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# Minimal Flavor Violation: from quarks to leptons

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# Outline

- The “Flavor Problem” and MFV in the quark sector

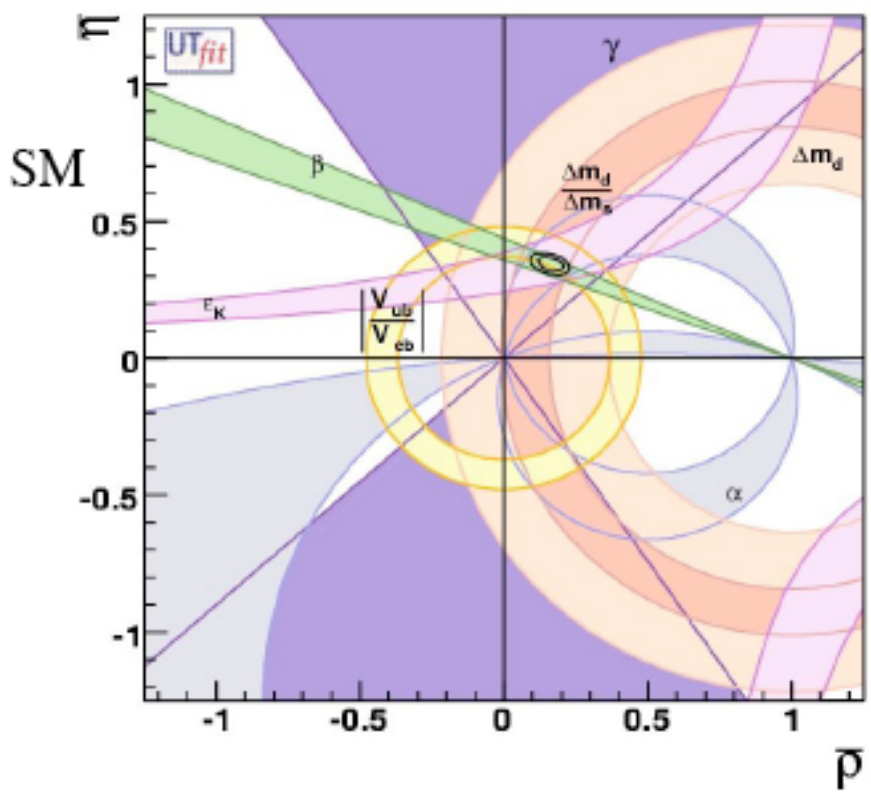
- MFV in the lepton sector ?
  - Identify two ‘minimal’ scenarios
  - Signatures: the role of $\mu \to e$ and $\tau \to \mu, e$ processes
  - Role of mu-to-e conversion


- VC, B. Grinstein, G. Isidori, M. Wise, Nucl.Phys. B 728 (2005) 121 (hep-ph/0507001)
- VC, B. Grinstein, Nucl. Phys. B 752 (2006) 18 (hep-ph/0601111)
- VC, B. Grinstein, G. Isidori, M. Wise, Nucl.Phys. B 763 (2007) 35 (hep-ph/0608123)

# The “Flavor Problem”:

- Clash between *theoretical expectation* of “new physics” at the ~TeV scale and *experimental observations in rare FCNC processes (K, B, $\mu$, $\tau$)*

- Quark Sector: the unreasonable success of the CKM paradigm!




$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$\longrightarrow$ $\Lambda_{NP} > 10^{3\text{-}4}$ TeV

# The “Flavor Problem”:

- Clash between *theoretical expectation* of “new physics” at the ~TeV scale and *experimental observations in rare FCNC processes (K, B, $\mu$, $\tau$)*

- Lepton sector: severe constraints from FCNC of charged leptons

$\longleftrightarrow$ $\frac{C_{\mu e}}{\Lambda^2} H^\dagger \, \bar{\mu}_R \sigma^{\mu\nu} L_L \, F_{\mu\nu}$

[$\mu \to e\,\gamma$ in SUSY]

$$BR(\mu \to e\gamma) < 1.2 \times 10^{-11}$$

$$\downarrow$$

$$\Lambda/\sqrt{C_{\mu e}} \; > \; 2 \times 10^4 \,\mathrm{TeV}$$

$\longleftarrow$ *(95% C.L.)*

# Evading the "Flavor Problem"

- $\Lambda \sim$ TeV + Symmetry Principle protecting FCNC $\rightarrow$ MFV hypothesis:

*The irreducible sources of flavor-symmetry breaking are aligned to fermion mass matrices (Yukawa couplings + …)*

Georgi-Chivukula 1987 Hall-Randall 1990 Buras et al 2001 D'Ambrosio et al 2002

Most conservative of the "symmetry principles":
no additional source of flavor breaking beyond what is needed to generate observed fermion masses and mixings

# Evading the "Flavor Problem"

- $\Lambda \sim$ TeV + Symmetry Principle protecting FCNC $\rightarrow$ MFV hypothesis:
- Flavor symmetry of $\mathcal{L}_{\text{Gauge}}$ [$G_F = SU(3)^5$] broken *only* by $\lambda_U$ and $\lambda_D$

$$\mathcal{L}_{\text{Gauge}} = \sum_i \bar{\psi}_i \, i \not{D}(A) \, \psi_i$$

$$\psi^i = \left(Q_L^i, u_R^i, d_R^i, L_L^i, e_R^i\right)$$

i=1,2,3

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$\bar{Q}_L^i \, \lambda_D^{ij} \, d_R^j \, H \qquad \bar{Q}_L^i \, \lambda_U^{ij} \, u_R^j \, H_c$$

$$\downarrow \qquad \downarrow$$

$$\frac{m_D^{\text{diag}}}{v} \qquad V_{\text{CKM}}^\dagger \frac{m_U^{\text{diag}}}{v}$$

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$$\downarrow \qquad \downarrow$$

$$\frac{m_D^{\text{diag}}}{v} \qquad V_{\text{CKM}}^\dagger \frac{m_U^{\text{diag}}}{v}$$

- Explore consequences of MFV in model-independent way:

1) Observe that mass terms are formally invariant if

$$\lambda_D \rightarrow V_L \, \lambda_D \, V_D^\dagger$$
$$\lambda_U \rightarrow V_L \, \lambda_U \, V_U^\dagger$$

2) Construct local operators (BSM physics) that are formally invariant under $G_F$

