Muon Physics in Anarchic RS Models

2nd Project-X Physics Workshop

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Based on work with
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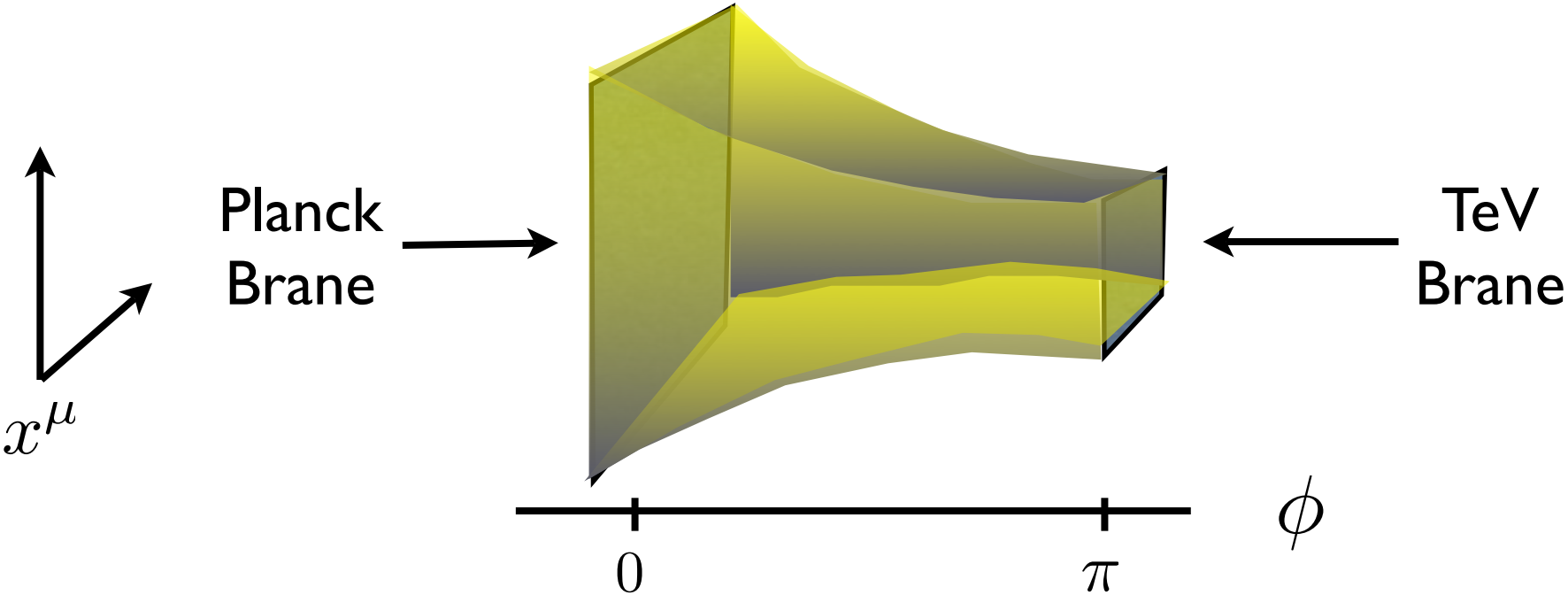
Outline

- Review of RS-1
- Review of the charged lepton flavor structure
- Parameter space of "Flavor-Anarchic RS"
- Study of CLFV Probes

Review of RS-1



$$ds^2 = e^{-2kr_c\phi}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2$$

Solves the Hierarchy Problem:

$$M_4(\phi) = M_5 e^{-kr_c\phi} \qquad kr_c \sim \mathcal{O}(10)$$

So on the "TeV Brane"

$$M_4(\pi) = M_5 e^{-kr_c\pi} \sim \mathrm{TeV}$$

Review of RS-1

- Original RS-1 Model had SM living on the "Planck Brane" and the Higgs boson living on the "TeV Brane."
- This does not explain flavor hierarchy, nor can it explain suppression of higher-dimension operators:

FCNC Operators:

$$\frac{1}{\Lambda_{\rm FCNC}^2}(\bar{s}d)(\bar{s}d) \Rightarrow \Lambda_{\rm FCNC} \sim 10^3 {\rm TeV}$$

EW Precision Operators:

$$\frac{1}{\Lambda_{\rm EWP}^2} F_{\mu\nu} W^{\mu\nu} H^\dagger H \Rightarrow \Lambda_{\rm EWP} \sim 5 - 10 {\rm TeV}$$

Possible Fix: SM in Bulk

Only the Higgs field needs to live on the TeV brane to solve the hierarchy problem, so put the rest of the SM fields in the bulk.

- Solves Flavor Hierarchy (Gerghetta-Pomarol)
- Fixes FCNCs (Huber-Shafi)
- Fixes EWP, but also generates new operators from KK-exchange, giving a bound: $M_{KK} > 5\text{-}10$ TeV (Hewett-Petriello-Rizzo)

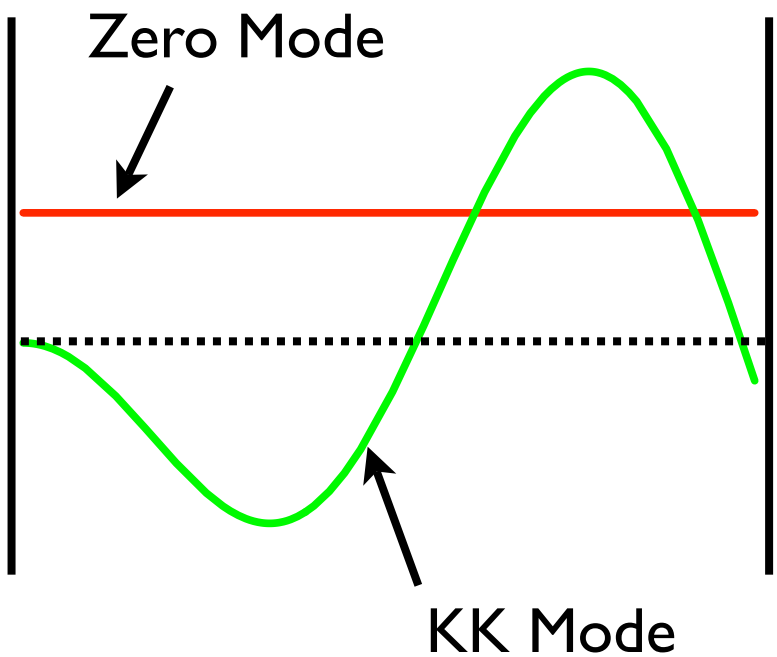
Possible Fix: Custodial Isospin in the Bulk (Agashe-Delgado-May-Sundrum).

This protects T-parameter and lowers bounds on $M_{KK} > 3$ TeV.

