Simple Representation for Combining

$$P(\nu_\mu \to \nu_e), \quad P(\bar{\nu}_\mu \to \bar{\nu}_e),$$

with the result from a reactor neutrino experiment

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LBNL

Disclaimer

- Perhaps nothing new here. Literature search consisted of dinner with Boris, Gary, and Mark Messier.
- Don't believe the precise numbers. I haven't had time to check anything.
- I am simply proposing a way to present the determination of $\sin^2 2\theta_{13}$ and δ.
- Original motivation was to generate a problem for the 2nd edition of my book with Gerson Goldhaber, in which the neutrino oscillation chapter will be the longest.

Simplest case: no matter effect

- Assume known:

$$\sin^2 2\theta_{23} = 1, \quad \sin^2 2\theta_{12} = 0.87$$
$$|\Delta m^2_{31}| = 2.5 \times 10^{-3}\,\mathrm{eV}^2, \quad \Delta m^2_{21} = 8 \times 10^{-5}\,\mathrm{eV}^2$$

- Unknown: hierarchy, $\sin^2 2\theta_{13}, \delta$

Choice of co-ordinates

Fix L=810 km, E=2 GeV.

$$\Delta_{ij} = \Delta m^2_{ij} L/(4E)$$

$$\begin{aligned} P(\nu_\mu \to \nu_e) = & \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta_{31} \\ & + \Delta_{21} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \Delta_{31} \cos(\Delta_{31} + \delta) \\ & + \Delta_{21}^2 \cos^2\theta_{23} \sin^2 2\theta_{12} \end{aligned} \quad (1)$$

Polar co-ordinates: $r = \sin 2\theta_{13}, \quad \phi = \delta$

Cartesian co-ordinates: $x = r\cos\phi; \quad y = r\sin\phi$

A Circle

$$A(x^2 + y^2) + B(x\cos\Delta_{31} - y\sin\Delta_{31}) + C = P_\nu$$

with

$$\begin{aligned} A &= \sin^2\theta_{23}\,\sin^2\Delta_{31} \\ B &= \Delta_{21}\sin 2\theta_{12}\,\sin 2\theta_{23}\sin\Delta_{31} \\ C &= \Delta_{21}^2\cos^2\theta_{23}\sin^2 2\theta_{12} \end{aligned}$$