# Evading the "Flavor Problem"

- $\Lambda \sim$ TeV + Symmetry Principle protecting FCNC $\rightarrow$ MFV hypothesis
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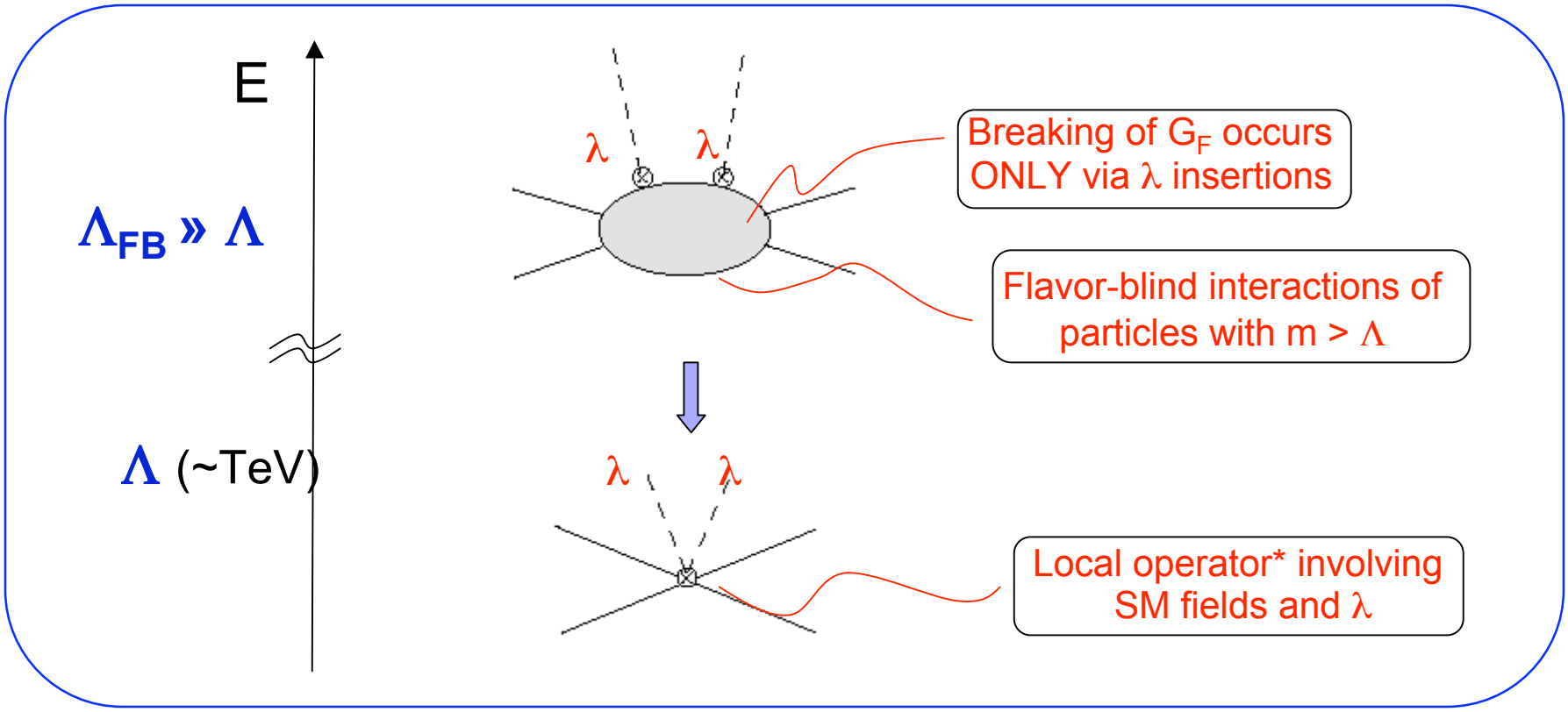



Group Theory + Effective Field Theory $\Rightarrow$ investigate consequences of MFV hypothesis in great generality

# How does it work for quarks ?

- Typical MFV operator mediating FCNC

D'Ambrosio et al 2002

$$O_{F1} = H^\dagger \, \bar{D}_R \, \sigma^{\mu\nu} \left( \lambda_D \, \lambda_U \lambda_U^\dagger \right) Q_L \, F_{\mu\nu} \longrightarrow \bar{d}_R^{i'} \, \sigma^{\mu\nu} \, m_D^i \Delta_{FC}^{ij} \, d_L^{j'} \, F_{\mu\nu}$$

$$\boxed{(\Delta_{FC})_{ij} = (\lambda_U \lambda_U^\dagger)_{ij} \simeq \left( \frac{m_t}{v} \right)^2 V_{3i}^* \, V_{3j}}$$

Normalization

Mixing pattern

1. FCNC suppression follows from Cabibbo hierarchy.
   Flavor problem essentially "solved": $\Lambda \sim$ TeV is now allowed

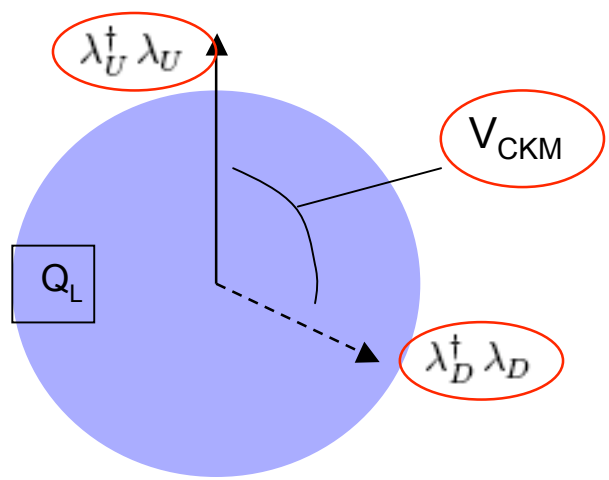
2. Highly predictive (=testable) framework, relates various $d_i \rightarrow d_j$ transitions.
   Tool to investigate structure of flavor-breaking. Far from being verified.

# A “geometric” point of view

- Mass matrices (Yukawas) select two distinct eigen-bases in $Q_L$ flavor space (related by $V_{CKM}$)

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad \lambda_U^\dagger \lambda_U \qquad \lambda_D^\dagger \lambda_D$$

Not diagonalized simultaneously




- MFV(q): new physics flavor structures do not select new “eigen-bases” in $Q_L$ flavor space → FCNC controlled by masses and $V_{CKM}$ (GIM + predictive)

# MFV in the lepton sector ?

- If MFV reflects a deep principle, it is worth exploring extensions to leptons

  - Does MFV($\ell$) alleviate the leptonic FCNC problem?

