupling governing $\ell_i \rightarrow \ell_j$ transitions: $\Delta_{FC} \rightarrow V_L \Delta_{FC} V_L^\dagger$

\[ \Delta_{FC} = g_\nu^\dagger g_\nu = \frac{\Lambda_{LN}^2}{v^4} U_{\text{PMNS}} \hat{m}_\nu^2 U_{\text{PMNS}}^\dagger \equiv \frac{\Lambda_{LN}^2}{v^2} \hat{a} \]

Up to scale factor, link between $\nu$ phenomenology and FCNC of charged leptons
MLFV: extended field content

- $\tilde{G}_{LF} = SU(3)_{LL} \times SU(3)_{ER} \times O(3)_{\nu R}$ broken only by $\lambda_e$, $\lambda_\nu$

Formally invariant under if

$$\mathcal{L}_{\text{Sym.Br.}} = -\lambda_{ij}^e \bar{e}_R^i (H^\dagger L^j_L) + i\lambda_{ij}^\nu \bar{\nu}_R^i (H^T \tau_2 L^j_L) + h.c.$$ 

Effective coupling governing $\ell_i \to \ell_j$ transitions: $\Delta_{FC} \to V_L \Delta_{FC} V_L^\dagger$

$$\Delta_{FC} = \lambda_\nu^\dagger \lambda_\nu = \frac{M_\nu}{v^2} U_{PMNS} \hat{m}_\nu^{1/2} H^2 \hat{m}_\nu^{1/2} U_{PMNS}^\dagger \equiv \frac{M_\nu}{v} \hat{b}$$

- Direct connection between FCNC and neutrino physics lost unless $H = I$ (CP limit)
Explicit form of LFV couplings:

1. Minimal field content: \( g_\nu^\dagger g_\nu \equiv \frac{\Lambda_{LN}^2}{v^2} \hat{a} \)

\[
\begin{align*}
a_{\mu e} &= \frac{1}{\sqrt{2} v^2} \left( s c \Delta m^2_{\text{sol}} \pm s_{13} e^{i\delta} \Delta m^2_{\text{atm}} \right) \\
a_{\tau e} &= \frac{1}{\sqrt{2} v^2} \left( -s c \Delta m^2_{\text{sol}} \pm s_{13} e^{i\delta} \Delta m^2_{\text{atm}} \right) \\
a_{\tau\mu} &= \frac{1}{2 v^2} \left( -c^2 \Delta m^2_{\text{sol}} \pm \Delta m^2_{\text{atm}} \right)
\end{align*}
\]

\(+ \leftrightarrow m_{\nu_1} < m_{\nu_2} \ll m_{\nu_3} \)

\(- \leftrightarrow m_{\nu_3} \ll m_{\nu_1} < m_{\nu_2} \)

Normal \( \rightarrow \) inverted \( \iff \delta \rightarrow \pi - \delta \)

2. Extended field content: \( \chi_\nu^\dagger \chi_\nu \equiv \frac{M_\nu}{v} \hat{b} \)

\[
\begin{align*}
b_{\mu e} &= \frac{1}{\sqrt{2} v} \left[ s c (m_{\nu_2} - m_{\nu_1}) \pm s_{13} (m_{\nu_3} - m_{\nu_1}) \right] \\
b_{\tau e} &= \frac{1}{\sqrt{2} v} \left[ -s c (m_{\nu_2} - m_{\nu_1}) \pm s_{13} (m_{\nu_3} - m_{\nu_1}) \right] \\
b_{\tau\mu} &= \frac{1}{2 v} \left[ -c^2 (m_{\nu_2} - m_{\nu_1}) + (m_{\nu_3} - m_{\nu_1}) \right]
\end{align*}
\]

\(+ \rightarrow \delta = 0 \)

\(- \rightarrow \delta = \pi \)

\((\sim \text{CP limit})\)
Alternative handles on LL-quark operators?