KK Analysis

Gauge Bosons:

$$A_\mu(x,\phi) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x) \frac{1}{\sqrt{r_c}} \chi_A^{(n)}(\phi)$$



KK Analysis

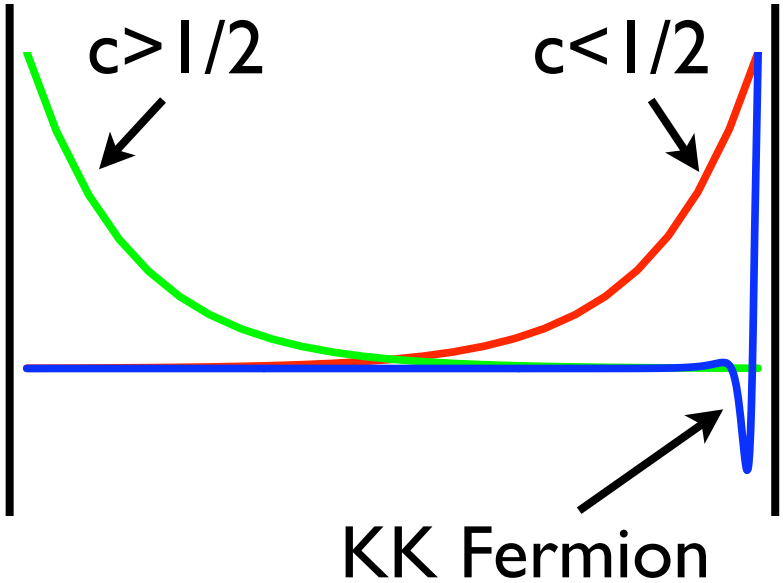
Fermions:

$$S = \int d^4x d\phi \sqrt{-G} \left[E_a^M \bar{\Psi} \gamma^a D_M \Psi - \mathrm{sgn}(\phi) kc \bar{\Psi} \Psi \right]$$

$$\Psi \equiv \Psi_L + \Psi_R$$

$$\Psi_{L,R}(x,\phi) = \sum_{n=0}^{\infty} \psi_{L,R}^{(n)}(x) \frac{e^{2kr_c|\phi|}}{\sqrt{r_c}} f_{L,R}^{(n)}(\phi; c)$$

All of the leptons will have $c_L = c_R > 1/2$, putting their zero-modes close to the Planck brane.



Flavor Violation from KK-Z

KK Gauge Bosons destroy (accidental) flavor symmetry of the Standard Model:

$$g_{L,R} Z^{(1)} \left(\bar{e}_F, \bar{\mu}_F, \bar{\tau}_F\right)_{L,R} \begin{pmatrix} \alpha_e^{L,R} & 0 & 0 \\ 0 & \alpha_\mu^{L,R} & 0 \\ 0 & 0 & \alpha_\tau^{L,R} \end{pmatrix} \begin{pmatrix} e_F \\ \mu_F \\ \tau_F \end{pmatrix}_{L,R}$$

$$\alpha_i^L = \int_0^\pi d\phi \chi^{(1)} [f_{L,i}^{(0)}]^2$$

We wish to go to the "Mass" Basis. We can do this with a Unitary transformation:

$$\begin{pmatrix} e_F \\ \mu_F \\ \tau_F \end{pmatrix}_{L,R} = U_{L,R}^\dagger \begin{pmatrix} e_{\rm ph} \\ \mu_{\rm ph} \\ \tau_{\rm ph} \end{pmatrix}_{L,R}$$

Thus there will be off-diagonal couplings with the $Z^{(1)}$ and the "physical" (mass-eigenstate) leptons.

"Anarchic-RS" Assumption

"Anarchic RS" has no flavor structure in the 5D:

$$Y_5 = \begin{pmatrix} Y_e & \lambda \\ \lambda & Y_\mu \end{pmatrix} \sim \mathcal{O}(1) \quad \Rightarrow \quad Y_4 = \begin{pmatrix} Y_e f_e^{(0)2} & \lambda f_e^{(0)} f_\mu^{(0)} \\ \lambda f_e^{(0)} f_\mu^{(0)} & Y_\mu f_\mu^{(0)2} \end{pmatrix}$$

Y_4 has Eigenvalues $\sim Y_e f_e^{(0)2}, \quad Y_\mu f_\mu^{(0)2}$ and is ***almost*** diagonalized by:

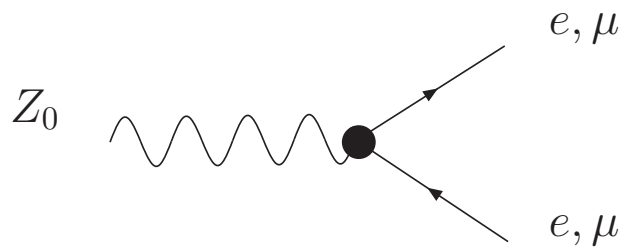
$$U \sim \begin{pmatrix} 1 & \sqrt{\frac{m_e}{m_\mu}} \\ -\sqrt{\frac{m_e}{m_\mu}} & 1 \end{pmatrix}$$

Therefore, we can choose "canonical values" for the mixing angles:

$$\boxed{\begin{aligned} U_{12}^{L,R} &\sim \sqrt{\frac{m_e}{m_\mu}} \\ U_{13}^{L,R} &\sim \sqrt{\frac{m_e}{m_\tau}} \\ U_{23}^{L,R} &\sim \sqrt{\frac{m_\mu}{m_\tau}} \end{aligned}}$$

Note that this is PURELY a consequence of the Anarchic RS assumption!!

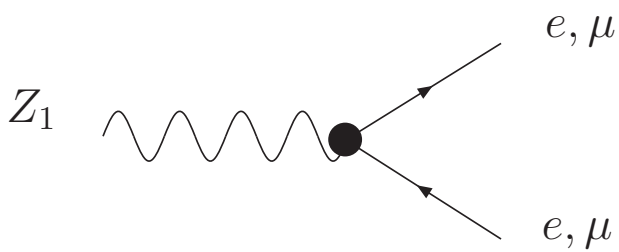
Z-Lepton Couplings



$$= \frac{g_{L,R}}{\cos\theta_W}$$

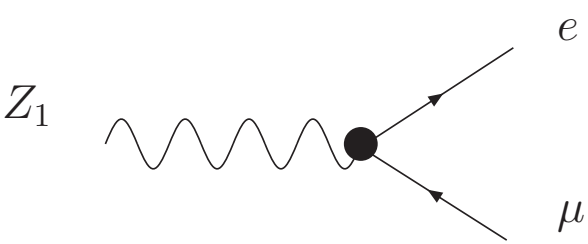
$$Z_0 = Z^{(0)} - f\frac{m_Z^2}{m_{Z'}^2}Z^{(1)}$$

$$Z_1 = Z^{(1)} + f\frac{m_Z^2}{m_{Z'}^2}Z^{(0)}$$

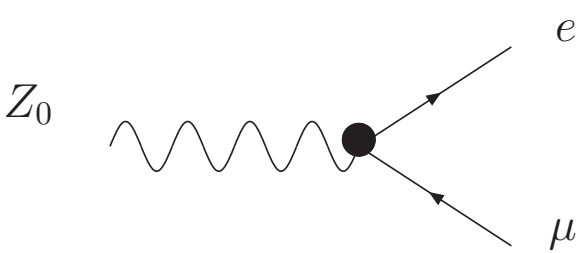


$$= \frac{g_{L,R}}{\cos\theta_W}\alpha_{e,\mu}$$

$$f = \sqrt{2\pi k r_c} \sim \mathcal{O}(10)$$



$$= \frac{g_{L,R}}{\cos\theta_W}g_{L,R}^{(1)\mu e}$$



$$= \frac{g_{L,R}}{\cos\theta_W}g_{L,R}^{(1)\mu e}\left(-f\frac{m_Z^2}{m_{Z'}^2}\right)$$

$$g_L^{(1)\mu e} \approx U_{12}^L U_{22}^{L*}(\alpha_\mu - \alpha_e) + U_{13}^L U_{23}^{L*}(\alpha_\tau - \alpha_e)$$