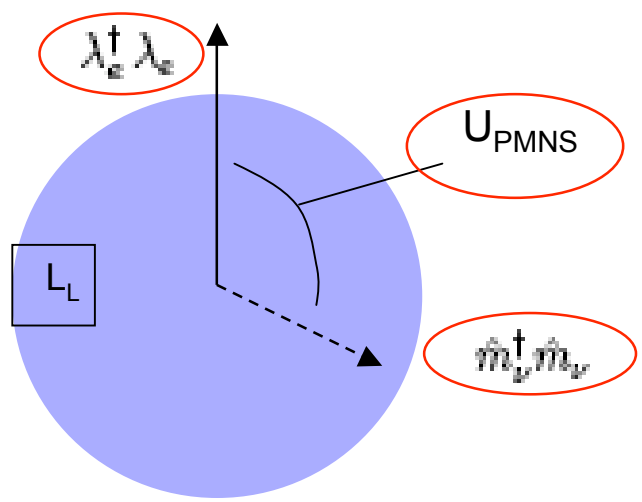
  - What pattern of FCNC is predicted? Can we test it?

In some sense, we want to use MFV($\ell$) as a tool to learn about the flavor-breaking structures in the lepton sector

# MFV in the lepton sector ?

- Our definition of MFV($\ell$) [based on mass matrices]: VC-Grinstein-Isidori-Wise

  - $m_\nu$ and $m_l$ select two distinct eigen-bases in $L_L$ space (related by $U_{PMNS}$)

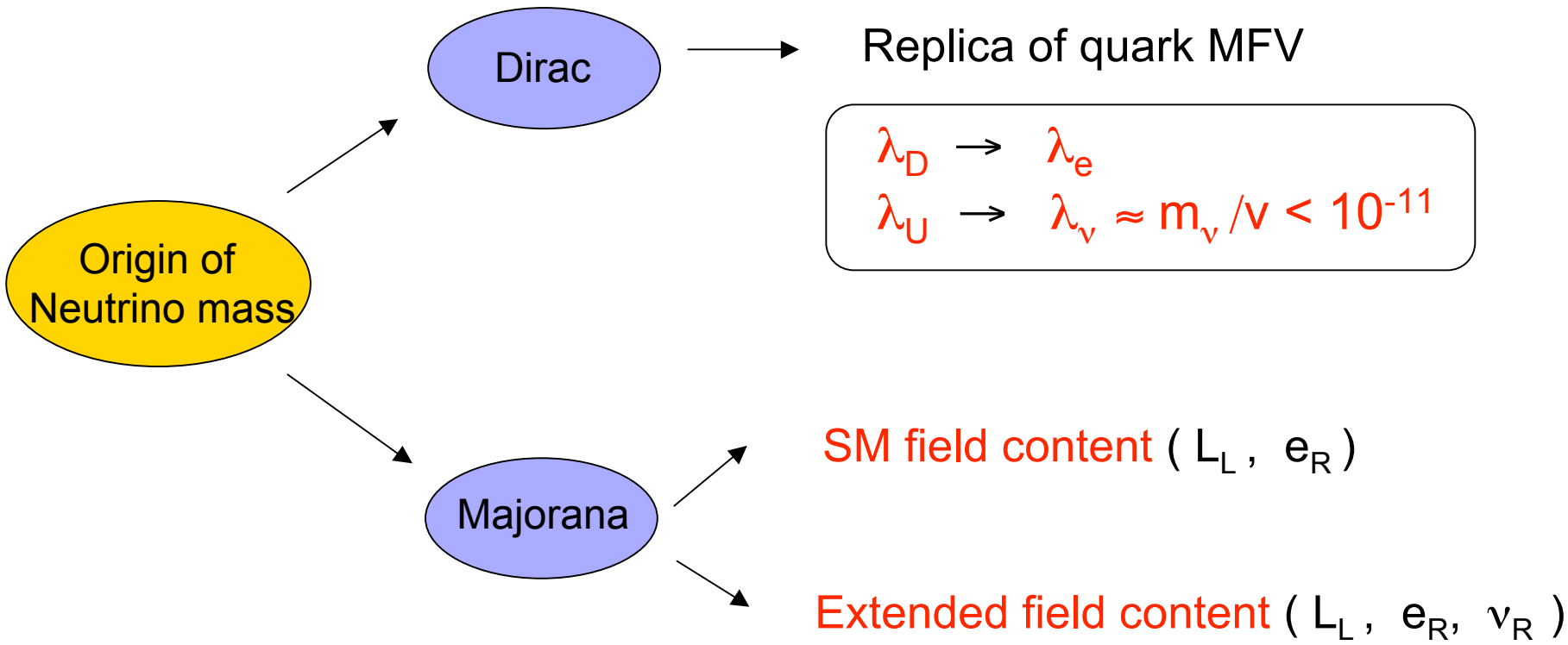



$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

  - New physics flavor structures do not select new "eigen-bases" in $L_L$ flavor space [ $\rightarrow$ FCNC controlled by lepton mass eigenvalues and $U_{PMNS}$ ]

# Discussion

- This scheme is predictive but quite restrictive [ ⊃ few explicit models]. Other definitions are possible [Davidson-Palorini 2006]
- Even with our restrictive definition, several options are available:




Focus on Majorana case(s)

- Majorana mass: $m_\nu^{ij} = g_\nu^{ij} \frac{v^2}{\Lambda_{\rm LN}}$ ← $\frac{g_\nu^{ij}}{\Lambda_{\rm LN}} (L_L^{Ti} C H_c^*)(H_c^\dagger L_L^j)$

Throughout, I *assume* that $U(1)_{LN}$ is broken at scale $\Lambda_{LN} > v_{ew}$

(so that EFT description in terms of dim5 operator is appropriate)

- Majorana mass: $m_\nu^{ij} = g_\nu^{ij} \frac{v^2}{\Lambda_{\rm LN}} \longleftarrow \frac{g_\nu^{ij}}{\Lambda_{\rm LN}} (L_L^{Ti} C H_c^*)(H_c^\dagger L_L^j)$

- Identify irr. sources of $G_{LF}$= $SU(3)_L$ x $SU(3)_E$ breaking satisfying MFV

Dim 4 Yukawa

$$\bar{L}_L^i \, \lambda_e^{ij} \, e_R^j \, H$$

Dim 5 $|\Delta L|=2$

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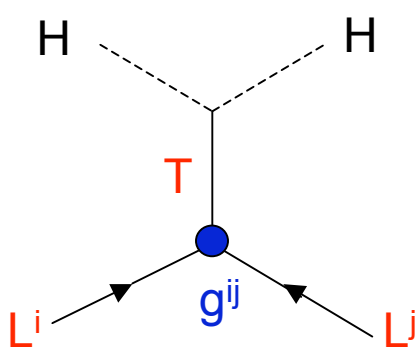
Treat $g_\nu$ as irreducible structure.
Most natural if underlying theory has
SM lepton field content



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$$g_\nu = \frac{\Lambda_{\rm LN}}{v^2} \, U_{\rm PMNS}^* \, \hat{m}_\nu \, U_{\rm PMNS}^\dagger$$