- Hadronic decays are predicted well below expt. sensitivities

\[
\text{Br}(\tau \to \mu \pi^0) = 6\pi^2 \frac{f_{\pi}^2}{m_{\tau}^2} \left( \frac{\Lambda_{\text{LN}}}{\Lambda} \right)^4 |a_{\mu\tau}|^2 |\tilde{C}_{LL}|^2  \quad \rightarrow \quad 10^{-15}
\]

\[
\frac{\Gamma(\gamma \to \tau\mu)}{\Gamma(\gamma \to ee)} = \frac{1}{e^2} \left( \frac{m_{\tau}}{2\nu} \right)^4 \left( \frac{\Lambda_{\text{LN}}}{\Lambda} \right)^4 |a_{\mu\tau}|^2 |\tilde{C}_{LL}'|^2  \quad \rightarrow \quad 10^{-20}
\]

- \( \mu \to 3e \) via loop effects:

  [relevant only if \( c_{4L}(\Lambda) < c_{LL}(\Lambda) \)]

\[
\text{Br}_{\mu \to 3e} = 6 \cdot 10^{-3} \times |c_{LL}(\Lambda)|^2 \times \left\{ \left( \frac{\Lambda_{\text{LN}}}{\Lambda} \right)^4 |a_{e\mu}|^2 \quad [\text{minimal}] \right. \\
\left. \left( \frac{\nu M_{\mu}}{\Lambda^2} \right)^2 |b_{e\mu}|^2 \quad [\text{extended}] \right\}
\]

\[\gg (\alpha/\pi)^2 \text{ due to large logs:} \]

\[
\log \frac{\Lambda}{M_Z} \quad \log \frac{M_Z}{1 \text{GeV}}
\]
BR formulae

\[ B_{\mu\to e\gamma} = 384\pi^2|A_R|^2 \]

\[ B_{\mu\to e} = \frac{2G_F^2m_\mu^5}{\Gamma_{\text{capt}}} |A_R^* D + \tilde{g}_{LV}^{(p)} V^{(p)} + \tilde{g}_{LV}^{(n)} V^{(n)}|^2 \]

Overlap integrals

\[ \tilde{g}_{LV}^{(p)} = -\frac{4v^2}{\Lambda_{LFV}^2} \Delta_{\mu e}^* \left[ \frac{1}{4} - s_w^2 \right] \left( c_{LL}^{(1)} + c_{LL}^{(2)} \right) + \frac{3}{2} c_{LL}^{(3)} + c_{LL}^{(4u)} + \frac{1}{2} c_{LL}^{(4d)} - \frac{1}{2} c_{LL}^{(5)} \right] \]

\[ \tilde{g}_{LV}^{(n)} = -\frac{4v^2}{\Lambda_{LFV}^2} \Delta_{\mu e}^* \left[ -\frac{1}{4} \left( c_{LL}^{(1)} + c_{LL}^{(2)} \right) + \frac{3}{2} c_{LL}^{(3)} + \frac{1}{2} c_{LL}^{(4u)} + c_{LL}^{(4d)} + \frac{1}{2} c_{LL}^{(5)} \right] \]

\[ A_R = \frac{ev^2}{\Lambda_{LFV}^2} \Delta_{\mu e} \left[ -c_{RL}^{(1)} + c_{RL}^{(2)} \right] \]
MFV in Grand Unified Theories?

- Truly MFV cannot be realized in Grand Unified Theories.
  - Quarks and leptons belong to same gauge multiplets
  - If $\Lambda_{FB} > \Lambda_{GUT}$, rad. corr. $[\Lambda_{FB} > E > \Lambda_{GUT}]$ induce cross-talk between quark & lepton flavor-breaking structures (related to mass matrices)
  - New mixing matrices appear (fewer indep. flavor rotations are possible)

We looked at $SU(5)_{\text{gauge}}$ with following assignments:

\[
\begin{align*}
\psi[\bar{5}] & \ni L_L, d_R^c \\
\chi[10] & \ni Q_L, u_R^c, e_R^c \\
N[1] & \ni \nu_R
\end{align*}
\]

$G_F^{\max} = U(3)_{\bar{5}} \times U(3)_{10} \times U(3)_1$
Interesting implications for $\ell_i \to \ell_j \gamma$