whose center is at

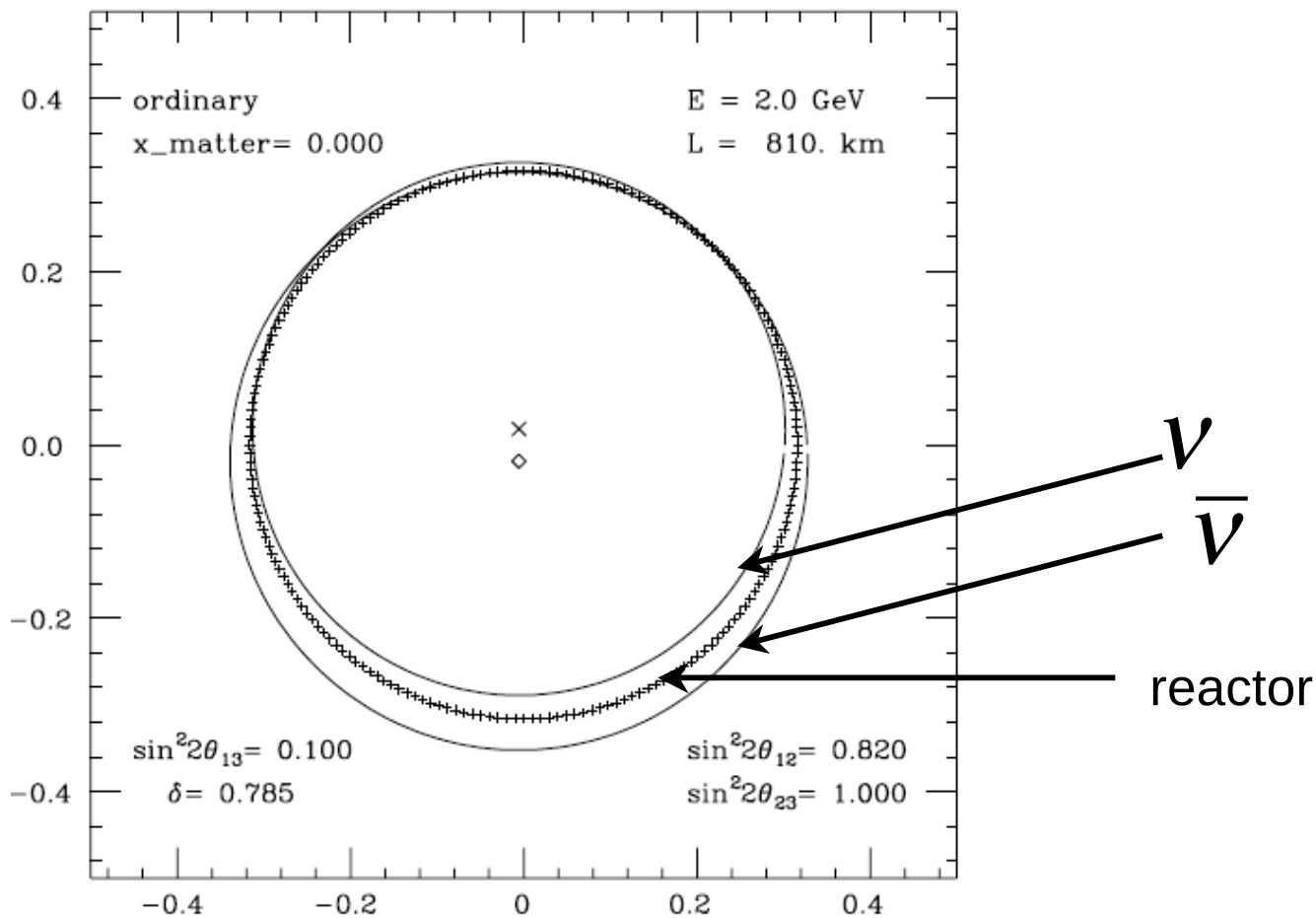
$$(x_{center}, y_{center}) = (-\frac{B}{2A}\cos\Delta_{31}, \frac{B}{2A}\sin\Delta_{31})$$

with radius squared

$$r^2 = \frac{P_\nu - C}{A} - \frac{B^2}{4A^2}$$

For antineutrino, $\delta \to -\delta$

NOvA in Vacuum!



Including matter and hierarchy

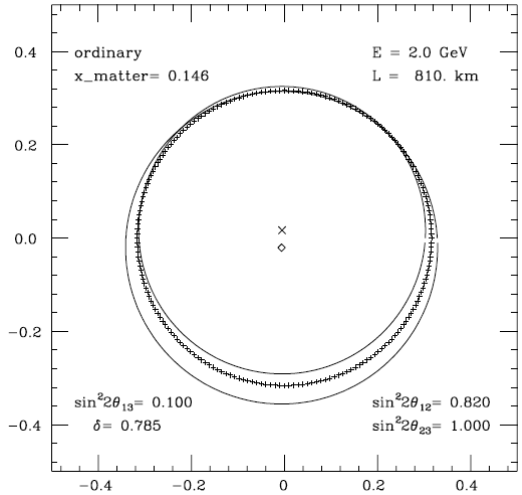
When matter effect is included, the result is

$$P(\nu_\mu \to \nu_e) = \sin^2\theta_{23}\,\sin^2 2\theta_{13}\,\frac{\sin^2(1-x)\Delta_{31}}{(1-x)^2}$$
$$+\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\sin 2\theta_{13}\sin 2\theta_{12}\,\sin 2\theta_{23}\frac{\sin[(1-x)\Delta_{31}]}{1-x}\frac{\sin x\Delta_{31}}{x}\cos(\Delta_{31}+\delta)$$
$$+\left(\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right)^2\cos^2\theta_{23}\sin^2 2\theta_{12}\frac{\sin^2(x\Delta_{31})}{x^2} \qquad (9)$$