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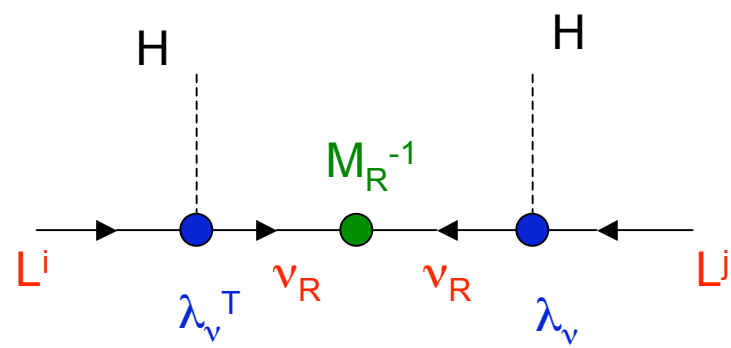
Treat $g_\nu$ as irreducible structure.
Most natural if underlying theory has
SM lepton field content

Treat $g_\nu$ as reducible.
Consider class of models with heavy $\nu_R$:
$g_\nu \sim \lambda_\nu^T M_R^{-1} \lambda_\nu \rightarrow$
treat $\lambda_\nu$ and $M_R$ as irreducible

$$g_\nu = \frac{\Lambda_{\rm LN}}{v^2} \, U_{\rm PMNS}^* \, \hat{m}_\nu \, U_{\rm PMNS}^\dagger$$



- Majorana mass: $m_\nu^{ij} = g_\nu^{ij} \frac{v^2}{\Lambda_{\rm LN}} \longleftarrow \frac{g_\nu^{ij}}{\Lambda_{\rm LN}} (L_L^{Ti} C H_c^*)(H_c^\dagger L_L^j)$

- Identify irr. sources of $G_{LF}$= $SU(3)_L$ x $SU(3)_E$ breaking satisfying MFV

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Satisfies MFV "alignment" only if
$M_R = M_\nu \times I$ and $\lambda_\nu = \lambda_\nu^*$

$$g_\nu = \frac{\Lambda_{\rm LN}}{v^2} \, U_{\rm PMNS}^* \, \hat{m}_\nu \, U_{\rm PMNS}^\dagger$$

$$\lambda_\nu = \frac{M_\nu^{1/2}}{v} \, \hat{m}_\nu^{1/2} \, U_{\rm PMNS}^\dagger$$

# Effective operator analysis

- Typical dim=6 operators:

$$O_{LL}^{(3)} = \bar{L}_L \gamma^\mu \Delta L_L \; \bar{Q}_L \gamma_\mu Q_L \longrightarrow \bar{\ell}_L^i \gamma^\mu \Delta_{FC}^{ij} \ell_L^j \; \bar{Q}_L \gamma_\mu Q_L$$

$$O_{RL}^{(F)} = H^\dagger \, \bar{e}_R \sigma^{\mu\nu} \left(\lambda_e \Delta\right) L_L \; F_{\mu\nu} \longrightarrow \bar{\ell}_R^i \sigma^{\mu\nu} \left(m_\ell^i \, \Delta_{FC}^{ij}\right) \ell_L^j \; F_{\mu\nu}$$

$$\Delta \to V_L \, \Delta \, V_L^\dagger \qquad [\; \Delta = \lambda_e^\dagger \lambda_e, \; g_\nu^\dagger g_\nu, \; \lambda_\nu^\dagger \lambda_\nu, \ldots \;]$$

- Effective coupling governing $\ell_i \to \ell_j$ transitions:

Diagonal ν mass matrix

PMNS matrix

$$\Delta_{FC} = \begin{cases} \dfrac{\Lambda_{\rm LN}^2}{v^4} \, \hat{U} m_\nu^2 \hat{U}^\dagger \equiv \dfrac{\Lambda_{\rm LN}^2}{v^2} \, \hat{a} & \leftarrow \quad g_\nu^\dagger g_\nu \quad [\text{minimal}] \\[2ex] \dfrac{M_\nu}{v^2} \, \hat{U}^* m_\nu \hat{U}^\dagger \equiv \dfrac{M_\nu}{v} \, \hat{b} & \leftarrow \quad \lambda_\nu^\dagger \lambda_\nu \quad [\text{extended}] \end{cases}$$

Controlled by $U_{PMNS}$ and $m_\nu^{diag}$ (up to overall normalization)

# Phenomenology of $\ell_i \to \ell_j \gamma$

$$B_{\ell_i \to \ell_j \gamma} = |C_{RL}^{(F)}|^2 \, I_{PS} \times \begin{cases} \left(\frac{\Lambda_{LN}}{\Lambda}\right)^4 \cdot |a_{ij}(U_{\rm PMNS}, \Delta m_\nu^2)|^2 \\ \frac{v^2 M_\nu^2}{\Lambda^4} \cdot |b_{ij}(U_{\rm PMNS}; m_{\rm min}; \Delta m_\nu^2)|^2 \end{cases}$$

Investigate: (i) overall normalization, size of CLFV rates

(ii) MFV signatures, falsifiable predictions

# Phenomenology of $\ell_i \to \ell_j \gamma$

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ia) Flavor problem "solved" for $\Lambda_{LN} < 10^{12\text{-}13}$ GeV (normalization of $g_\nu$ and $\lambda_\nu$)

ib) Signals within reach of future facilities are expected only for large hierarchy between scale of $U(1)_{LN}$ breaking and $\Lambda$

$$B_{\mu \to e(\gamma)} \sim 10^{-13} \quad \underset{c_i \sim O(1)}{\Leftrightarrow} \quad \begin{array}{l} \Lambda_{\rm LN} \sim 10^{12}\,{\rm GeV} \times \Lambda/1\,{\rm TeV} \\ M_\nu \sim 10^{12}\,{\rm GeV} \times (\Lambda/10\,{\rm TeV})^2 \end{array}$$