Naive Estimates

We can estimate the size of the FVC by plugging in our "canonical values" for the parameters:

$$U_{ii} = 1 \quad U_{ij} = -U_{ji} = \sqrt{\frac{m_i}{m_j}} \; (i < j)$$

$Y_5 = 2$

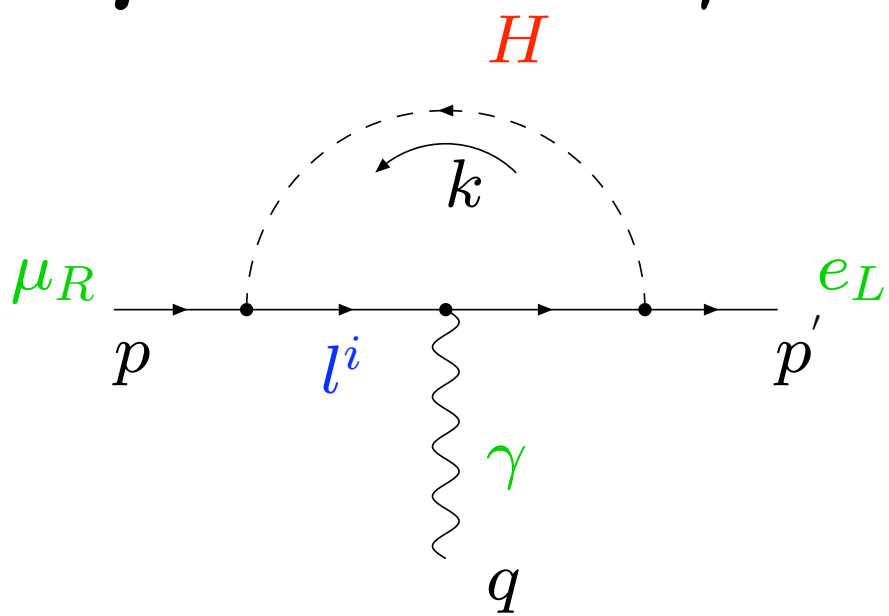
No relative phases

$$\left.\begin{aligned} \alpha_\mu - \alpha_e &\sim 0 \\ \alpha_\tau - \alpha_\mu &\sim 0.04 \\ \alpha_\tau - \alpha_e &\sim 0.04 \end{aligned}\right\} \quad \begin{aligned} g^{(1)\mu e} &\sim 10^{-4} \\ g^{(1)\tau\mu} &\sim 10^{-2} \\ g^{(1)\tau e} &\sim 10^{-3} \end{aligned}$$

$\mu \to 3e$	$\Rightarrow$	$m_{Z'} > 2.5$ TeV	
$\mu \to e$ Conv.	$\Rightarrow$	$m_{Z'} > 5.9$ TeV	(Brane Higgs)
$\mu \to e$ Conv.	$\Rightarrow$	$m_{Z'} > 4.7$ TeV	(Bulk Higgs)

Tau decays are much weaker: $m_{Z'}$ > few hundred GeV

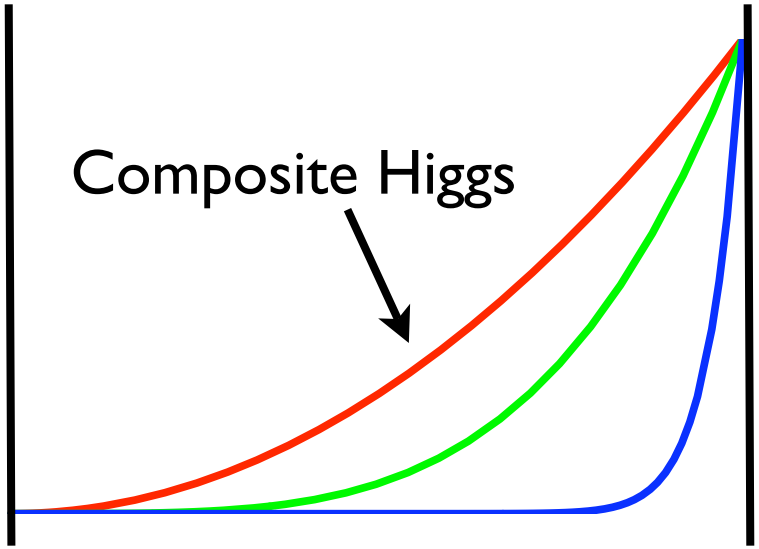
$\mu \longrightarrow e\gamma$



- Sensitive to the Higgs, and therefore makes a good compliment to mu-e conversion.
- Brane-Higgs is naively suppressed, but is also sensitive to cutoff effects; so we only consider Bulk-Higgs.
- We model a generic Bulk-Higgs using the formalism presented in Davoudiasl-Lillie-Rizzo.

Tension Between Yukawa vs Gauge FVC

Bulk Higgs Profiles:



$$Y_4 \sim Y_5 \int d\phi \left[f_l^{(0)}(\phi)\right]^2 \chi_H^{(0)}(\phi) \quad \Rightarrow \quad Y_5 \sim \frac{1}{\left[f_l^{(0)}(\pi)\right]^2}$$

$$\alpha \sim \int d\phi \left[f_l^{(0)}(\phi)\right]^2 \chi_Z^{(1)}(\phi) \quad \Rightarrow \quad \alpha \sim \left[f_l^{(0)}(\pi)\right]^2$$

Therefore: $\boxed{\alpha \sim \frac{1}{Y_5}}$

Rule of thumb.
***NOT** an equality!!*

$\mu \to e\gamma$

Bulk Higgs introduces new parameter, parametrizing where the Higgs must live:

$\nu = 0$ corresponds (*almost*) to the "Composite Higgs" Model of Agashe-Contino-Pomarol