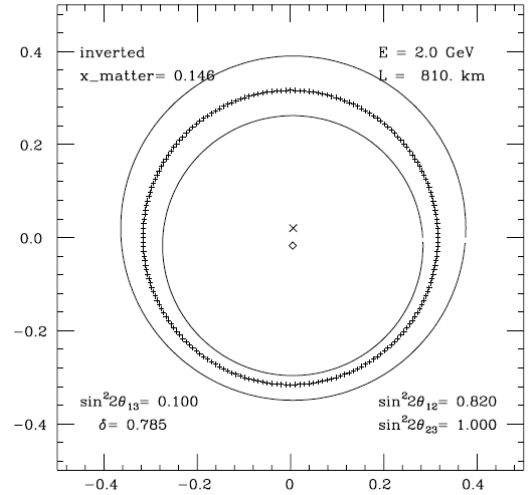
where $x = 2\sqrt{2}G_F N_e E/\Delta m^2_{31}$ and where non-leading terms in $\Delta m^2_{21}/\Delta m^2_{31}$ and θ_{13} have been neglected.

Matter and Hierarchy

- For antineutrinos
  - $\delta \to -\delta$
  - $x \to -x$
- For inverted hierarchy
  - $m_{13}^2 \to -m_{13}^2$
  - $\Delta_{31} \to -\Delta_{31}$
  - $x \to -x$
- Without matter effect, if $(\Delta m_{31}^2, \delta)$ is solution so is $(-\Delta m_{31}^2, \pi - \delta)$