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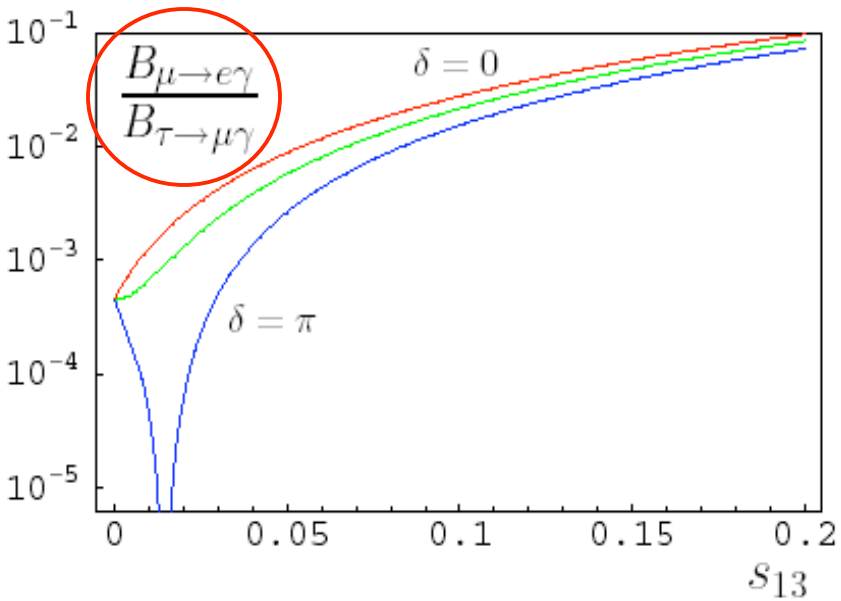
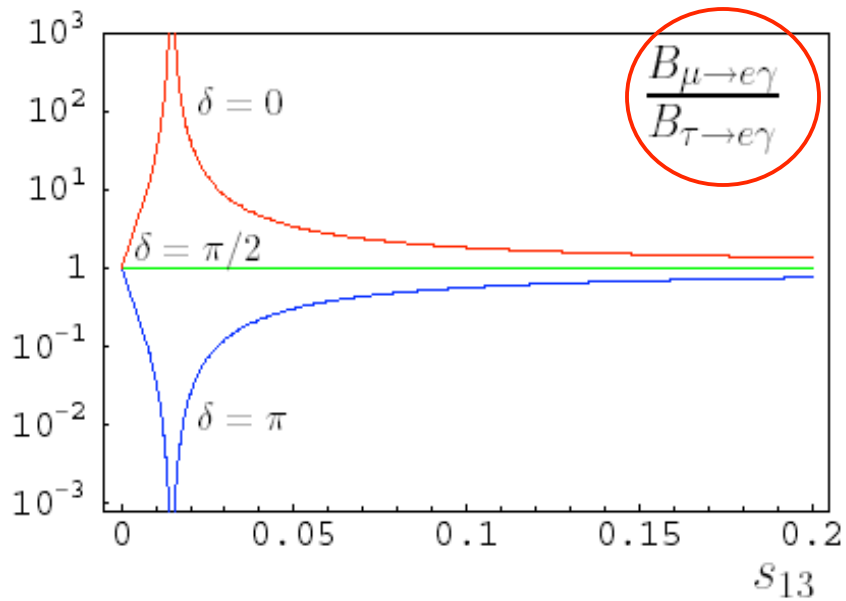
ii) MLFV *predicts* ratios of $B(\ell_a\to\ell_b\gamma)$ ($c_{RL}$ and $\Lambda$ cancel out, PS is known)

$$B(\tau\to\mu\gamma) >> B(\tau\to e\gamma) \sim B(\mu\to e\gamma)$$

(with $\mu\to e/\tau\to\mu$ suppression increasing as $s_{13} \to 0$)

# Illustration: minimal field content




$$a_{\mu e} = \frac{1}{\sqrt{2}\,v^2}\left(s\,c\,\Delta m^2_{\rm sol} \pm s_{13}\,e^{i\delta}\,\Delta m^2_{\rm atm}\right) \quad \text{vs} \quad a_{\tau\mu} = \frac{1}{2\,v^2}\left(-c^2\,\Delta m^2_{\rm sol} \pm \Delta m^2_{\rm atm}\right)$$

Sine and cosine of solar mixing angle

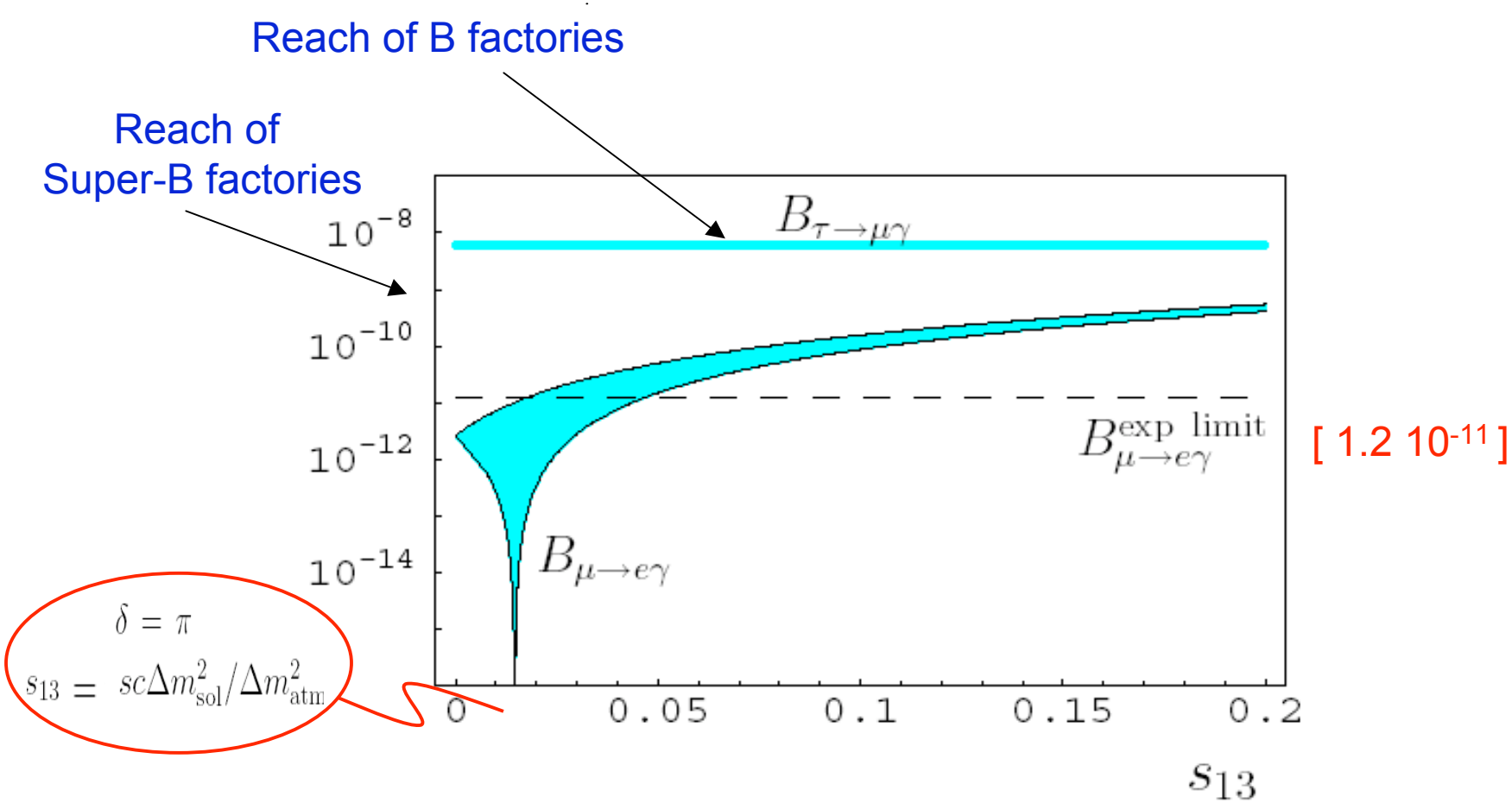
Normal/inverted hierarchy

Pattern entirely determined by:

- $\Delta m^2_{atm} >> \Delta m^2_{sol}$
- $\theta_{atm}$ , $\theta_{sol} >> \theta_{13}$

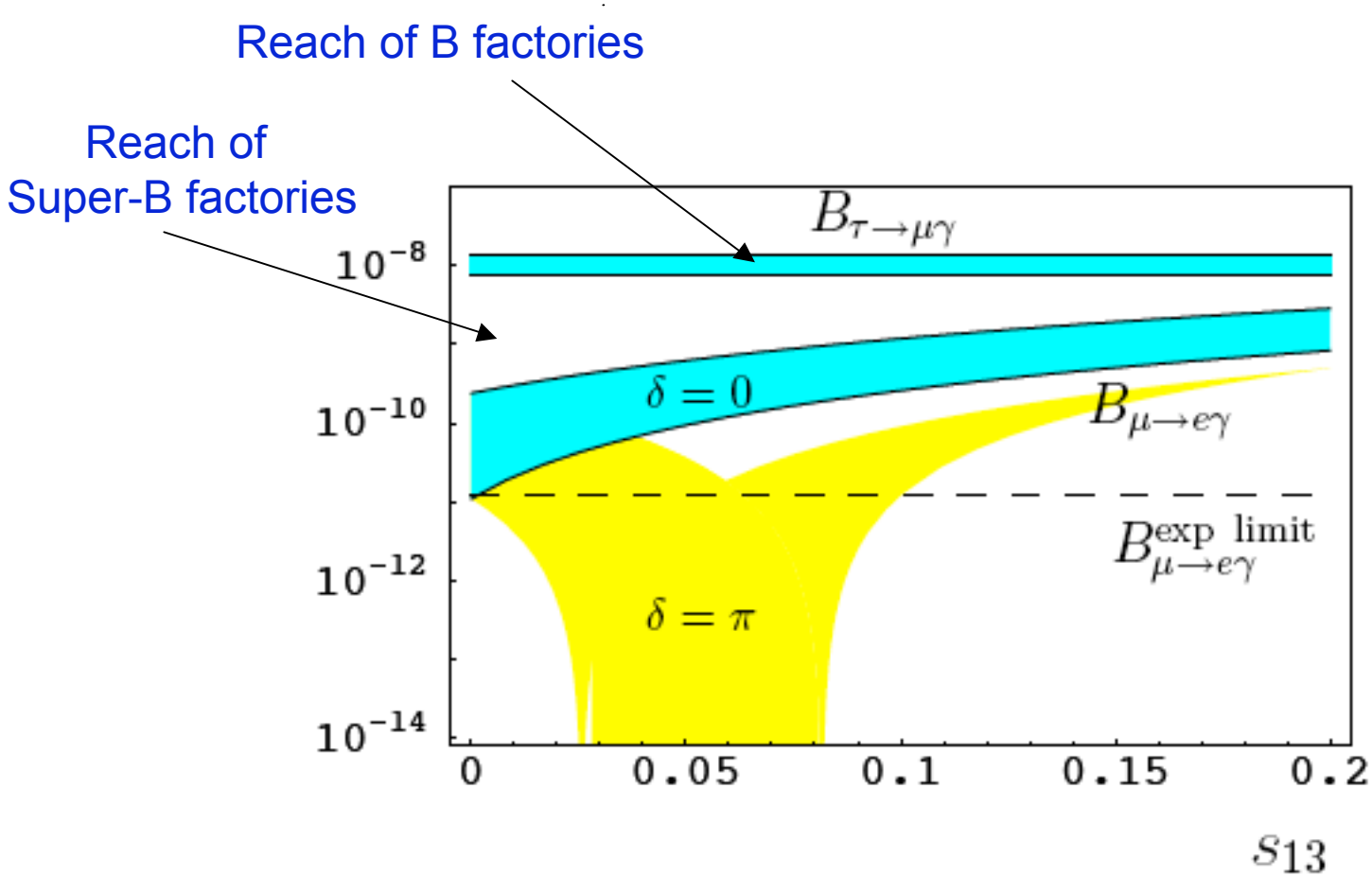
The framework can be tested:

If $s_{13} \geq 0.05$, limits on $B(\mu \to e\gamma)$ preclude observing $\tau \to \mu\gamma$ at B factories




Shading corresponds to different values of the phase $\delta$ and normal/inverted spectrum

Similar conclusion holds in the case of extended field content

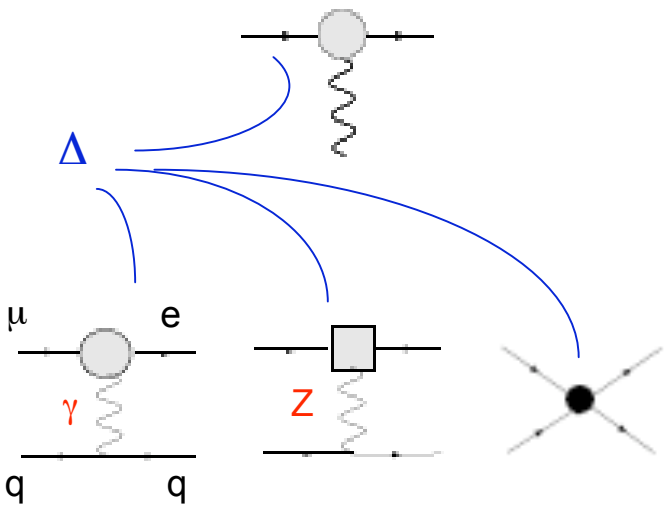



Hierarchy is milder (lower power of $m_{sol}/m_{atm}$)

Shading corresponds to different values of the lightest neutrino mass

# The role of $\mu \to e$ conversion

- $\mu \to e$ conversion rate depends on the UV details of the theory (relative strength of magnetic dipole vs 4-fermion operators)




- Z-dependence of conversion rates and/or comparison with $\mu \to e\gamma$ in principle allows one to disentangle relative size of operators: learn about underlying dynamics !