$\nu \to \infty$ corresponds to Brane-Higgs

For Composite Higgs:

$$\left.\begin{array}{lcl} \mu - e \text{ Conv} & \Rightarrow & m_{Z'} > 6.7 \text{ TeV} \\ \mu \to e\gamma & \Rightarrow & m_{Z'} > 6.6 \text{ TeV} \end{array}\right\} Y_5 = 1$$

$$\left.\begin{array}{lcl} \mu - e \text{ Conv} & \Rightarrow & m_{Z'} > 4.7 \text{ TeV} \\ \mu \to e\gamma & \Rightarrow & m_{Z'} > 13 \text{ TeV} \end{array}\right\} Y_5 = 2$$

Parameter Space

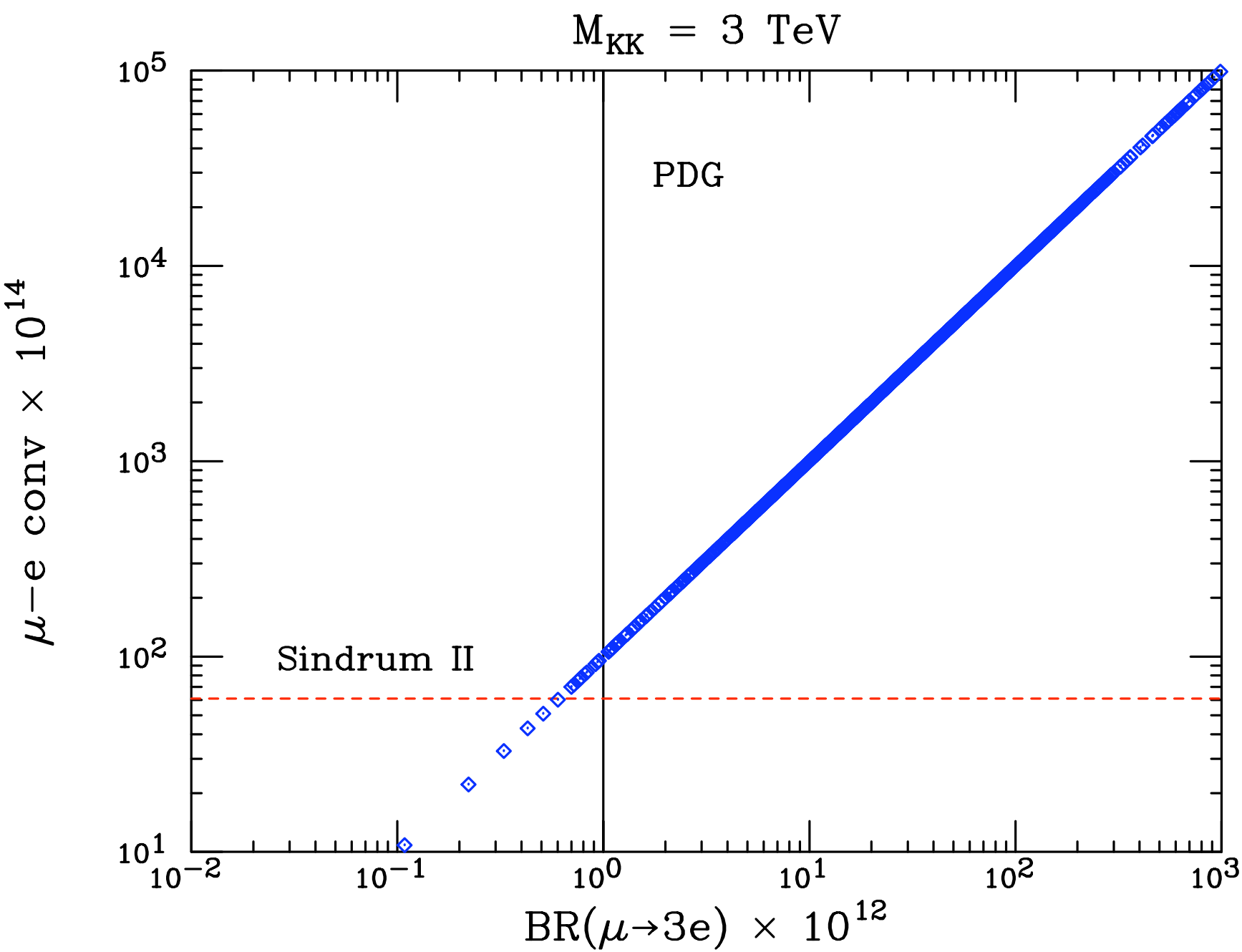
- Three 5D Yukawas
- 6 Mixing Angles + 6 Phases $\left(U_{ij}^{L,R} = \beta_{ij}^{L,R}\sqrt{\frac{m_i}{m_j}}\right)$
- M_{KK}

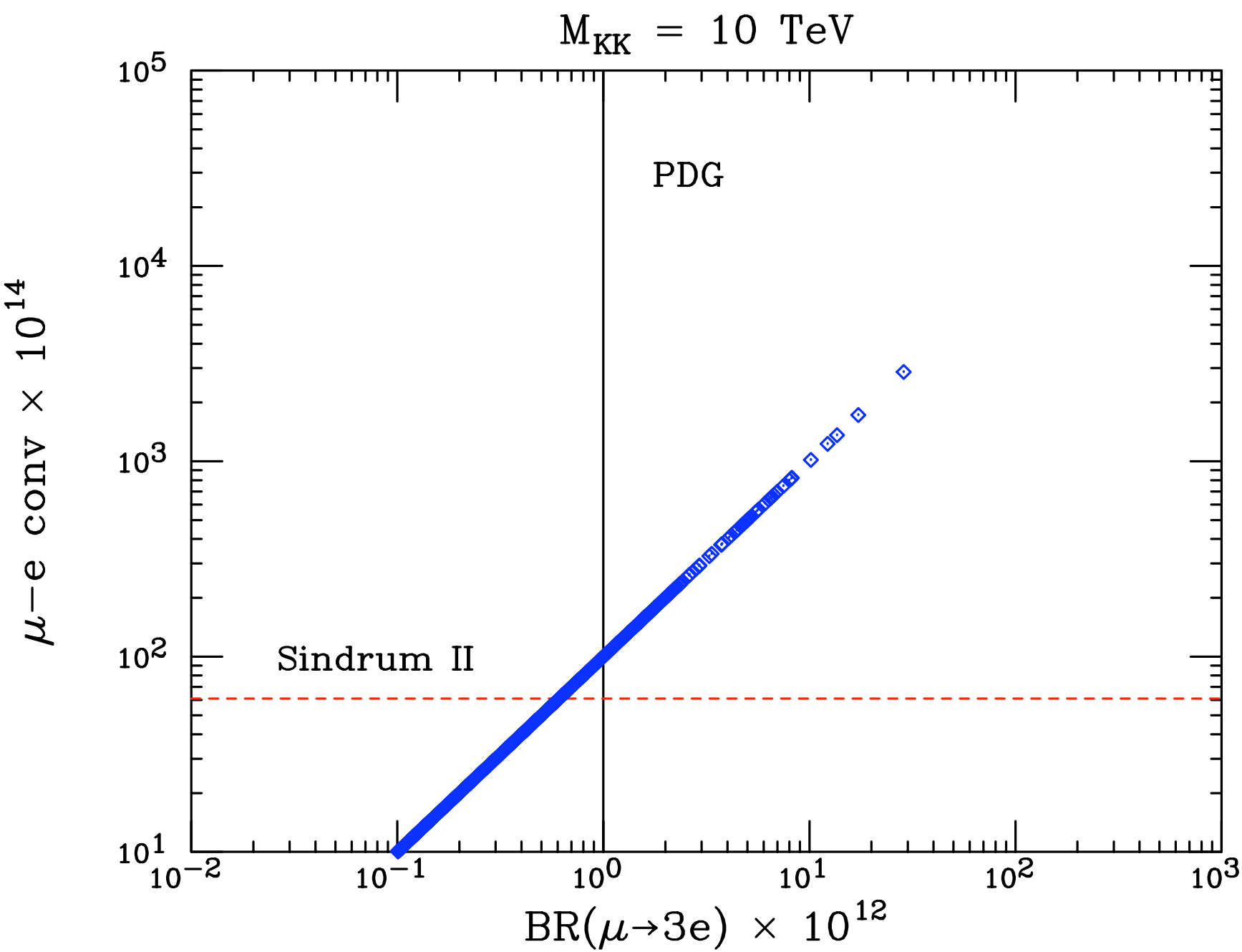
Ranges:

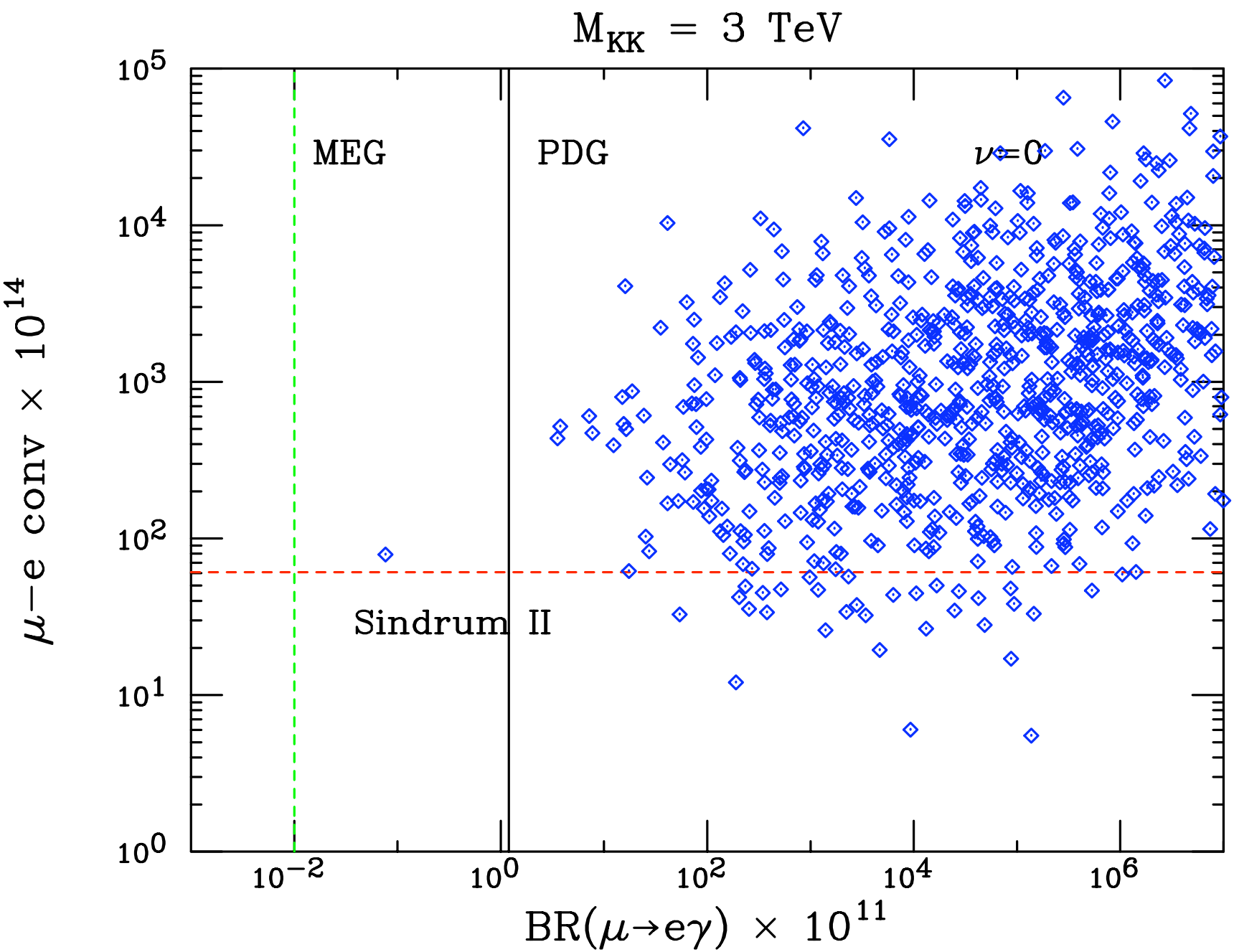
$$\frac{1}{4} \leq Y_i \leq 4$$

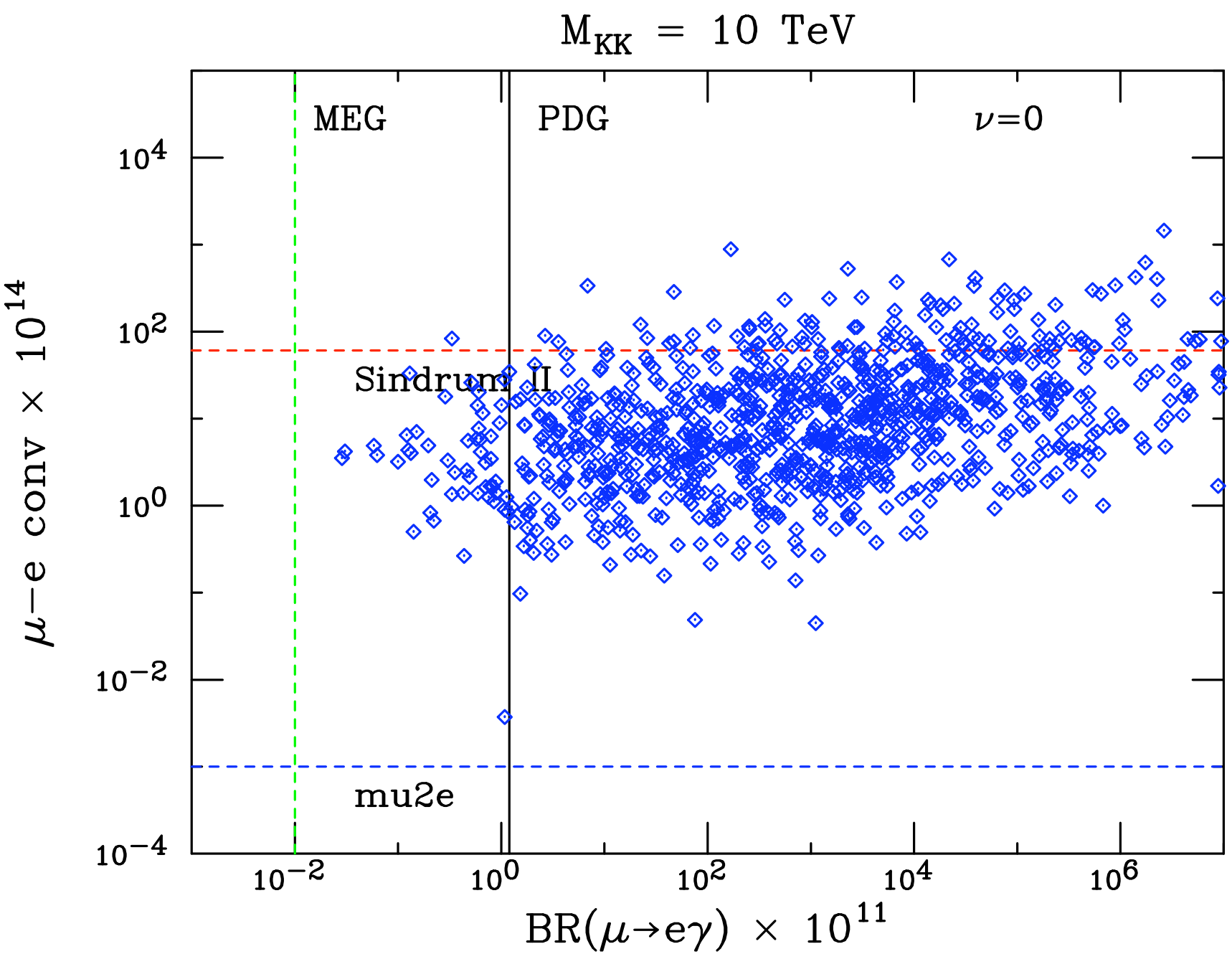
$$\frac{1}{4} \leq |\beta_{ij}^{L,R}| \leq 4$$