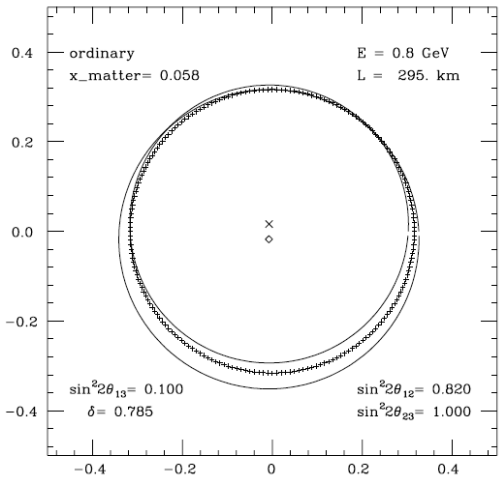


NOvA

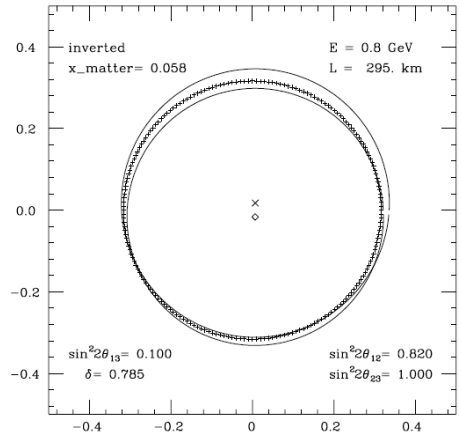


normal

inverted



T2K

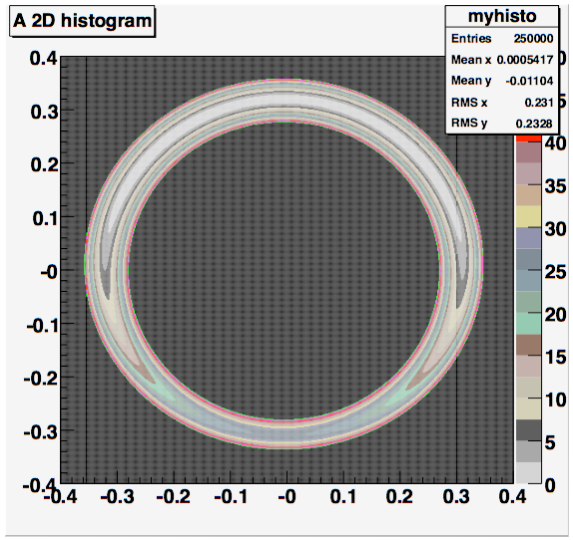


χ^2 expected

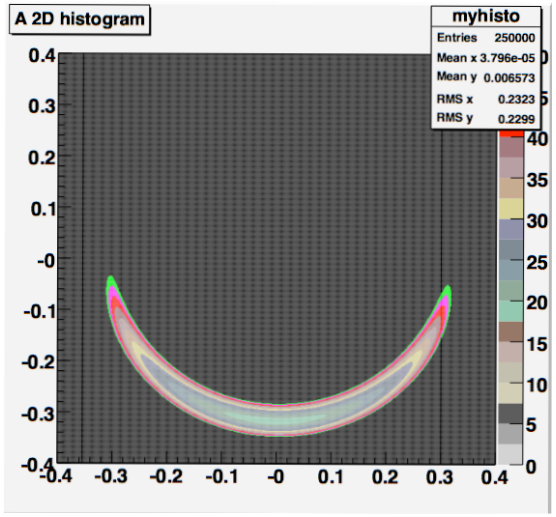
$$\chi^2_{eff}(\sin^2 2\theta_{13},\delta) = \left(\frac{P_\nu(\sin^2 2\theta_{13},\delta) - P_\nu(\sin^2 2\theta^{true}{}_{13},\delta^{true})}{\sigma_\nu}\right)^2 + \left(\frac{P_{\bar{\nu}}(\sin^2 2\theta_{13},\delta) - P_{\bar{\nu}}(\sin^2 2\theta^{true}{}_{13},\delta^{true})}{\sigma_{\bar{\nu}}}\right)^2$$

$$\text{assume } \sigma_\nu = \sigma_{\bar{\nu}} = 0.005$$

Input: normal hierarchy, $\sin^2 2\theta_{13}=0.1$, $\delta=\pi/4$, NOvA

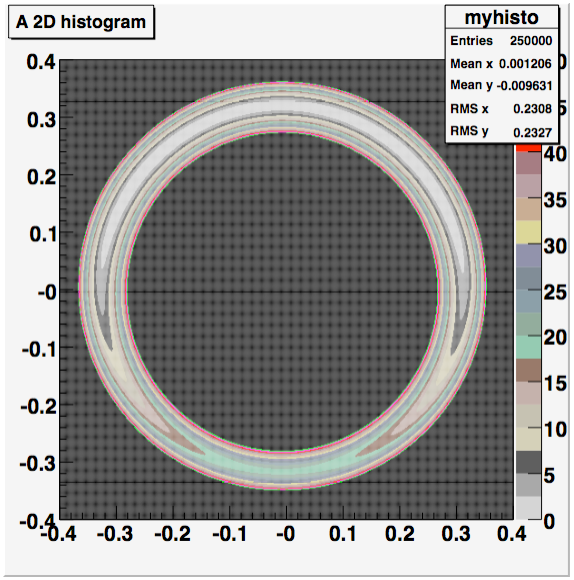


NOvA



normal

inverted



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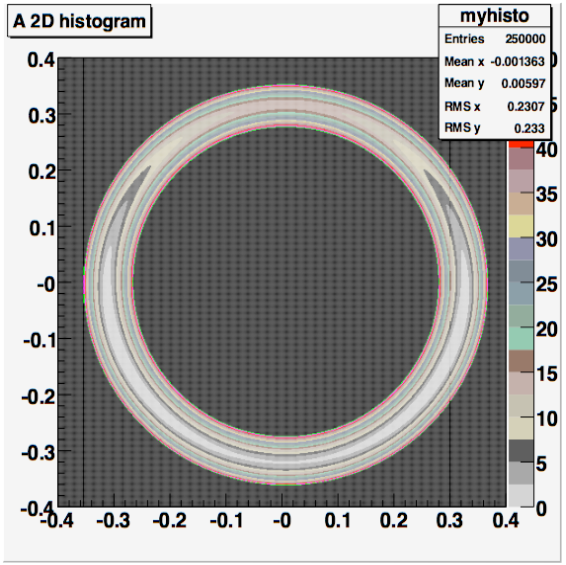

Beyond Monoenergetic Beams ?

- With detectors at first two maxima, get two circles each for neutrino and antineutrino.
- Broad-band beam could be accommodated by binning in energy.