- In the best case scenario, the Z dependence alone would allow one to reconstruct size of Wilson coeff. $\Rightarrow$ test MFV by comparing $\mu \to e$ conversion with $\tau \to \mu/e\,\gamma$ transitions, without $\mu \to e\gamma$ !!!

# Conclusions

- The notion of MFV can be extended to the lepton sector.

  Working hypothesis, to investigate the nature/structure of LFV sources:

  - are $m_e$ and $m_\nu$ the only irreducible sources of LFV?
  - if not, is $M_R$ flavor blind?
  - ….

- Two scenarios emerge (with/without $\nu_R$). Phenomenology highlights:

  - normalization of rates depends on $\Lambda_{LN}/\Lambda$ ($\mu \to e$ observable if $\Lambda_{LN}/\Lambda > 10^{10}$)
  - pattern of predictions for ratios of LFV transitions $\mu \to e/\tau \to \mu$, … is governed by measured leptonic mass matrices and mixing angles

- Role of mu-to-e conversion:

  - probe details of underlying UV dynamics (strength of different operators)
  - through Z-dependence of rates, extract $\Delta_{\mu e}$ $\Rightarrow$ test MFV with $\tau \to \mu/e\ \gamma$

# Additional Material

# MLFV: minimal field content

- $G_{\rm LF} = SU(3)_{L_L} \times SU(3)_{E_R}$ broken only by $\lambda_e$ , $g_\nu$

$$\mathcal{L}_{\rm Sym.Br.} = -\lambda_e^{ij}\, \bar{e}_R^i (H^\dagger L_L^j) - \frac{1}{2\Lambda_{\rm LN}}\, g_\nu^{ij} (\bar{L}_L^{ci} \tau_2 H)(H^T \tau_2 L_L^j)$$

Formally invariant under $\begin{array}{l} L_L \to V_L\, L_L \\ e_R \to V_R\, e_R \end{array}$ if $\begin{array}{l} \lambda_e \to V_R\, \lambda_e V_L^\dagger \\ g_\nu \to V_L^*\, g_\nu V_L^\dagger \end{array}$

- Effective coupling governing $\ell_i \to \ell_j$ transitions: $\Delta_{\rm FC} \to V_L\, \Delta_{\rm FC}\, V_L^\dagger$

$$\Delta_{\rm FC} = g_\nu^\dagger\, g_\nu = \frac{\Lambda_{\rm LN}^2}{v^4}\, U_{\rm PMNS}\, \hat{m}_\nu^2\, U_{\rm PMNS}^\dagger \equiv \frac{\Lambda_{\rm LN}^2}{v^2}\, \hat{a}$$

Up to scale factor, link between
ν phenomenology and FCNC of charged leptons

# MLFV: extended field content

- $\bar{G}_{\rm LF} = SU(3)_{L_L} \times SU(3)_{E_R} \times O(3)_{\nu_R}$ broken only by $\lambda_e$ , $\lambda_\nu$

$$\mathcal{L}_{\rm Sym.Br.} = -\lambda_e^{ij}\, \bar{e}_R^i (H^\dagger L_L^j) + i\lambda_\nu^{ij} \bar{\nu}_R^i (H^T \tau_2 L_L^j) + {\rm h.c.}$$

Formally invariant under $\begin{array}{l} L_L \to V_L\, L_L \\ e_R \to V_R\, e_R \\ \nu_R \to O_\nu\, \nu_R \end{array}$ if $\begin{array}{l} \lambda_e \to V_R\, \lambda_e V_L^\dagger \\ \lambda_\nu \to O_\nu\, \lambda_\nu V_L^\dagger \end{array}$

- Effective coupling governing $\ell_i \to \ell_j$ transitions: $\Delta_{\rm FC} \to V_L\, \Delta_{\rm FC}\, V_L^\dagger$

$$\Delta_{\rm FC} = \lambda_\nu^\dagger \lambda_\nu = \frac{M_\nu}{v^2}\, U_{\rm PMNS}\, \hat{m}_\nu^{1/2}\, H^2\, \hat{m}_\nu^{1/2}\, U_{\rm PMNS}^\dagger \equiv \frac{M_\nu}{v}\, \hat{b}$$

$$\lambda_\nu = \frac{M_\nu^{1/2}}{v}\, H\, \hat{m}_\nu^{1/2}\, U_{\rm PMNS}^\dagger$$

Direct connection between FCNC and neutrino physics lost unless *H* = *I* (CP limit)

# Explicit form of LFV couplings:

1. Minimal field content: $g_\nu^\dagger g_\nu \equiv \frac{\Lambda_{\rm LN}^2}{v^2}\,\hat{a}$

$$
\begin{aligned}
a_{\mu e} &= \frac{1}{\sqrt{2}\,v^2}\left(s\,c\,\Delta m^2_{\rm sol} \pm s_{13}\,e^{i\delta}\,\Delta m^2_{\rm atm}\right)\\
a_{\tau e} &= \frac{1}{\sqrt{2}\,v^2}\left(-s\,c\,\Delta m^2_{\rm sol} \pm s_{13}\,e^{i\delta}\,\Delta m^2_{\rm atm}\right)\\
a_{\tau\mu} &= \frac{1}{2\,v^2}\left(-c^2\,\Delta m^2_{\rm sol} \pm \Delta m^2_{\rm atm}\right)
\end{aligned}
$$

$$
\begin{aligned}
+ &\leftrightarrow m_{\nu_1} < m_{\nu_2} \ll m_{\nu_3}\\
- &\leftrightarrow m_{\nu_3} \ll m_{\nu_1} < m_{\nu_2}
\end{aligned}
$$

Normal → inverted ⇔

$$\delta \longrightarrow \pi - \delta$$

2. Extended field content: $\lambda_\nu^\dagger \lambda_\nu \equiv \frac{M_\nu}{v}\,\hat{b}$

$$
\begin{aligned}
b_{\mu e} &= \frac{1}{\sqrt{2}\,v}\left[s\,c\,(m_{\nu_2} - m_{\nu_1}) \pm s_{13}\,(m_{\nu_3} - m_{\nu_1})\right]\\
b_{\tau e} &= \frac{1}{\sqrt{2}\,v}\left[-s\,c\,(m_{\nu_2} - m_{\nu_1}) \pm s_{13}\,(m_{\nu_3} - m_{\nu_1})\right]\\
b_{\tau\mu} &= \frac{1}{2\,v}\left[-c^2\,(m_{\nu_2} - m_{\nu_1}) + (m_{\nu_3} - m_{\nu_1})\right]
\end{aligned}
$$

$$+ \longrightarrow \delta = 0$$

$$- \longrightarrow \delta = \pi$$

(~ CP limit)