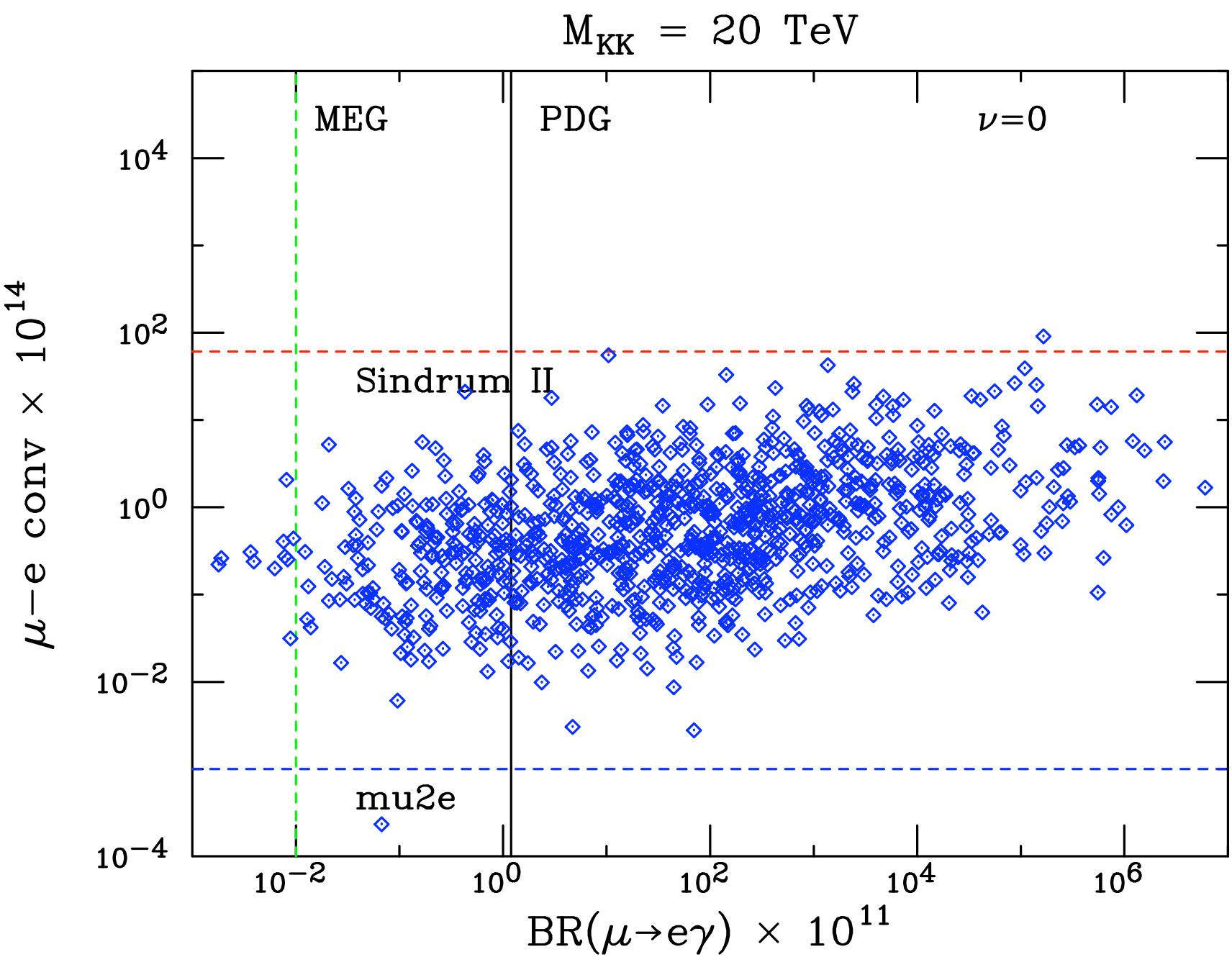
Phases scanned on $[0, 2\pi]$

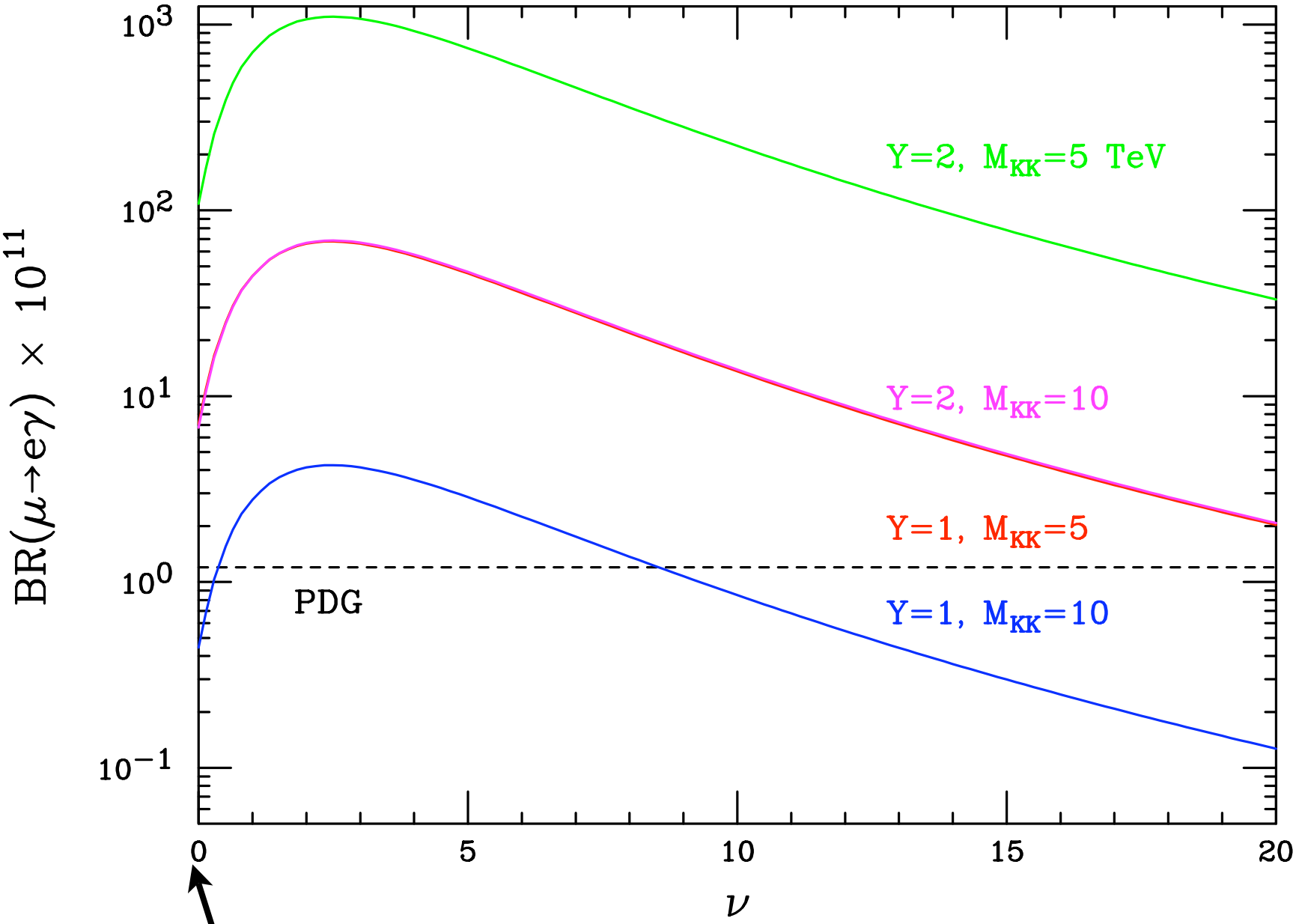

M_{KK} = 3 TeV
PDG
Sindrum II
μ−e conv × 10^{14}
BR(μ→3e) × 10^{12}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}


M_{KK} = 10 TeV
PDG
Sindrum II