- Strength of leptonic FCNC is governed by two effective operators:

$$\frac{v}{\Lambda^2} \bar{e}^i_R \left( \lambda_e \lambda^i \lambda^j \right) \sigma^{\mu\nu} \epsilon^j_L F_{\mu\nu}$$  \hspace{1cm} \text{PMNS mixing pattern} \hspace{1cm} M_\nu > 10^{12} \text{ GeV}

$$\frac{v}{\Lambda^2} \bar{e}^i_R \left( \lambda_U \lambda^i \lambda^j \right) \sigma^{\mu\nu} \epsilon^j_L F_{\mu\nu}$$  \hspace{1cm} \text{CKM mixing pattern} \hspace{1cm} M_\nu < 10^{12} \text{ GeV}

[~ Barbieri-Hall-Strumia ‘95]
Interesting implications for $\ell_i \rightarrow \ell_j \gamma$

- Strength of leptonic FCNC is governed by two effective operators:
  \[
  \frac{v}{\Lambda^2} \bar{e}_R^i \left( \lambda_e \lambda_{\nu}^\dagger \lambda_{\nu} \right)^{ij} \sigma^{\mu \nu} e_L^i F_{\mu \nu} \quad \text{PMNS mixing pattern} \quad M_\nu > 10^{12} \text{ GeV}
  \]
  \[
  \frac{v}{\Lambda^2} \bar{e}_R^i \left( \lambda_U \lambda_{U}^\dagger \lambda_{D}^T \right)^{ij} \sigma^{\mu \nu} e_L^i F_{\mu \nu} \quad \text{CKM mixing pattern} \quad M_\nu < 10^{12} \text{ GeV}
  \]
  [~ Barbieri-Hall-Strumia '95]

- Normalization: cannot be suppressed by lowering $M_\nu < 10^{12} \text{ GeV}$. GUT induced term (controlled by top Yukawa and CKM) sets in !!

\[
\Lambda < 10 \text{ TeV} \rightarrow \begin{cases} 
B(\mu \rightarrow e \gamma) > 10^{-13} \\
B_{Al,Au}(\mu \rightarrow e) > 5 \times 10^{-16}
\end{cases}
\]

within reach of next generation expts.
Interesting implications for $\ell_i \rightarrow \ell_j \gamma$

- Strength of leptonic FCNC is governed by two effective operators:

$$\frac{v}{\Lambda^2} \bar{e}^i_R \left( \lambda_e \lambda_{\nu} \lambda_{\nu} \right)^{ij} \sigma^{\mu \nu} e^j_L F_{\mu \nu}$$

$\Rightarrow$ PMNS mixing pattern \hspace{1cm} $M_\nu > 10^{12}$ GeV

$$\frac{v}{\Lambda^2} \bar{e}^i_R \left( \lambda_U \lambda_{U} \lambda_{D} \right)^{ij} \sigma^{\mu \nu} e^j_L F_{\mu \nu}$$

$\Rightarrow$ CKM mixing pattern \hspace{1cm} $M_\nu < 10^{12}$ GeV

[~ Barbieri-Hall-Strumia '95]

- Pattern of BRs:

\[
B(\tau \rightarrow \mu \gamma) : B(\tau \rightarrow e \gamma) : B(\mu \rightarrow e \gamma) = \min \left[ s_{13}^{-2}, \left( \frac{\Delta m_{\text{atm}}^2}{\Delta m_{\text{sol}}^2} \right) \right] = 1 : 1 \sim 10 - 100 : 1 : 1
\]

\[
\lambda_C^{-6} : \lambda_C^{-4} = 1 \sim 10^4 : 500 : 1
\]

If GUT-induced term dominates, $\tau \rightarrow \mu \gamma$ is within reach of super-B factories
Conclusions on MFV+GUT

- MFV can be married with GUTs, but result is NOT MFV(q) + MFV(ℓ)
  - “guaranteed” signal in $\mu \rightarrow e$ transitions if $\Lambda < 10$ TeV
  - $\tau \rightarrow \mu, e$ transitions very useful to discriminate minimal vs GUT scenario