# Alternative handles on LL-quark operators?

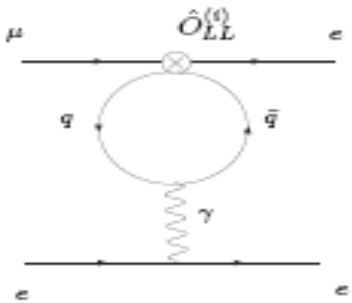
- Hadronic decays are predicted well below expt. sensitivities

$$\mathrm{Br}(\tau \to \mu\pi^0) = 6\pi^2 \frac{f_\pi^2}{m_\tau^2} \left(\frac{\Lambda_{\mathrm{LN}}}{\Lambda}\right)^4 |a_{\mu\tau}|^2 |\tilde{C}_{LL}|^2 \longrightarrow 10^{-15}$$

$C_{LL}$~1 ,
$\Lambda_{LN}/\Lambda$ ~$10^9$

$$\frac{\Gamma(\Upsilon \to \tau\mu)}{\Gamma(\Upsilon \to ee)} = \frac{1}{e^2}\left(\frac{m_\Upsilon}{2v}\right)^4 \left(\frac{\Lambda_{\mathrm{LN}}}{\Lambda}\right)^4 |a_{\mu\tau}|^2 |\tilde{C}'_{LL}|^2 \longrightarrow 10^{-20}$$

- $\mu \to 3e$ via loop effects:
  [relevant only if $c_{4L}(\Lambda) < c_{LL}(\Lambda)$]




$$\mathrm{Br}_{\mu\to 3e} = 6\cdot 10^{-3} \times \left|c_{LL}^{(5)}(\Lambda)\right|^2 \times \begin{cases} \left(\frac{\Lambda_{\mathrm{LN}}}{\Lambda}\right)^4 |a_{e\mu}|^2 & [\text{minimal}] \\ \left(\frac{vM_\mu}{\Lambda^2}\right)^2 |b_{e\mu}|^2 & [\text{extended}] \end{cases}$$

>> $(\alpha/\pi)^2$ due to large logs: $\log\frac{\Lambda}{M_Z}$ $\log\frac{M_Z}{1\mathrm{GeV}}$

# BR formulae

$$B_{\mu\to e\gamma} = 384\pi^2 |A_R|^2$$

$$B_{\mu\to e} = \frac{2G_F^2 m_\mu^5}{\Gamma_{\rm capt}} |A_R^* D + \tilde{g}_{LV}^{(p)} V^{(p)} + \tilde{g}_{LV}^{(n)} V^{(n)}|^2$$

Overlap integrals

$$\tilde{g}_{LV}^{(p)} = -\frac{4v^2}{\Lambda_{\rm LFV}^2} \Delta_{\mu e}^* \left[ (\frac{1}{4} - s_w^2)(c_{LL}^{(1)} + c_{LL}^{(2)}) + \frac{3}{2} c_{LL}^{(3)} + c_{LL}^{(4u)} + \frac{1}{2} c_{LL}^{(4d)} - \frac{1}{2} c_{LL}^{(5)} \right]$$

$$\tilde{g}_{LV}^{(n)} = -\frac{4v^2}{\Lambda_{\rm LFV}^2} \Delta_{\mu e}^* \left[ -\frac{1}{4}(c_{LL}^{(1)} + c_{LL}^{(2)}) + \frac{3}{2} c_{LL}^{(3)} + \frac{1}{2} c_{LL}^{(4u)} + c_{LL}^{(4d)} + \frac{1}{2} c_{LL}^{(5)} \right]$$

$$A_R = \frac{ev^2}{\Lambda_{\rm LFV}^2} \Delta_{\mu e} \left[ -c_{RL}^{(1)} + c_{RL}^{(2)} \right]$$

# MFV in Grand Unified Theories ?

- Truly MFV cannot be realized in Grand Unified Theories.

  - Quarks and leptons belong to same gauge multiplets

  - If $\Lambda_{FB} > \Lambda_{GUT}$, rad. corr. $[\Lambda_{FB} > E > \Lambda_{GUT}]$ induce cross-talk between quark & lepton flavor-breaking structures (related to mass matrices)

  - New mixing matrices appear (fewer indep. flavor rotations are possible)

- We looked at $SU(5)_{gauge}$ with following assignments:

$$\begin{array}{lcl} \psi[\bar{5}] & \supset & L_L\,,\, d_R^c \\ \chi[10] & \supset & Q_L\,,\, u_R^c\,,\, e_R^c \\ N[1] & \supset & \nu_R \end{array} \longrightarrow \boxed{G_F^{\max} = U(3)_{\bar{5}} \times U(3)_{10} \times U(3)_1}$$

# Interesting implications for $\ell_i \to \ell_j \gamma$

- Strength of leptonic FCNC is governed by two effective operators:

$$\frac{v}{\Lambda^2}\, \bar{e}_R^i \left(\lambda_e \lambda_\nu^\dagger \lambda_\nu\right)^{ij} \sigma^{\mu\nu} e_L^j \; F_{\mu\nu}$$

→ PMNS mixing pattern $M_\nu > 10^{12}$ GeV

$$\frac{v}{\Lambda^2}\, \bar{e}_R^i \left(\lambda_U \lambda_U^\dagger \lambda_D^T\right)^{ij} \sigma^{\mu\nu} e_L^j \; F_{\mu\nu}$$

→ CKM mixing pattern $M_\nu < 10^{12}$ GeV

[~ Barbieri-Hall-Strumia '95]

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- Normalization: cannot be suppressed by lowering $M_\nu < 10^{12}$ GeV. GUT induced term (controlled by top Yukawa and CKM) sets in !!

$$\Lambda < 10\,\text{TeV} \longrightarrow \begin{cases} B(\mu \to e\gamma) > 10^{-13} \\ B_{Al,Au}(\mu \to e) > 5\times 10^{-16} \end{cases}$$

within reach of next generation expts.

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- Pattern of BRs:

$$B(\tau \to \mu\gamma) : B(\tau \to e\gamma) : B(\mu \to e\gamma) \longrightarrow \begin{cases} \text{Min}\left[s_{13}^{-2}\,,\ \dfrac{\Delta m^2_{\text{atm}}}{\Delta m^2_{\text{sol}}}\right] : 1 : 1 \sim 10 - 100 : 1 : 1 \\ \lambda_C^{-6} : \lambda_C^{-4} : 1 \sim 10^4 : 500 : 1 \end{cases}$$

If GUT-induced term dominates, $\tau \to \mu\gamma$ is within reach of super-B factories

# Conclusions on MFV+GUT

- MFV can be married with GUTs, but result is NOT MFV(q) + MFV($\ell$)
  - "guaranteed" signal in $\mu \to e$ transitions if $\Lambda$ < 10 TeV
  - $\tau \to \mu, e$ transitions very useful to discriminate minimal vs GUT scenario