μ−e conv × 10^{14}
BR(μ→3e) × 10^{12}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

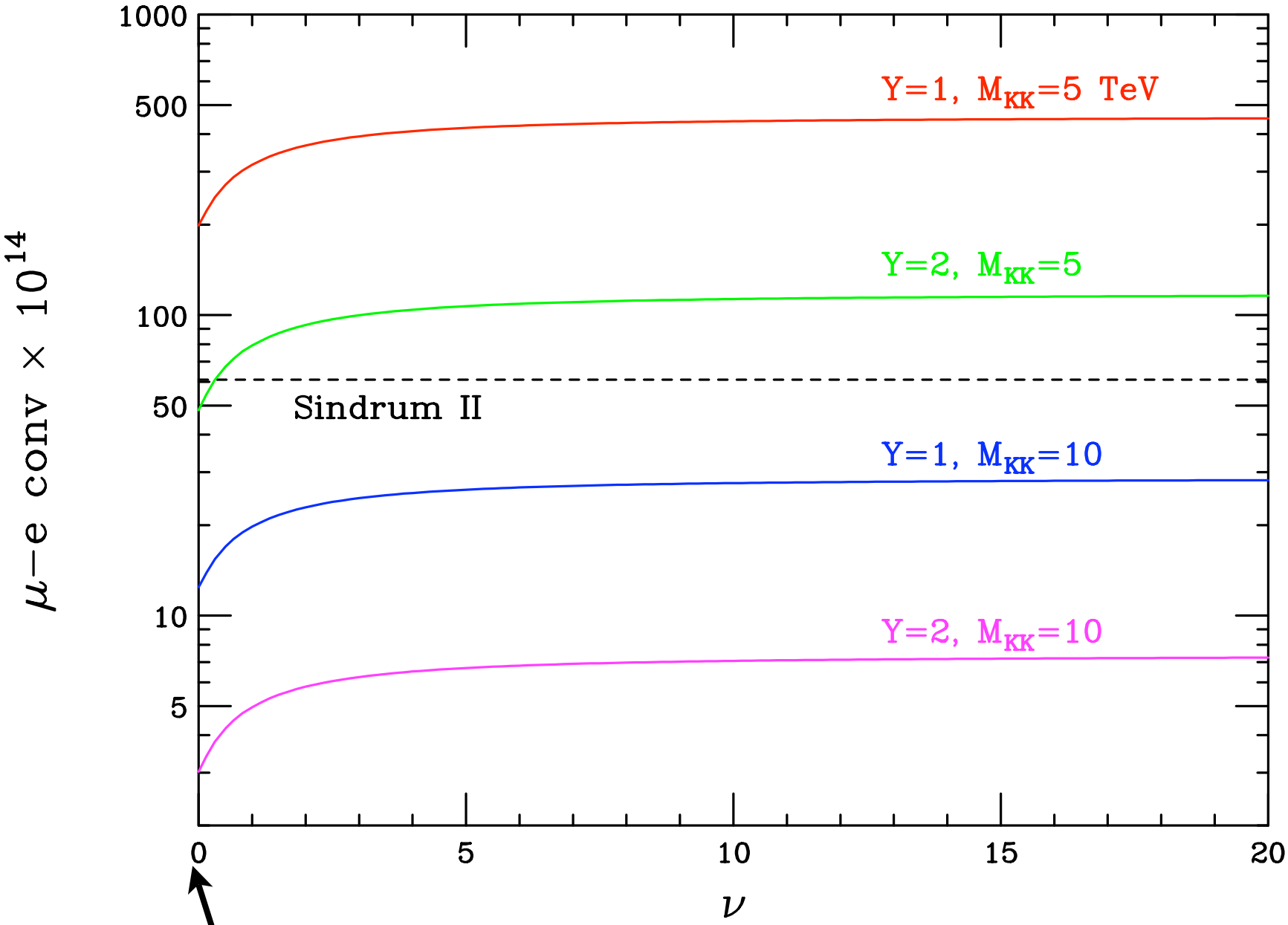

M_{KK} = 3 TeV
MEG
PDG
ν=0
Sindrum II
μ−e conv × 10^{14}
BR($\mu\to e\gamma$) × 10^{11}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}


M_{KK} = 10 TeV
MEG
PDG
ν=0
Sindrum II
mu2e
10^4
10^2
10^0
10^{-2}
10^{-4}
μ−e conv × 10^{14}
10^{-2}
10^0
10^2
10^4
10^6
BR($\mu\to e\gamma$) × 10^{11}


M_{KK} = 20 TeV
MEG
PDG
ν=0
Sindrum II
mu2e
10^4
10^2
10^0
10^{-2}
10^{-4}
$\mu-e$ conv $\times 10^{14}$
10^{-2}
10^0
10^2
10^4
10^6
BR($\mu\to e\gamma$) $\times 10^{11}$



Composite Higgs



Composite Higgs

Conclusions

- RS-1 with the SM in the bulk and low M_{KK} has much interesting phenomenology for the upcoming experiments
- "Anarchic RS" - Low-energy solution to the flavor hierarchy!
- Strongly constrained by CLFV experiments on both sides.
- Between these experiments and the LHC, it is a very exciting time!

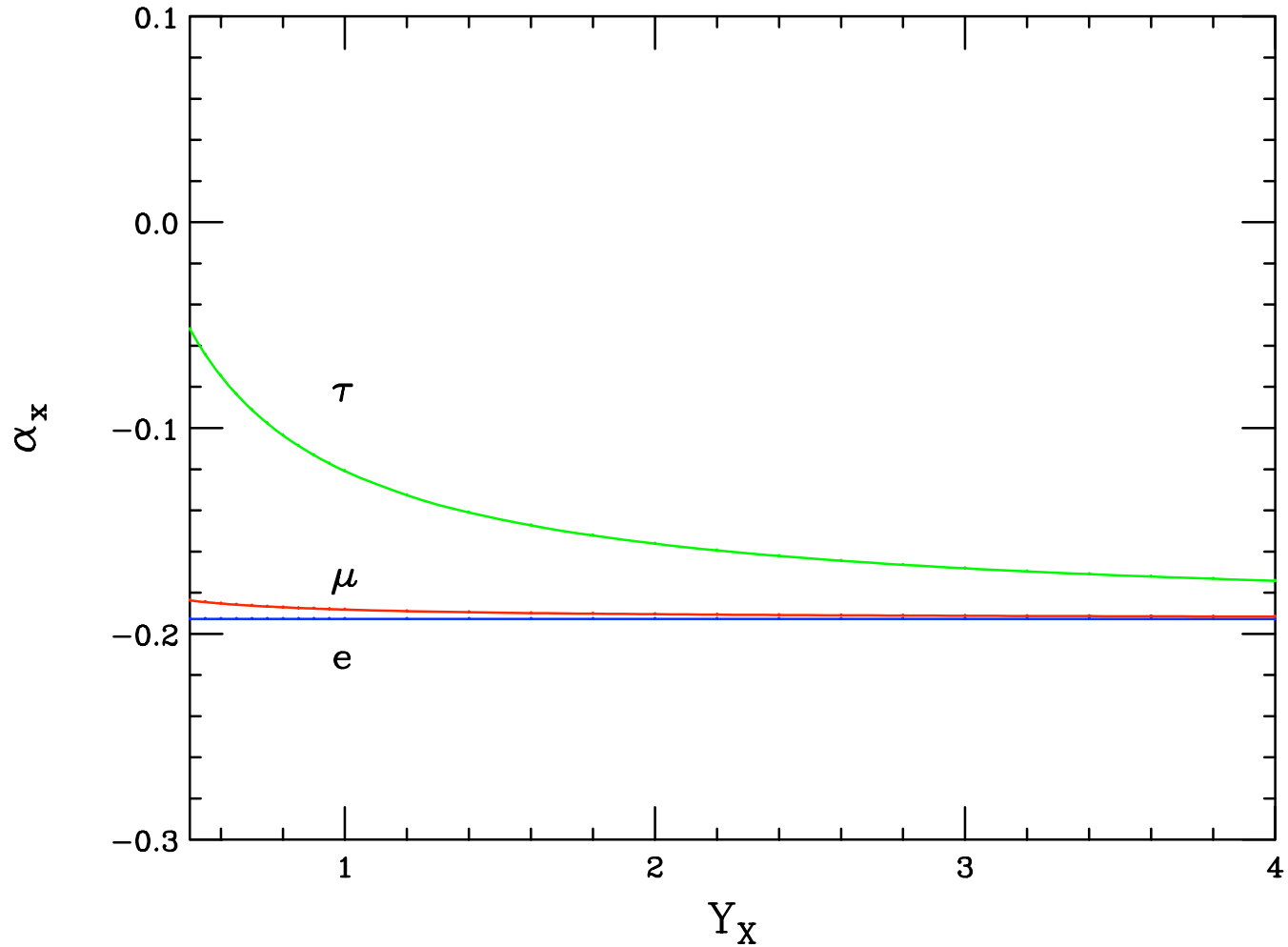
Slides I don’t have time for!

Behavior of FVC

Recall: $g_{L,R}^{(1)\mu e}$ is related to the difference of gauge couplings α_i

There is ***no*** choice of Y_5 such that $g_{L,R}^{(1)\mu e} = 0$

Flavor violation is always present!



(t,b) sector suggests $Y_5 \sim 2$ is the most "natural" choice.

Matching Calculation

We can write an effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}\left[g_3(\bar{e}_R\gamma^\beta\mu_R)(\bar{e}_R\gamma_\beta e_R) + \cdots\right]$$

Tree-level KK Exchange

$$+\frac{1}{2m_\mu}\left[C_R\bar{e}_R\sigma^{\rho\nu}F_{\rho\nu}\mu_L + \cdots\right]$$

Dipole Operators (loop)

Example of Matching:

$$g_3 = -2g_R\frac{m_Z^2}{m_{Z'}^2}(f - \alpha_e)g_R^{(1)\mu e}$$

Z_0 Z_1

From this we see that Z_0-exchange is more important than Z_1-exchange.

Possible Extensions

1. Shifting $SU(2)_R$ Q.N. of the fermions and readjusting c_L and c_R can kill off mu-e conversion but not $\mu \to e\gamma$
2. Our treatment of the "Composite Higgs" model was a little too casual. A more careful analysis suggests mu-e conversion is unchanged, but a suppression for $\mu \to e\gamma$ due to an additional flavor-alignment.